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The electric and magnetic disordered Maxwell equations as eigenvalue problem

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We consider Maxwell's equations in a 3-dimensional material, in which both, the electric permittivity, as well as the magnetic permeability, fluctuate in space. Differently from all previous treatments of the *disordered* electromagnetic problem, we transform Maxwell's equations and the electric and magnetic fields in such a way that the linear operator in the resulting secular equations is manifestly Hermitian, in order to deal with a proper eigenvalue problem. As an application of our general formalism, we use an appropriate version of the Coherent-Potential approximation (CPA) to calculate the photon density of states and scattering-mean-free path. Applying standard localization theory, we find that in the presence of both electric and magnetic disorder the spectral range of Anderson localization appears to be much larger than in the case of electric (or magnetic) disorder only. Our result could explain the absence of experimental evidence of 3D Anderson localization of light (all the existing experiments has been performed with electric disorder only) and pave the way towards a successful search of this, up to now, elusive phenomenon.

INTRODUCTION

Understanding the propagation and scattering of electromagnetic radiation in random media, especially visible light, is an issue, which is important in different parts of science [1–7]. A particularly interesting feature of waves in a disordered environment is the possibility of localization, i.e. the absence of diffusion, demonstrated first for electron wave functions by Anderson [8]. Anderson localization (AL) arises from the interference of the waves scattered by the random inhomogeneities of the medium [6, 9–12]. This phenomenon occurs with all kinds of waves, including atomic-matter and gravitational waves [13–15].

Localization of classical waves has first been discussed by John et al. [16, 17] for acoustical and later for electromagnetic waves (light) [18, 19]. The successful observation of weak localization of light (the back-scattering cone) [20] created an impact for looking for strong AL of light [21–25]. It was realized [26, 27] that the chances for the observation of this phenomenon are much higher in dimensionally reduced systems. This has been successfully demonstrated in paraxial structures with transverse (2-dimensional) disorder [28, 29] and two-dimensional photonic crystals [30]. In 3-dimensional media with a spatially fluctuating permittivity, however, until now, AL has not been found [22, 31-35]. Indeed, 3D localization effects are often obscured by absorption or fluorescence processes, making its experimental demonstration extremely elusive [35]. Recently, the possibility of obtaining Anderson localization in 3D systems has been made plausible in numerical simulations of (i) hyperuniform amorphous photonic materials [36, 37], and (ii) systems with overlapping spherical, perfectly conducting obstacles [38].

On the other hand, the theoretical description of AL of light is, until now, built on the ground of a mathematically questionable mapping of Maxwell's equations to Anderson's Schrödinger equation of an electron in a random potential [18, 19]. This mapping, which was taken over by the subsequent literature [6, 27, 39–41], started with the Helmholtz equation for a stationary frequency-dependent electric field $\mathbf{E}(\mathbf{r}, \omega)$ in the presence of a spatially fluctuating permittivity $\epsilon(\mathbf{r})$, derived from Maxwell's equations¹

$$\omega^{2} \epsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}, \omega) = \frac{1}{\mu_{0}} \nabla \times \left[\nabla \times \mathbf{E}(\mathbf{r}, \omega) \right], \qquad (1)$$

This equation was transformed in the following way: (i) the double curl was converted to $-\nabla^2$, ignoring that $\nabla \cdot \mathbf{E} \neq 0$ for $\nabla \epsilon \neq 0$, (ii) the coefficient of \mathbf{E} on the LHS, which features the spectral parameter ω^2 of the eigenvalue equation, was rewritten as $\omega^2 \epsilon_0 + \omega^2 [\epsilon(\mathbf{r}) - \epsilon_0]$, where the second term was re-interpreted as an ω dependent potential (changing completely the physical content), and (*iii*) the eigenvalue problem associated with Eq. (1) was neither formulated nor solved properly (see below).

In the case of transverse localization this truncated and ill-posed equation (called "potential-type approach" by Schirmacher *et al.* [42]) produced results, which were at variance with experiment: A wavelength dependence of the localization length, predicted on base of this equation [27, 43], was not observed experimentally and is not

¹ Here $\mu_0 = 1/\epsilon_0 c_0^2$ is the magnetic permeability of the vacuum, ϵ_0 is the electric permittivity of the vacuum and c_0 is the vacuum light velocity.

predicted by a proper treatment [42]. In view of all these inconsistencies it is apparent that a mean-field theory, based on a consistently formulated Hermitian eigenvalue problem of Maxwell theory in the presence of disorder, is urgently called for.

