Mössbauer neutrinos

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in collaboration with E. Kh. Akhmedov and M. Lindner
based on JHEP **0805** (2008) 005 (arXiv:0802.2513),
arXiv:0803.1424, and work in progress

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Outline

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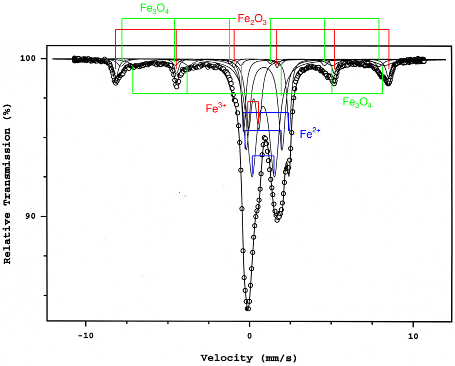

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A similar effect should exist for neutrino emission/absorption in bound state β decay and induced electron capture processes.

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Proposed experiment:

$$\begin{aligned} \text{Production:} \quad & {}^3\text{H} && \rightarrow {}^3\text{He}^+ + \bar{\nu}_e + e^-(\text{bound}) \\ \text{Detection:} \quad & {}^3\text{He}^+ + e^-(\text{bound}) + \bar{\nu}_e && \rightarrow {}^3\text{H} \end{aligned}$$

^{3}H and ^{3}He are embedded in metal crystals (metal hydrides).

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Physics goals:

- Neutrino oscillations on a laboratory scale: $E = 18.6$ keV, $L_{\text{atm}}^{\text{osc}} \sim 20$ m.
- Gravitational interactions of neutrinos
- Study of solid state effects with unprecedented precision

Mössbauer neutrinos (2)

Mössbauer neutrinos have very special properties:

- Neutrino receives *full* decay energy: $Q = 18.6$ keV
- Natural line width: $\gamma \sim 1.17 \times 10^{-24}$ eV
- Atucal line width: $\gamma \gtrsim 10^{-11}$ eV
  - Inhomogeneous broadening (Impurities, lattice defects)
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Experimental challenges:

- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
- Can a linewidth $\gamma \gtrsim 10^{-11}$ eV be achieved?
- Can the resonance condition be fulfilled?

Mössbauer neutrinos (3)

Recent controversy:

- Does the small energy uncertainty prohibit oscillations of Mössbauer neutrinos?
- Do oscillating neutrinos need to have equal energies resp. equal momenta?

S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G34** (2007) 987, hep-ph/0611285

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$\Rightarrow$ Careful treatment with as few assumptions as possible is needed

$\Rightarrow$ Answer to the above questions will be No.

Outline

Textbook derivation of the oscillation formula

Diagonalization of the mass terms of the charged leptons and neutrinos gives

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} \left(\bar{e}_{\alpha L} \gamma^\mu U_{\alpha j} \nu_{jL} \right) W^-_\mu \; + \; \text{diag. mass terms} \; + \; h.c.$$

(flavour eigenstates: $\alpha = e, \mu, \tau$, mass eigenstates: $j = 1, 2, 3$)

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Assume, at time $t = 0$ and location $\vec{x} = 0$, a flavour eigenstate

$$|\nu(0,0)\rangle = |\nu_\alpha\rangle = \sum_i U^*_{\alpha j} |\nu_j\rangle$$

is produced. At time t and position $\vec{x}$, it has evolved into

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Oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \Big| \left\langle \nu_\beta | \nu(t,\vec{x}) \right\rangle \Big|^2 = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i(E_j - E_k)t + i(\vec{p}_j - \vec{p}_k)\vec{x}}$$

Equal energies or equal momenta?

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These are *assumptions* or *approximations*, not fundamental principles!

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- In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.

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Energy-momentum conservation for emission of mass eigenstate $|\nu_i\rangle$:

$$E_j^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_j^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^4}{4m_\pi^2}$$

$$p_j^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_j^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^4}{4m_\pi^2}$$

For massless neutrinos: $E_j = p_j = E \equiv \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\ \mathrm{MeV}$.

To first order in m_i^2:

$$E_j \simeq E + \xi\frac{m_j^2}{2E}\,, \qquad p_j \simeq E - (1-\xi)\frac{m_j^2}{2E}\,, \qquad \xi \approx \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

Oscillation probability for Mössbauer neutrinos

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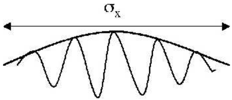
Beuthe, Giunti, Grimus, Kiers, Kim, Lee, Mohanty, Nussinov, Stockinger, Weiss, ...

