Time Series Analysis in Sociophysics and Econophysics

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Time Series

Series of numbers (X_t)

$$X_1, X_2, X_3, \ldots, X_N$$

Mean value

$$\langle X \rangle = \frac{1}{N} \sum_{i=1}^{N} X_i$$

n-th moment

$$\langle X^n \rangle = \frac{1}{N} \sum_{i=1}^{N} X_i^n$$

Variance

$$\mathrm{Var}(X) = \frac{1}{N}\sum_{i=1}^{N}(X_i - \langle X \rangle)^2$$

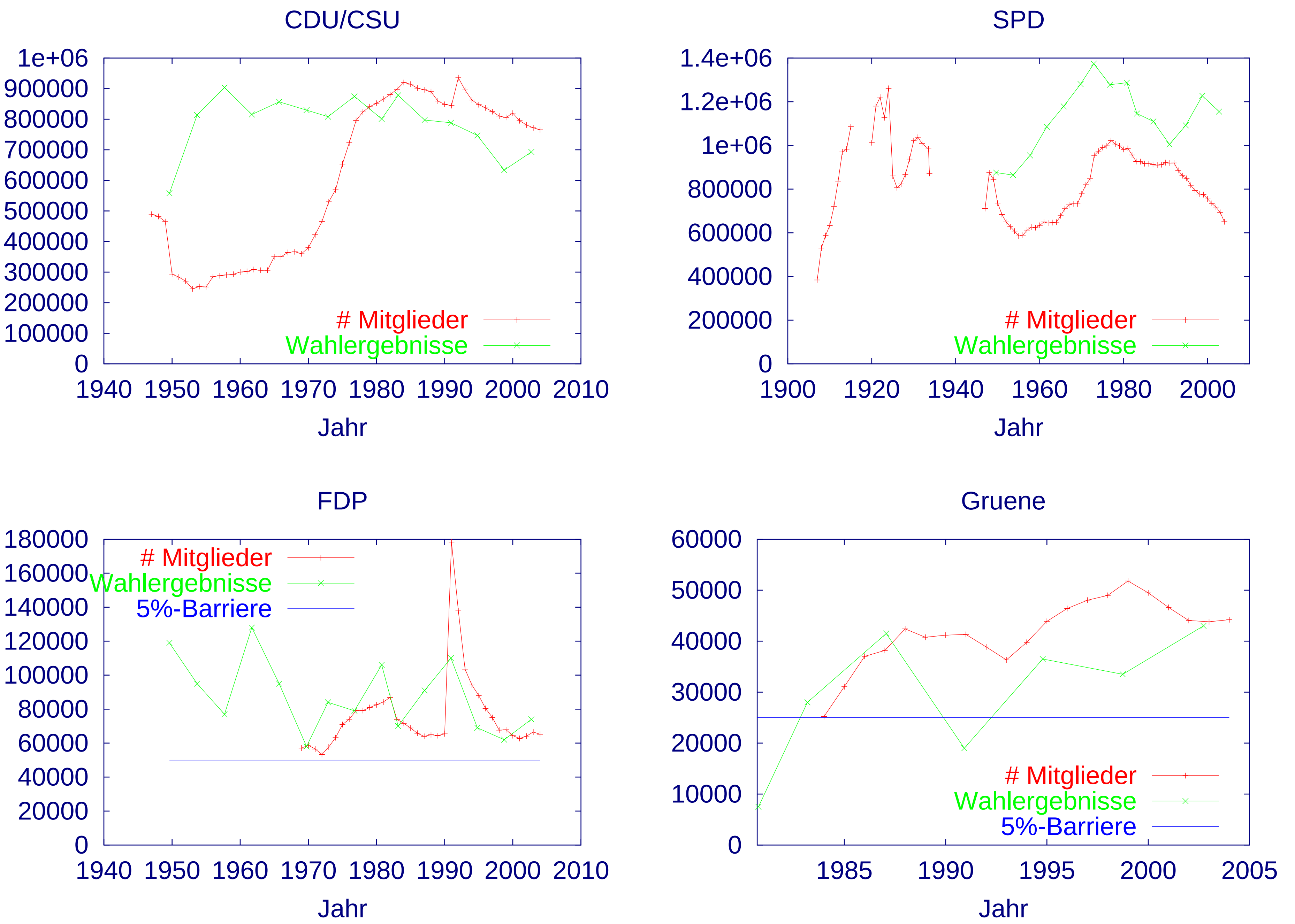
$$\mathrm{Var}(X) = \frac{1}{N}\sum_{i=1}^{N} X_i^2 - 2\langle X \rangle \frac{1}{N}\sum_{i=1}^{N} X_i + \langle X \rangle^2 = \langle X^2 \rangle - \langle X \rangle^2$$