Here, we present such a mean-field theory of disorder, based on a properly formulated eigenvalue problem. In this theory we allow both for electric (spatially varying electric permittivity $\epsilon(\mathbf{r})$) and magnetic disorder (spatially varying magnetic permeability $\mu(\mathbf{r})$). The theory is an appropriate version of the coherent-potential approximation (CPA), derived by S. Köhler and two of the present authors for elastic waves in the presence of disorder [44].

Applying the CPA results for the scattering mean-free path and the density of states to standard localization theory suggests that by combining electric and magnetic disorder the chances for observing AL of light in three dimensions are greatly enhanced with respect to the case where only one quantity ($\epsilon(\mathbf{r}) \ or \ \mu(\mathbf{r})$) is left to vary.

We start by defining dimensionless electric and magnetic moduli $M_{\epsilon}(\mathbf{r}) := \epsilon_0/\epsilon(\mathbf{r})$ and $M_{\mu}(\mathbf{r}) := \mu_0/\mu(\mathbf{r})$. The generalization of (1) for including magnetic disorder takes the form²

$$\frac{\omega^2}{c_0^2} \mathbf{E}(\mathbf{r}, \omega) = M_{\epsilon}(\mathbf{r}) \nabla \times [M_{\mu}(\mathbf{r}) \nabla \times \mathbf{E}(\mathbf{r}, \omega)]$$
$$=: \mathcal{L}_{\mathbf{E}} \mathbf{E}(\mathbf{r}, \omega).$$
(2)

The operator $\mathcal{L}_{\mathbf{E}}$ on the RHS of this equation is not Hermitian, if the ("naive") definition of the scalar product $\langle \mathbf{E}_1 | \mathbf{E}_2 \rangle = \int d^3 \mathbf{r} \mathbf{E}_1^*(\mathbf{r}) \cdot \mathbf{E}_2(\mathbf{r})$ is used. Only if we define [46]

$$<\mathbf{E}_1|\mathbf{E}_2>:=\int d^3\mathbf{r} M_{\epsilon}^{-1}(\mathbf{r})\mathbf{E}_1^*(\mathbf{r})\cdot\mathbf{E}_2(\mathbf{r})\,,\qquad(3)$$

the operator $\mathcal{L}_{\mathbf{E}}$ has the Hermitian property:

$$<\mathbf{E}_{1}|\mathcal{L}_{\mathbf{E}}\mathbf{E}_{2}>=\int d^{3}\mathbf{r}\,\mathbf{E}_{1}^{*}(\mathbf{r})\cdot\left[\nabla\times M_{\mu}(\mathbf{r})[\nabla\times\mathbf{E}_{2}(\mathbf{r})]\right]$$
$$=\int d^{3}\mathbf{r}\,M_{\mu}(\mathbf{r})\left[\nabla\times\mathbf{E}_{1}^{*}(\mathbf{r})\right]\cdot\left[\nabla\times\mathbf{E}_{2}(\mathbf{r})\right]$$
$$=\int d^{3}\mathbf{r}\,\mathbf{E}_{2}(\mathbf{r})\cdot\left[\nabla\times M_{\mu}(\mathbf{r})[\nabla\times\mathbf{E}_{1}^{*}(\mathbf{r})]\right]$$
$$\stackrel{!}{=}<\mathbf{E}_{2}|\mathcal{L}_{\mathbf{E}}\mathbf{E}_{1}>^{*}.$$
(4)

The second line guarantees the positiveness of the spectrum. It is easily verified that for the scalar product without the fluctuating permittivity included, $\mathcal{L}_{\mathbf{E}}$ is not Hermitian, because extra terms involving ∇M_{ϵ} are obtained.

Similarly an equation for the magnetic field can be derived from Maxwell's equations

$$\frac{\omega^2}{c_0^2} \mathbf{H}(\mathbf{r},\omega) = M_{\mu}(\mathbf{r}) \nabla \times M_{\epsilon}(\mathbf{r}) [\nabla \times \mathbf{H}(\mathbf{r},\omega)]$$

=: $\mathcal{L}_{\mathbf{H}} \mathbf{H}(\mathbf{r},\omega)$. (5)

Here, the operator $\mathcal{L}_{\mathbf{H}}$ is Hermitian, if the scalar product includes a factor $M_{\mu}^{-1}(\mathbf{r})$. In the case of pure electric disorder ($M_{\mu} = const$) no special definition of the scalar product is needed. This (properly defined) eigenvalue equation for electric disorder was used recently for treating transverse two-dimensional AL [42].