Conditions for oscillations in a wave packet approach



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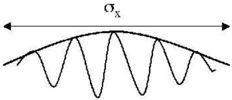

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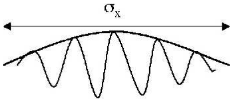

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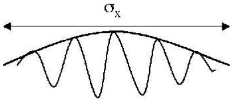

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$$\sigma_m^2 = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2} > \Delta m^2$$

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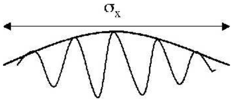

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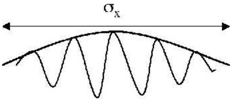
This is easily fulfilled for Mössbauer neutrinos, since

$$\sigma_E \sim 10^{-11} \text{ eV}$$
$$\sigma_p = 1/2\sigma_x \sim 1/\text{interatomic distance} \sim 10 \text{ keV}$$
$$E = p = 18.6 \text{ keV}$$

Conditions for oscillations in a wave packet approach

- Coherence in production and detection processes
- Coherence maintained during propagation



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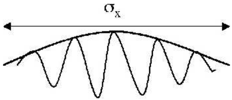

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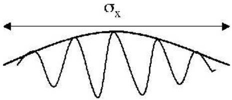

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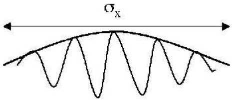

Decoherence could be caused by wave packet separation

It can be shown that, for Mössbauer neutrinos, σ_p is small enough, so that

$$L^{\text{osc}} \ll L^{\text{coh}}.$$

$\Rightarrow$ Stanard oscillation formula is approximately recovered:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\Big[-2\pi i \frac{L}{L^{\text{osc}}_{jk}} \Big]$$

$$L^{\text{osc}}_{jk} = \frac{4\pi E}{\Delta m^2_{jk}}$$

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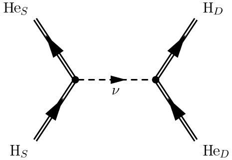

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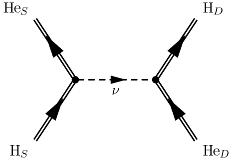

External particles reside in harmonic oscillator potentials.
E.g. for ^{3}H atoms in the source:

$$\psi_{H,S}(\vec{x}, t) = \left[\frac{m_H \omega_{H,S}}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_H \omega_{H,S} |\vec{x} - \vec{x}_S|^2\right] \cdot e^{-iE_{H,S} t}$$

Oscillation amplitude

$$i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1}$$
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$$\cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu *} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2 - t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)}$$
$$\cdot \bar{u}_{e,S}\gamma_\mu(1-\gamma^5)\, \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon}\, (1+\gamma^5)\gamma_\nu u_{e,D}.$$

Oscillation amplitude

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1} \\
\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_{\rm He}\omega_{{\rm He},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{+iE_{{\rm He},S}t_1} \\
\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_{\rm He}\omega_{{\rm He},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{-iE_{{\rm He},D}t_2} \\
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\end{aligned}
$$

Evaluation:

- $dt_1\, dt_2$-integrals $\rightarrow$ energy-conserving δ functions $\rightarrow$ p_0-integral trivial
- $d^3x_1\, d^3x_2$-integrals are Gaussian
- d^3p-integral: Use Grimus-Stockinger theorem (for large $L = |\vec{x}_D - \vec{x}_S|$).

W. Grimus, P. Stockinger, Phys. Rev. **D54** (1996) 3414, hep-ph/9603430

The Grimus-Stockinger theorem

Let $\psi(\vec{p})$ be a three times continuously differentiable function on $\mathbb{R}^3$, such that ψ itself and all its first and second derivatives decrease at least like $1/|\vec{p}|^2$ for $|\vec{p}| \to \infty$. Then, for any real number $A > 0$,

$$\int d^3p \, \frac{\psi(\vec{p}) \, e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}| \to \infty} -\frac{2\pi^2}{L} \psi(\sqrt{A}\tfrac{\vec{L}}{L}) e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}}).$$

$\Rightarrow$ Quantification of requirement of on-shellness for large $L = |\vec{L}|$.