Standard deviation: $\mathrm{std}(X) = \sqrt{\mathrm{Var}(X)}$

Taking into account recency of the values

$$\langle X \rangle_1 = X_1, \langle X \rangle_t = \alpha X_t + (1-\alpha)\langle X \rangle_{t-1}$$

Problems with Real Data



Wegscheider Trend Analysis

(R. Schlittgen und B. H. J. Streitberg, Zeitreihenanalyse

(Oldenbourg, München, 1997))

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
p_t	—	—	—	—	—	—	—	—	—	—	—	—

Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	—	1	6	5	9	8	4	—	—	5	2
p_t	—	0	—	—	—	—	—	—	0	0	—	—

Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	—	1	6	5	9	8	4	—	—	5	2
	2	—	1	6	5	9	—	4	—	—	5	2
p_t	—	0	—	—	—	—	0	—	0	0	—	—

Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	–	1	6	5	9	8	4	–	–	5	2
	2	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	6	5	9	–	4	–	–	5	2
p_t	-1	0	–	–	–	–	0	–	0	0	–	–

Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	–	1	6	5	9	8	4	–	–	5	2
	2	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	–	–	9	–	4	–	–	5	2
p_t	-1	0	–	-1	1	–	0	–	0	0	–	–

Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	–	1	6	5	9	8	4	–	–	5	2
	2	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	–	–	9	–	4	–	–	5	2
	–	–	1	–	–	9	–	–	–	–	–	2
p_t	-1	0	–	-1	1	–	0	1	0	0	-1	–

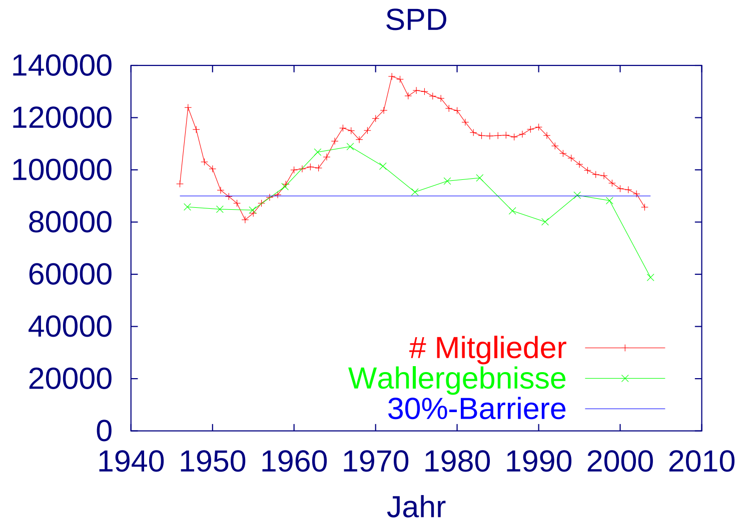
Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	–	1	6	5	9	8	4	–	–	5	2
	2	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	6	5	9	–	4	–	–	5	2
	–	–	1	–	–	9	–	4	–	–	5	2
	–	–	1	–	–	9	–	–	–	–	–	2
	–	–	1	–	–	9	–	–	–	–	–	–
p_t	-1	0	–	-1	1	–	0	1	0	0	-1	-7

Wegscheider Trend Analysis

t	1	2	3	4	5	6	7	8	9	10	11	12
A_t	2	1	1	6	5	9	8	4	5	5	5	2
	2	—	1	6	5	9	8	4	—	—	5	2
	2	—	1	6	5	9	—	4	—	—	5	2
	—	—	1	6	5	9	—	4	—	—	5	2
	—	—	1	—	—	9	—	4	—	—	5	2
	—	—	1	—	—	9	—	—	—	—	—	2
	—	—	1	—	—	9	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—
p_t	-1	0	8	-1	1	-8	0	1	0	0	-1	-7

