A1 $d((x_1,y_1),(x_2,y_2)) = \max\{|x_2-x_1|, |y_2-y_1|\}$

1) Abstandsfunktion

(1) klar

(2) klar

(3) $d((x_1,y_1),(x_2,y_2)) = \max\{|x_2-x_1|, |y_2-y_1|\}$

$= \max\{|x_2-x_1+x_3-x_3|, |y_2-y_1+y_3-y_3|\}$

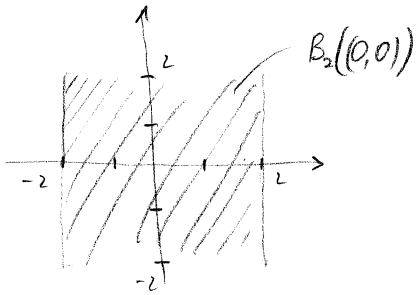
$= \max\{|(x_3-x_1)+(x_2-x_3)|, |(y_3-y_1)+(y_2-y_3)|\}$

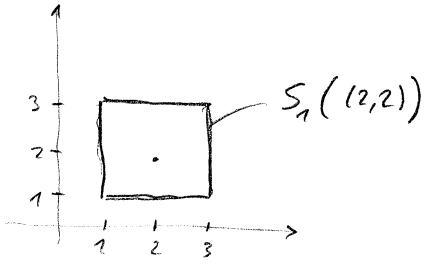
$\leq \max\{|x_3-x_1|+|x_2-x_3|, |y_3-y_1|+|y_2-y_3|\}$

$\leq \max\{|x_3-x_1|, |y_3-y_1|\} + \max\{|x_2-x_3|, |y_2-y_3|\}$

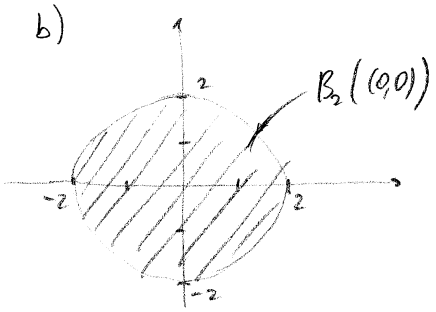
$= d((x_1,y_1),(x_3,y_3)) + d((x_3,y_3),(x_2,y_2))$ □

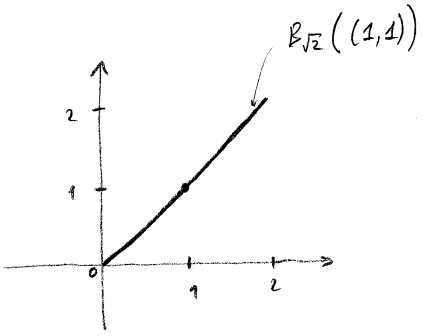
2)

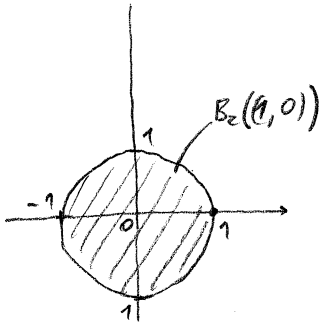




A2. b)







A3 $X = \{0,1\}^n = \mathbb{F}_2^n$

$d(v,w) = \#\{i \mid v_i \neq w_i\}$

a) Abstandsfunktion

(1) klar

(2) klar

(3) $d(v,w) = \#\{i \mid v_i \neq w_i\} \overset{\text{Hinweis}}{\leq} \#\{i \mid v_i \neq u_i \text{ oder } u_i \neq w_i\}$

$= \#\{i \mid v_i \neq u_i\} + \#\{i \mid u_i \neq w_i\} = d(v,u) + d(u,w)$ □

b) $S_1((0,0,0,0,0)) = \{(1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1)\}$

$S_2((0,0,0,0,0)) = \{(k_1,\dots,k_5) \mid \text{genau 2 Einsen}\}$

$B_2((0,0,0,0,0)) = \{(k_1,\dots,k_5) \mid \text{maximal 2 Einsen}\}$

$S_5((0,1,1,1,0)) = \{(1,0,0,0,1)\}$

A4 $X = C(0,1)$

a) $d(f,g) = \max_{x\in[0,1]} |g(x)-f(x)|$

(1) $d(f,g) = 0 \Leftrightarrow \max_{x\in[0,1]} |f(x)-g(x)| = 0 \Leftrightarrow f(x)=g(x) \ \forall x\in[0,1] \Leftrightarrow f=g$

(2) klar

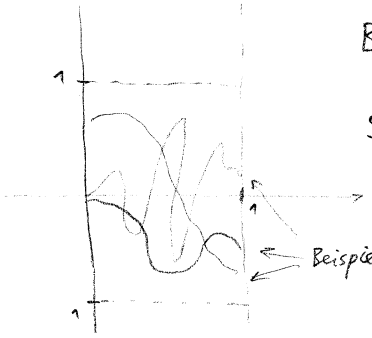
(3) $d(f,g) = \max_{x\in[0,1]} |g(x)-f(x)| = \max_{x\in[0,1]} |g(x)-f(x)+h(x)-h(x)|$

$= \max_{x\in[0,1]} |(h(x)-f(x)) + (g(x)-h(x))|$

$\leq \max_{x\in[0,1]} \left(|h(x)-f(x)| + |g(x)-h(x)| \right)$

$\leq \max_{x\in[0,1]} |h(x)-f(x)| + \max_{x\in[0,1]} |g(x)-h(x)|$

$= d(f,h) + d(h,g)$ □



$B_1(0) = \{ f \mid |f(x)| \leq 1 \ \forall x\in[0,1] \}$

$S_2(0) = \{ f \mid |f(x)| \leq 2 \ \forall x\in[0,1] \text{ und } \exists x\in[0,1] \text{ mit } |f(x)|=2 \}$

b) $d(f,g) = \int_0^1 |g(x)-f(x)|\,dx$

(1) klar

(2) klar

(3) $d(f,g) = \int_0^1 |g(x)-f(x)|\,dx = \int_0^1 |g(x)-f(x)+h(x)-h(x)|\,dx$

$\leq \int_0^1 \left(|h(x)-f(x)| + |g(x)-h(x)| \right) dx$

$= \int_0^1 |h(x)-f(x)|\,dx + \int_0^1 |g(x)-f(x)|\,dx$

$= d(f,h) + d(h,g)$ □

$B_1(0) = \{ f \mid \int_0^1 |f(x)|\,dx \leq 1 \} = \{ f \mid$ Fläche unter den Graphen von $|f|$ ist $\leq 1 \}$

$S_2(0) = \{ f \mid \int_0^1 |f(x)|\,dx = 2 \} = \{ f \mid$ Fläche unter den Graphen von $|f|$ ist $2 \}$