

Antworten zu den Beispielaufgaben zum Thema **Fourierreihen I**

Aufgabe 9.1 Bestimmen Sie die Fourierreihenentwicklung der Funktion $f(x)$.

a) $f(x) = x, -\pi < x < \pi.$

$$f(x) \sim 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx.$$

b) $f(x) = \begin{cases} 1, & 0 \leq x < \pi, \\ 0, & -\pi < x < 0. \end{cases}$

$$f(x) \sim \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}.$$

c) $f(x) = \begin{cases} \frac{\pi}{4}, & 0 \leq x < \pi, \\ -\frac{\pi}{4}, & -\pi < x < 0. \end{cases}$

$$f(x) \sim \sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{2n+1}.$$

d) $f(x) = \pi + x, -\pi < x < \pi.$

$$f(x) \sim \pi + 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx.$$

e) $f(x) = |x|, -\pi < x < \pi.$

$$f(x) \sim \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^2}.$$

f) $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x, & 0 \leq x < \pi. \end{cases}$

$$f(x) \sim \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\pi n^2} \cos nx + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}.$$

g) $f(x) = \begin{cases} 1, & 0 < x < \pi, \\ 0, & x = 0, \\ -1, & -\pi < x < 0. \end{cases}$

$$f(x) \sim \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}.$$

Aufgabe 9.2 Bestimmen Sie die Fourierreihenentwicklung der Funktion $f(x)$.

a) $f(x) = \pi^2 - x^2, -\pi < x < \pi.$

$$f(x) \sim \frac{2}{3}\pi^2 + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx.$$

b) $f(x) = x^3, -\pi < x < \pi.$

$$f(x) \sim \sum_{n=1}^{\infty} (-1)^n \left(\frac{12}{n^3} - \frac{2\pi^2}{n} \right) \sin nx.$$

c) $f(x) = x \sin x, -\pi < x < \pi.$

$$f(x) \sim 1 - \frac{1}{2} \cos x + 2 \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^2 - 1} \cos nx.$$

d) $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ \sin x, & 0 \leq x < \pi. \end{cases}$

$$f(x) \sim \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}.$$