Trend Reversals for Bavarian SPD



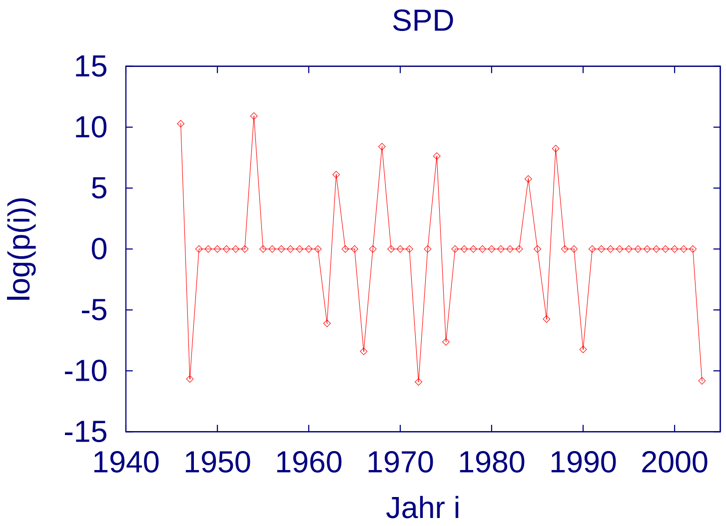
28,2% (1954) – 31.2% (1958) – 35.6% (1962)

31.2% (1958) – 35.6% (1962) – 36.3% (1966)

36.3% (1966) – 33.8% (1970) – 30.5% (1974)

30.5% (1974) – 31.9% (1978) – 32.3% (1982)

32.3% (1982) – 28.1% (1986) – 26.7% (1990)



1962 negative, 1963 positive

1966 negative, 1968 positive

1974 positive, 1975 negative

1984 positive

1990 negative

Covariance between two time series

two time series: $(A_i): A_1, A_2, \ldots, A_X$

$(B_i): B_1, B_2, \ldots, B_X$

Covariance:

$$\mathrm{Cov}(A,B) = \langle A \cdot B \rangle - \langle A \rangle \cdot \langle B \rangle = \frac{1}{X}\sum_{i=1}^{X} A_i \cdot B_i - \frac{1}{X}\sum_{i=1}^{X} A_i \cdot \frac{1}{X}\sum_{i=1}^{X} B_i$$

Correlation:

$$\varrho(A,B) = \frac{\mathrm{Cov}(A,B)}{\sqrt{\mathrm{Var}(A) \cdot \mathrm{Var}(B)}}$$

Correlation is restricted to the interval $[-1; 1]$.

Can Election Results be Predicted by Opinion Polls?