From the amplitude to the transition rate

Amplitude:

$$i\mathcal{A} = \frac{-i}{2L}\mathcal{N}\,\delta(E_S - E_D)\,\exp\left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2}\right]\sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*}|U_{ej}|^2\, e^{i\sqrt{E_S^2 - m_j^2}L}$$
$$\cdot\, \bar{u}_{e,S}\gamma_\mu \tfrac{1-\gamma^5}{2}(\not{p}_j + m_j)\tfrac{1+\gamma^5}{2}\gamma_\nu u_{e,D},$$

$$\sigma_p^{-2} = (m_{\mathrm{H}}\omega_{\mathrm{H},S} + m_{\mathrm{He}}\omega_{\mathrm{He},S})^{-1} + (m_{\mathrm{H}}\omega_{\mathrm{H},D} + m_{\mathrm{He}}\omega_{\mathrm{He},D})^{-1}$$

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Transition rate: Integrate $|\mathcal{A}|^2$ over densities of initial and final states

$$\Gamma \propto \int_0^\infty dE_{\mathrm{H},S}\, dE_{\mathrm{He},S}\, dE_{\mathrm{He},D}\, dE_{\mathrm{H},D}$$
$$\cdot\,\delta(E_S - E_D)\rho_{\mathrm{H},S}(E_{\mathrm{H},S})\,\rho_{\mathrm{He},D}(E_{\mathrm{He},D})\,\rho_{\mathrm{He},S}(E_{\mathrm{He},S})\,\rho_{\mathrm{H},D}(E_{\mathrm{H},D})$$
$$\cdot\sum_{j,k}|U_{ej}|^2|U_{ek}|^2\,\underbrace{\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right]}_{\text{Analogue of Lamb-Mössbauer factor (Recoil-free fraction)}}\;\underbrace{e^{i\left(\sqrt{E_S^2-m_j^2}-\sqrt{E_S^2-m_k^2}\right)L}}_{\text{Oscillation phase}}$$

The Lamb-Mössbauer factor

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Convenient reformulation:

$$\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right] = \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

where $(p_{jk}^{\min})^2 = E_S^2 - \max(m_j^2, m_k^2)$.

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$\Rightarrow$ Localization condition

$$4\pi\sigma_x E/\sigma_p \lesssim L_{jk}^{\rm osc},$$

(with $\sigma_x = 1/2\sigma_p$) is satisfied if $L_{jk}^{\rm osc} \gtrsim 2\pi\sigma_x$, which is easily fulfilled in realistics situations.

Outline

Inhomogeneous line broadening

Energy levels of ^{3}H and ^{3}He in the source and detector are smeared e.g. due to crystal impurities, lattice defects, etc.

R. S. Raghavan, hep-ph/0601079
W. Potzel, Phys. Scripta **T127** (2006) 85
B. Balko, I. W. Kay, J. Nicoll, J. D. Silk, G. Herling, Hyperfine Int. **107** (1997) 283

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Good approximation:

$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

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Result for two neutrino flavours ($m_2 > m_1$):

$$\Gamma \propto \exp\left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m^2|}{2\sigma_p^2}\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}} \cdot \left\{1 - 2s^2c^2\left[1 - \frac{1}{2}(e^{-L/L_S^{\text{coh}}} + e^{-L/L_D^{\text{coh}}})\cos\left(\pi\frac{L}{L^{\text{osc}}}\right)\right]\right\}$$

$$L_{S,D}^{\text{coh}} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}$$

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$$L_{S,D}^{\mathrm{coh}} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}$$

In realistic cases: $L_{S,D}^{\mathrm{coh}} \gg L^{\mathrm{osc}}$ $\Rightarrow$ Decoherence is not an issue.

Outline

Homogeneous line broadening

- Fluctuating electromagnetic fields in solid state crystal
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  - Experimentally indistinguishable
  - We expect a result similar to that for the case of inhomogeneous broadening
- Ansatz: Introduce modulation factors of the form

$$f_{A,B}(t) = \exp\left[-i \int_0^t dt' \left[E_{A,B}(t') - E_{A,B,0}\right] t' \right]$$

in the ^{3}H and ^{3}He wave functions ($A = \mathrm{H}, \mathrm{He}$, $B = S, D$).

J. Odeurs, Phys. Rev. **B52** (1995) 6166

Transition amplitude for homogeneous line broadening

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 & \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1} \\
& \cdot \left(\frac{m_{\rm He}\omega_{{\rm He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm He}\omega_{{\rm He},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{+iE_{{\rm He},S}t_1} \\
& \cdot \left(\frac{m_{\rm He}\omega_{{\rm He},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm He}\omega_{{\rm He},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{-iE_{{\rm He},D}t_2} \\
& \cdot \left(\frac{m_{\rm H}\omega_{{\rm H},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm H}\omega_{{\rm H},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{+iE_{{\rm H},D}t_2} \\
& \cdot \sum_j \mathcal{M}_S^{\mu}\mathcal{M}_D^{\nu *}|U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} \exp\left[-ip_0(t_2 - t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)\right] \\
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Evaluation:

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Transition rate $\Gamma \propto \langle \mathcal{A}\mathcal{A}^* \rangle$
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$\Rightarrow B_S(t_1, \tilde{t}_1) = \exp\Big[-\tfrac{1}{2}\gamma_S |t_1 - \tilde{t}_1| \Big]$.