It is remarkable [45] that for $\omega \neq 0$ Eqs. (2) and (5) automatically guarantee the transversality conditions

$$\nabla \cdot \left[\mathbf{E}(\mathbf{r}, \omega) / M_{\epsilon}(\mathbf{r}) \right] = 0; \quad \nabla \cdot \left[\mathbf{H}(\mathbf{r}, \omega) / M_{\mu}(\mathbf{r}) \right] = 0.$$
(6)

In order to formulate an analytic theory for the disorder-averaged physical quantities in a system described by (2) and (5) it is rather disadvantageous to work with the disorder dependent scalar product. This can be avoided using symmetrized fields [45, 47, 48] $\widetilde{\mathbf{E}} := \mathbf{E}/\sqrt{M_{\epsilon}(\mathbf{r})}$ and $\widetilde{\mathbf{H}} := \mathbf{H}/\sqrt{M_{\mu}(\mathbf{r})}$ which obey the symmetrized Helmholtz equations

$$\frac{\omega^2}{c_0^2} \widetilde{\mathbf{E}}(\mathbf{r},\omega) = M_{\epsilon}^{1/2}(\mathbf{r}) \nabla \times M_{\mu}(\mathbf{r}) [\nabla \times M_{\epsilon}^{1/2}(\mathbf{r}) \widetilde{\mathbf{E}}(\mathbf{r},\omega)]$$
$$=: \mathcal{L}_{\widetilde{\mathbf{E}}} \widetilde{\mathbf{E}}(\mathbf{r},\omega) , \qquad (7)$$

$$\frac{\omega^2}{c_0^2} \widetilde{\mathbf{H}}(\mathbf{r},\omega) = M_{\mu}^{1/2}(\mathbf{r}) \nabla \times M_{\epsilon}(\mathbf{r}) [\nabla \times M_{\mu}^{1/2}(\mathbf{r}) \widetilde{\mathbf{H}}(\mathbf{r},\omega)]$$
$$=: \mathcal{L}_{\widetilde{\mathbf{H}}} \widetilde{\mathbf{H}}(\mathbf{r},\omega).$$
(8)

Eqs. (7) and (8) now constitute conventional eigenvalue equations with operators $\mathcal{L}_{\widetilde{\mathbf{E}}}, \mathcal{L}_{\widetilde{\mathbf{H}}}$ that are Hermitian with respect to the scalar products $\langle \widetilde{\mathbf{E}}_1 | \widetilde{\mathbf{E}}_2 \rangle = \int d^3 \mathbf{r} \widetilde{\mathbf{E}}_1^*(\mathbf{r}) \cdot \widetilde{\mathbf{E}}_2(\mathbf{r})$ and $\langle \widetilde{\mathbf{H}}_1 | \widetilde{\mathbf{H}}_2 \rangle = \int d^3 \mathbf{r} \widetilde{\mathbf{H}}_1^*(\mathbf{r}) \cdot \widetilde{\mathbf{H}}_2(\mathbf{r})$.

In this transformed way the differential operators are manifestly Hermitian with respect to the conventional definition of the scalar product. In this form the eigenvalue problem can be dealt with in the usual way, using functional integrals and replica theory [44, 49].

Generalizing the derivation of Köhler *et al.* [44] we establish a coherent-potential approximation (CPA), based on Eqs. (7), (8), along the lines of our peviou s work on elasticity.

The CPA arises as a saddle-point equation of an effective field theory, constructed by field-theoretic methods [44]. This variational derivation is equivalent to the traditional method [50] requiring that the scattering T matrix of the "perturbation" $M_{\alpha,i} - M_{\alpha}(z)$, ($\alpha = \epsilon, \mu$) be zero on the average. In the CPA the disordered system is replaced by an effective medium, in which the fluctuating quantities (in our case $M_{\epsilon}(\mathbf{r})$ and $M_{\mu}(\mathbf{r})$) are replaced by uniform, but frequency-dependent, complex quantities

² Exactly the same equation is obtained for the vector potential $\mathbf{A}(\mathbf{r},\omega)$, defined as $\nabla \times \mathbf{A}(\mathbf{r},\omega) = \mu(\mathbf{r})\mathbf{H}(\mathbf{r},\omega)$, if the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ is applied [45].