Crosscorrelations

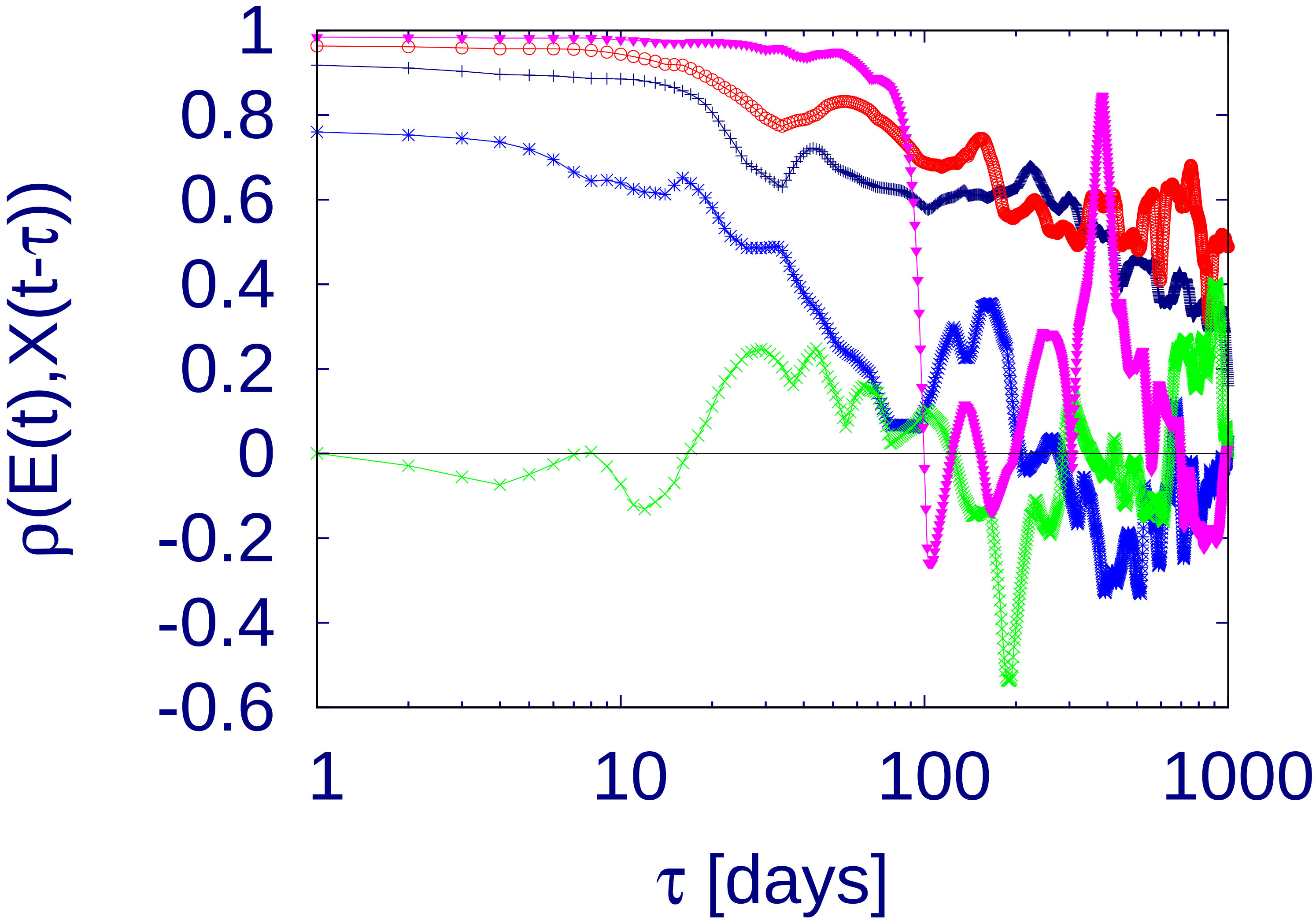
time series $(E(t))$ of election results

time series $(X(t))$ of poll results

crosscorrelation $\rho_\tau = \rho(E(t), X(t-\tau))$

$$\rho_\tau = \frac{\langle E(t) \times X(t-\tau)\rangle - \langle E(t)\rangle \times \langle X(t-\tau)\rangle}{\sqrt{\mathrm{Var}(E(t)) \times \mathrm{Var}(X(t-\tau))}}$$