Transition rate for homogeneous line broadening (2)

Result:

$$\Gamma \propto \exp\left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m^2|}{2\sigma_p^2}\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}}$$
$$\cdot \left\{1 - 2s^2c^2\left[1 - \frac{1}{2}(e^{-L/L_S^{\rm coh}} + e^{-L/L_D^{\rm coh}})\cos\left(\pi\frac{L}{L^{\rm osc}}\right)\right]\right\}$$

...identical to the result for inhomogeneous line broadening.

Outline

Amplitude for broadening by natural line width

Take into account the instability of ^{3}H in the source and the detector.

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} &\exp\left[-\tfrac{1}{2} m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1} \\
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\cdot \bar{u}_{e,S}\gamma_\mu \tfrac{1-\gamma^5}{2} &\frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \tfrac{1+\gamma^5}{2}\gamma_\nu u_{e,D}
\end{aligned}
$$

(correctness of this formula can be verified in the Wigner-Weisskopf approach)

E. Akhmedov, J. Kopp, M. Lindner, JHEP **0805** (2008) 005 (arXiv:0802.2513)

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\cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu *} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2 - t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \\
\cdot \bar{u}_{e,S}\gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2}\gamma_\nu u_{e,D}
\end{aligned}
$$

(correctness of this formula can be verified in the Wigner-Weisskopf approach)

E. Akhmedov, J. Kopp, M. Lindner, JHEP **0805** (2008) 005 (arXiv:0802.2513)

Probability for broadening by natural line width

$$\mathcal{P} \propto \sum_{j,k} \theta(T_{jk})\,|U_{ej}|^2 |U_{ek}|^2$$
$$\cdot \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right] e^{i\left(\sqrt{\bar{E}^2 - m_j^2} - \sqrt{\bar{E}^2 - m_k^2}\right)L}$$
$$\cdot\, e^{-\gamma T_{jk}}\, e^{-L/L_{jk}^{\mathrm{coh}}}\, \frac{\sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j})\right] \sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k})\right]}{(E_S - E_D)^2}$$

where $T_{jk} = \min\left(T - \frac{L}{v_j},\, T - \frac{L}{v_k}\right)$ and $L_{jk}^{\mathrm{coh}} = \frac{4\bar{E}^2}{\gamma |\Delta m_{jk}^2|}$

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- Lamb-Mössbauer factor: $\exp\left[-(p_{jk}^{\min})^2/\sigma_p^2\right]$
- Localization term: $\exp\left[-|\Delta m_{jk}^2|/2\sigma_p^2\right]$
- Coherence term: $e^{-L/L_{jk}^{\mathrm{coh}}}$

Probability for broadening by natural line width (2)

- Resonance term

$$\frac{\sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j})\right] \sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k})\right]}{(E_S - E_D)^2}$$

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- Analogy: subnatural spectroscopy in quantum optics
  - Atom is excited instantaneously to state $|b\rangle$.
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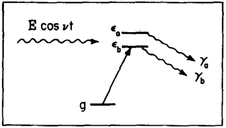

P. Meystre, O. Scully, H. Walther, Optics Communications **33** (1980) 153

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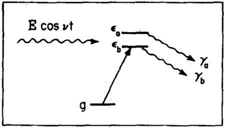

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- Here:
  - $|b\rangle \Leftrightarrow$ ^{3}H atom in the source, ^{3}He atom in the detector
  - $|a\rangle \Leftrightarrow$ ^{3}He atom in the source, ^{3}H atom in the detector
  - Excitation of $|b\rangle \Leftrightarrow$ Production of source
  - Transition $|b\rangle \rightarrow |a\rangle \Leftrightarrow$ neutrino production, propagation and absorption

Probability for broadening by natural line width (3)

- Time dependence for $E_S = E_D$: $T^2\, e^{-\gamma T}/4$.