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 $M_{\epsilon}(z)$ and $M_{\mu}(z)$, where $z = \frac{1}{c_0}\omega + i\eta$, (η is an infinitesimal positive real number), except inside a cavity around the midpoint \mathbf{r}_i . The volume of the cavity is V_c , and in this region $M_{\epsilon,\mu}$ take their fluctuating values evaluated at $\mathbf{r}_i \ M_{\epsilon,i} \doteq M_{\epsilon}(\mathbf{r}_i)$ and $M_{\mu,i} \doteq M_{\mu}(\mathbf{r}_i)$. Within CPA these quantities are assumed to be uncorrelated³, which means that V_c must be larger than the correlation volume ξ^3 , where ξ is the correlation length. This. naturally introduces an ultraviolet wavenumber cutoff $k_{\xi} \propto \xi^{-1}$ into the effective medium. In our treatment, this cutoff replaces the radius of the first Brillouin zone (in crystals) and the Debye cutoff (in glasses) for the definition of the density of states $g(\omega)$ which samples the states relevant for the disorder scattering:

$$g(\omega) = 2\omega\rho(\lambda) = 2\omega\frac{1}{\pi}\text{Im}\{G(z)\},\qquad(9)$$

where $\rho(\lambda)$ is the density of levels (eigenvalues), G(z) is the local Green's function

$$G(z) = \frac{3}{k_{\xi}^3} \int_0^{k_{\xi}} dk k^2 G(k, z) , \qquad (10)$$

and G(k, z) is the wavenumber dependent Green's function of the effective medium

$$G(k,z) = \frac{1}{-z^2 + k^2 M_{\varepsilon}(z) M_{\mu}(z)}.$$
 (11)

We emphasize that – in contrast to the PT treatment using the nonlinear-sigma-model theory [18, 19] - in CPA the small parameter for justifying the saddle-point approximation is not the relative variance of the fluctuating quantities [49], but the ratio V_c/V between the cavity volume and the volume V of the sample [44]. This enables to treat the case of strong disorder, where the relative variance may take any value.

The CPA equations read [44]

$$0 = \left\langle \frac{M_{\epsilon,i} - M_{\epsilon}(z)}{1 + q \left(M_{\epsilon,i} - M_{\epsilon}(z) \right) \Lambda_{\epsilon}(z)} \right\rangle_{\epsilon}$$
(12)

and

$$0 = \left\langle \frac{M_{\mu,i} - M_{\mu}(z)}{1 + q \left(M_{\mu,i} - M_{\mu}(z) \right) \Lambda_{\mu}(z)} \right\rangle_{\mu}$$
(13)

with $q = V_c k_{\xi}^3/3\pi^2$. The parameter q must be smaller than 1 and can be interpreted as a mean-field critical percolation threshold [44]. Because the critical percolation threshold for 3-dimensional continuum percolation is around 0.3, we take q = 0.3 in the numerical calculations that we performed to show graphically the effect of the disorder.

The quantities $\Lambda_{\epsilon,\mu}(z)$ are defined by

$$\Lambda_{\epsilon,\mu}(z) = \frac{1}{M_{\epsilon,\mu}(z)} \left[1 + z^2 G(z) \right] \tag{14}$$

We note that the CPA equations (12) and (13) are completely symmetric with respect to ϵ and μ , i.e. they hold for both, Eqs (7) and (8). We further note that if the distributions of the two spatially fluctuating quantities are the same, $\mathcal{P}(M_{\epsilon,i}) = \mathcal{P}(M_{\mu,i})$, it results $M_{\epsilon}(z) =$ $M_{\mu}(z)$. Therefore the CPA equations reduce to the ones one would obtain if one would take $M_{\epsilon}(\mathbf{r}) = M_{\mu}(\mathbf{r})$ from the outset [53].

The averages $\langle \ldots \rangle_{\epsilon,\mu}$ are to be performed with distribution densities $\mathcal{P}_{\epsilon}(M_{\epsilon,i})$ and $\mathcal{P}_{\mu}(M_{\mu,i})$. For our calculations, in order to be able to treat the case of strong disorder, we take log-normal distributions $[44] \ \mathcal{P}(x) = [\sqrt{2\pi}\sigma x]^{-1}e^{-\ln^2(x/x^{(0)})/2\sigma^2}$ with medians $x^{(0)} = M_{\epsilon}^{(0)} = M_{\mu}^{(0)} = 1$. The relative variances of the two distributions $\gamma_{\epsilon} = \langle (M_{\epsilon} - \langle M_{\epsilon} \rangle)^2 \rangle / \langle M_{\epsilon} \rangle^2 = e^{\sigma_{\epsilon}^2} - 1$ and $\gamma_{\mu} = \langle (M_{\mu} - \langle M_{\mu} \rangle)^2 \rangle / \langle M_{\mu} \rangle^2 = e^{\sigma_{\mu}^2} - 1$ are the control parameters of the theory.