Crosscorrelations between Election Resolts and Poll Results



Be careful with correlations

Example: (X_t)	-2	-1	0	1	2
(Y_t)	4	1	0	1	4

$$\varrho(X, Y) = 0, \text{ but } Y_t = X_t^2$$

External influences on time series

Bavaria of the 19th century: correlation between numbers of monks and of babies

Partial Correlations

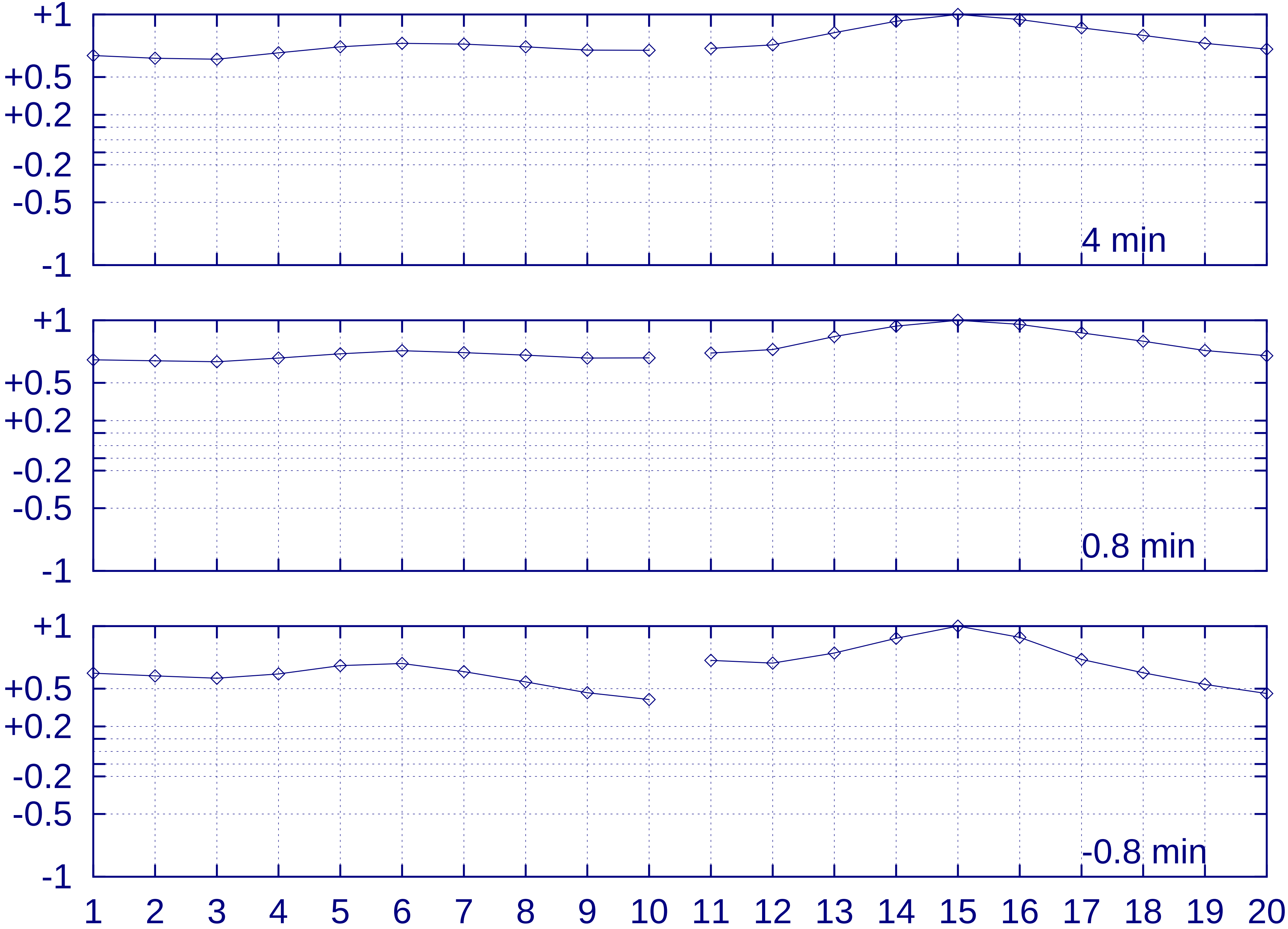
Correlation matrix between two sets $\mathcal{X}$ and $\mathcal{Y}$ of observables

$$\varrho(\mathcal{X},\mathcal{Y}) = \begin{pmatrix} \varrho(X_1,Y_1) & \dots & \varrho(X_1,Y_m) \\ \vdots & \vdots & \vdots \\ \varrho(X_n,Y_1) & \dots & \varrho(X_n,Y_m) \end{pmatrix}$$