  Classical argument for this behaviour:

  - Numbers of ^{3}H nuclei in the detector, N_D, and in the source, N_S, obey

    $$\dot{N}_D = -\dot{N}_S N_0 P_{ee} \frac{\sigma(T)}{4\pi L^2} - \gamma N_D\,,$$

    where N_0 is the number of ^{3}He atoms in the detector.
  - Absorption cross section $\sigma(T) \simeq s_0\, T$, because Heisenberg principle requires resonance condition to be fulfilled to an accuracy $\sim T^{-1}$. For Lorentzian emission an absorption lines, the overlap integral is then proportional to T.
  - Using $N_S = N_{S,0} \exp(-\gamma T)$, the solution to the above equation is

    $$N_D = \frac{N_{S,0} N_0 \gamma P_{ee} s_0}{8\pi L^2}\, T^2 e^{-\gamma T}.$$

Outline

The time-energy uncertainty relation

Mandelstam-Tamm relation:

$$\Delta E \Delta O \geq \frac{1}{2} \left| \frac{d}{dt} \overline{O}(t) \right| .$$

Here, $\overline{O}(t) = \langle \psi(t) | O | \psi(t) \rangle$, for any operator O and QFT Fock state $|\psi(t)\rangle$.

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Choose $O \equiv |\nu_\alpha\rangle\langle\nu_\alpha|$ (projection in 3d flavour space) and $\psi(t)$ a neutrino state

$$\Delta E \geq \frac{1}{2}\frac{|\frac{d}{dt}P(t)|}{\sqrt{P(t) - P^2(t)}},$$

with $P(t) = |\langle\nu_\alpha|\psi(t)\rangle|^2$.

S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G35** (2008) 095003 (arXiv:0803.0527)

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Problem: Interpretation of $P(t)$.

- Would be the oscillation probability for a completely delocalized detector.
- Imagine wave packet which is large compared to L^{osc}: Delocalized detector averages out oscillations.
- More realistic: Projection in flavour space and in coordinate space:

$$O_{\vec{x}} \equiv |\nu_\alpha\rangle |\vec{x}\rangle\langle\vec{x}| \langle\nu_\alpha|$$

The time-energy uncertainty relation (2)

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$$\Rightarrow \Delta E \geq |E_1 - E_2|.$$

"Energy difference of mass eigenstates must be smaller than energy uncertainty."

Easily fulfilled for Mössbauer neutrinos due to large *momentum* uncertainty.

Outline

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- The time energy uncertainty relation does not inhibit oscillations of Mössbauer neutrinos.

Thank you!

Oscillation formula for neutrino wave packets

Assume Gaussian wave packets:

$$|\nu_\alpha(x,t)\rangle = \frac{1}{(2\pi\sigma_{pS}^2)^{1/4}} \sum_j U_{\alpha j}^* \int \frac{dp}{\sqrt{2\pi}}\, e^{-(p-p_{jS})^2/4\sigma_{pS}^2}\, e^{-i\sqrt{p^2+m_j^2}\,t+ipx} |\nu_j\rangle$$

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Oscillation formula:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-2\pi i \frac{L}{L_{jk}^{\rm osc}} - \left(\frac{L}{L_{jk}^{\rm coh}}\right)^2 - 2\pi^2\xi^2\left(\frac{1}{2\sigma_p L_{jk}^{\rm osc}}\right)^2\right]$$

C. Giunti, C. W. Kim, U. W. Lee, Phys. Rev. **D44** (1991) 3635; C. Giunti, C. W. Kim, Phys. Lett. **B274** (1992) 87
K. Kiers, S. Nussinov, N. Weiss, Phys. Rev. **D53** (1996) 537, hep-ph/9506271
C. Giunti, C. W. Kim, Phys. Rev. **D58** (1998) 017301, hep-ph/9711363, C. Giunti, Found. Phys. Lett. **17** (2004) 103, hep-ph/0302026

with

$L_{jk}^{\rm osc} = 4\pi E/\Delta m_{jk}^2$	Oscillation length
$L_{jk}^{\rm coh} = 2\sqrt{2}E^2/\sigma_p|\Delta m_{jk}^2|$	Coherence length
E	Energy that a massless neutrino would have
ξ	quantifies the deviation from E (tiny for Mössbauer neutrinos)
σ_p	Effective wave packet width (tiny for Mössbauer neutrinos)

Results from the wave packet treatment

- Decoherence term

  $$\exp\left[-\frac{L}{L_{jk}^{\mathrm{coh}}}\right]$$

  cannot inhibit oscillations because

  $$L_{jk}^{\mathrm{coh}}/L_{jk}^{\mathrm{osc}} \sim E/\sigma_p \sim 10^{15}.$$

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$\Rightarrow$ Our expectation is confirmed:

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i\frac{\Delta m_{jk}^2 L}{2E}}.$$

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We are looking for a formalism, in which these quantities are automatically determined from the properties of the source and the detector.