From the Green's function (11) we can read off the formula for the (scattering) mean-free path

$$\frac{1}{\ell(\omega)} = \frac{2\omega}{c_0} \operatorname{Im}\left\{\frac{1}{\left[M_{\epsilon}(z)M_{\mu}(z)\right]^{1/2}}\right\}$$
(15)

and the speed of light inside the medium:

$$v(\omega) = c_0 \operatorname{Re}\left\{\left[M_{\epsilon}(z)M_{\mu}(z)\right]^{1/2}\right\}$$
(16)

In turn, from these quantities we can calculate the frequency-dependent (unrenormalized) diffusivity

$$D_0(\omega) = \frac{1}{3}v(\omega)\ell(\omega). \qquad (17)$$

Before we use the CPA for estimating the localization properties of disordered electromagnetic systems at finite frequency ω , we would like to comment on the limit $\omega \to 0$. As pointed out by Köhler *et al.* [44], in this limit the effective-medium expression of Bruggeman [54] for the permittivity of mixed dielectric materials is obtained. Contrary to this, the CPA applied to the potential-type

³ A generalization of the traditional CPA for electrons for the inclusion of correlated disorder exists [51]. In this treatment the k integral in Eq. (10) up to the cutoff $k_{\xi} \propto \xi^{-1}$ has to be replaced by an integral over the Green's function G(k, z), multiplied by the k dependent correlation function C(k), normalized by its value at k = 0. This function equals 1 for wavenumbers $k \ll k_{\xi}$ and then smoothly decays near $k = k_{\xi}$. So, the present CPA just replaces this "smooth cutoff" of the correlated treatment by a sharp one. The essential ingredient of the spatial correlations, namely the correlation length is included in the present version of the CPA. Long-range correlations, which are relevant in hyperuniform materials [36, 37, 52], and which govern the $k \to 0$ behavior of the correlations, are not included.



FIG. 1. "Conductivity" $\tilde{g}(\omega) = g(\omega)D(\omega)$ against frequency, calculated in CPA for a log-normal distribution of M_{ϵ} and M_{μ} , truncated at $M_{\epsilon} = M_{\mu} = 0$. Dashed blue lines: Only one quantity, say $M_{\epsilon}(\mathbf{r})$ is fluctuating, the relative variance $\gamma_{\epsilon} = e^{\sigma_{\epsilon}^2} - 1$ increases as $\gamma_{\epsilon} = 0.25, 0.5, 1., 1.5, 2.2.5$.

Continuous red lines: Both quantities $M_{\epsilon}(\mathbf{r})$ and $M_{\mu}(\mathbf{r})$ are fluctuating, one of the variances, say, γ_{ϵ} is held fixed at 2.5, the other variance increases from $\gamma_{\mu} = 0.25$ to $\gamma_{\mu} = 2.5$. in steps as before.

Inset: density of eigenvalues $\rho(\lambda)$ for the same CPA calculations.

The full circles in the main panel mark the end of the spectrum, given by $\rho(\lambda)$ in the inset.

treatment of Maxwell's equation [6], mentioned in the beginning, gives just the arithmetic average of the permittivity in the $\omega \to 0$ limit, because the non-trivial influence of the disorder in this approach is multiplied by ω^2 and just vanishes in the DC limit. This shows once more that a proper treatment of Maxwell's equations is necessary.

