Homework 1

11 May 2017

Please hand in your solutions before the start of next lecture, 18 May 2017 at 12:15.

(1) Give an example of a surjective morphism of sheaves $\varphi\colon \mathcal{F} \to \mathcal{G}$ and an open subset U such that $\varphi(U)\colon \mathcal{F}(U) \to \mathcal{G}(U)$ is not surjective.

(2) Let A be a ring. Show that the following conditions are equivalent:

(i) $\operatorname{Spec} A$ is disconnected;
(ii) there exist nonzero elements $e_1, e_2 \in A$ such that $e_1 e_2 = 0$, $e_1^2 = e_1$, $e_2^2 = e_2$, and $e_1 + e_2 = 1$ (these elements are called *orthogonal idempotents*);
(iii) A is isomorphic to a direct product $A_1 \times A_2$ of two nonzero rings.

(3) Let $\mathbb{Z}$ denote the ring of integers. Describe $\operatorname{Spec} \mathbb{Z}[X]$.

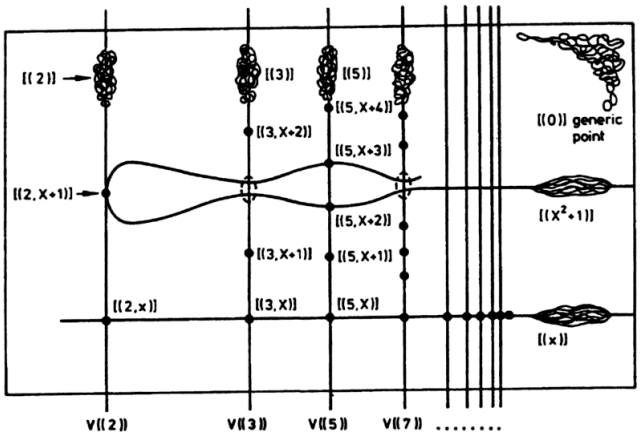

Figure 1: May this picture inspire you.

(4) Let S be a graded ring. Show that $\operatorname{Proj} S = \emptyset$ if and only if every element of S_+ is nilpotent.