We now turn to the discussion of the impact of electrical and magnetic disorder on Anderson localization of light. This phenomenon is known [9] to arise from interference of closed scattering paths. According to the selfconsistent theory of Anderson localization [12, 55, 56] in the version used for classical waves [52, 57–59] the renormalized diffusion coefficient, which includes the localization phenomena, is given by

$$D(\Omega, \omega) = D_0(\omega) - D(\Omega, \omega) P_0(\Omega, \omega)$$
(18)

Here Ω denotes the frequency corresponding to the diffusion dynamics of the radiation, and $P_0(\Omega, \omega)$ denotes the return probability

$$P_0(\Omega,\omega) = \frac{1}{\pi g(\omega)} \sum_{|\mathbf{q}| < q_0} \frac{1}{-i\Omega + q^2 D(\Omega,\omega)} \,. \tag{19}$$

The upper cutoff q_0 has been introduced, because the interference is only effective in the **q** region, where the diffusion approximation holds. In the original papers on electron localization [12, 55, 56] the inverse mean-free path ℓ^{-1} has been taken for q_0 , in the literature on phonon localization [58, 59] the Debye cutoff k_D , instead. Here we choose to take the correlation cutoff $q_0 = k_{\xi}$ as upper cutoff. The self-consistent Eq. (18) can now be written in the form

$$D(\Omega,\omega) = D_0(\omega) - \frac{3}{\pi k_{\xi}^3 g(\omega)} \int_0^{k_{\xi}} dq \frac{q^2}{q^2 - \frac{i\Omega}{D(\Omega,\omega)}} \quad (20)$$

Localization or otherwise is now defined to occur if the quantity

$$\lim_{\Omega \to 0} D(\Omega, \omega) \tag{21}$$

vanishes or not.

We now assume that a frequency ω^* exists (mobility edge) which separates the extended states ($\omega < \omega^*$) from the localized ones ($\omega > \omega^*$). In the localized regime the quantity $-i\Omega/D(\Omega,\omega)$ becomes a real quantity, namely the square of the inverse localization length. Right at the mobility edge $\omega = \omega^*$, this quantity becomes zero, and we have

$$D(\Omega,\omega) = \left[1 - \frac{3}{\pi k_{\xi}^2 g(\omega) D_0(\omega)}\right]$$
(22)

On the other hand, at the mobility edge, $D(\Omega, \omega) = 0$, so that the dimensionless quantity ("conductivity")

$$\tilde{g}(\omega) = k_{\xi}^2 g(\omega) D_0(\omega) \tag{23}$$

has to be equal to $3/\pi \approx 1$ at the mobility edge. Values of $\tilde{g}(\omega)$ larger than ~1, therefore, lead to delocalization, values smaller than ~1 to localization.

In Fig. 1 we have plotted this quantity, calculated in CPA against the dimensionless spectral parameter $\lambda = \omega^2/c_0^2 k_{\varepsilon}^2$. We consider two scenarios:

- (i) Only one of the moduli, say, $M_{\epsilon}(\mathbf{r})$ is considered to have spatial fluctuations with variance γ_{ϵ} increasing from 0.25 to 2.5: *electric (or magnetic) disorder only* (dashed blue lines).
- (ii) Setting one of the variances, say, $\gamma_{\mu} = 2.5$ and increasing the other, γ_{ϵ} from 0.25 to 2.5: Combined electric and magnetic disorder (continuous red lines).

It is seen that in the case of the combined electric and magnetic disorder the values of \tilde{g} are much lower and also the *spectral range* for which \tilde{g} is smaller than ~ 1 is much more extended. Fig. 1 comprises the central result of the present contribution. Our results may explain, why with electric disorder only (or perhaps magnetic disorder only) it is very hard to obtain Anderson localization, whereas for the combination of both, the odds for observing Anderson localization of light in three dimension are increased appreciably. We therefore recommend for meeting the challenge of experimentally observing 3D Anderson localization the consideration of disordered materials with both electric and magnetic disorder. Such materials could be e.g. polymeric materials with superparamagnetic inclusions [60].

Let us now discuss the recent numerical results of Yamilov *et al.* [38] in the light of our findings. The authors considered two cases of systems with the disorder induced by overlapping spherical obstacles. These spheres were designed to have in the first case a high electric permittivity, in the second case perfect electric conduction inside the spheres. In their first system with high dielectric permittivity of the spheres they consider the case of electric disorder only. In agreement with our results they find no localization. On the other hand, by using perfectly conducting obstacles they completely expel the time-varying electric and magnetic fields from the obstacles, just effectively introducing a combination of electric and magnetic disorder. Thus their numerical observation of Anderson localization for the perfectly conducting obstacles corresponds to our prediction of localization for the case of combined electric and magnetic disorder.

Summarizing, we have presented a mean-field theory for combined electric and magnetic disorder based on eigenvalue equations derived from Maxwell's equations, which involve manifestly Hermitian operators. The results for the dimensionless conductance suggest systems with combined electric and magnetic disorder as candidates for 3D Anderson localization.

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