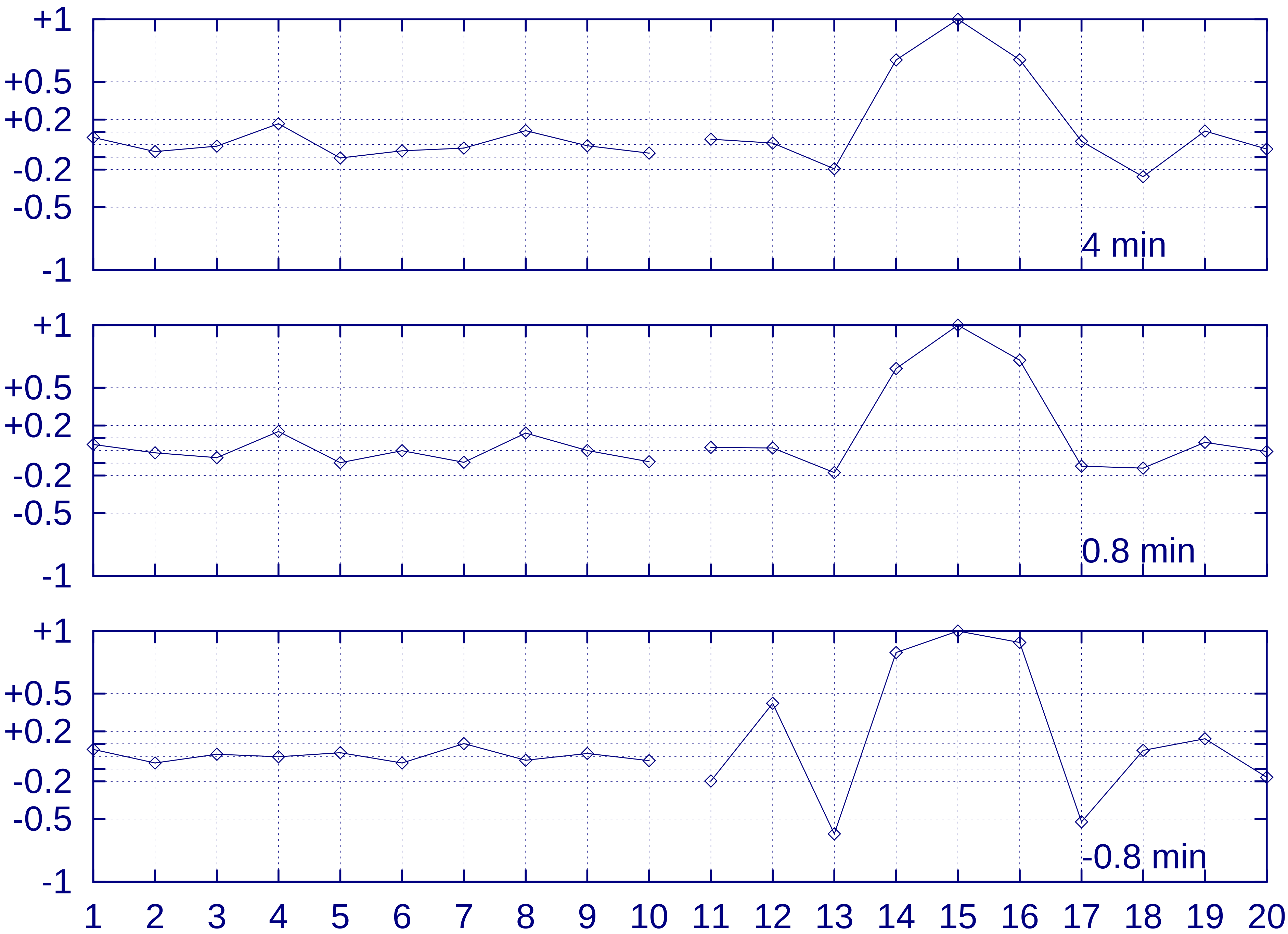
$$\varrho(X,Y|\mathcal{Z}) = \frac{\varrho(X,Y) - \varrho(X,\mathcal{Z})\cdot\varrho^{-1}(\mathcal{Z},\mathcal{Z})\cdot\varrho(\mathcal{Z},Y)}{\sqrt{(1-\varrho(X,\mathcal{Z})\cdot\varrho^{-1}(\mathcal{Z},\mathcal{Z})\cdot\varrho(\mathcal{Z},X))\ \text{(same for } Y)}}$$

If $\mathcal{Z} = \{Z\}$:

$$\varrho(X,Y|Z) = \frac{\varrho(X,Y) - \varrho(X,Z)\varrho(Y,Z)}{\sqrt{(1-\varrho^2(X,Z))(1-\varrho^2(Y,Z))}}$$



+1
+0.5
+0.2
-0.2
-0.5
-1
4 min
0.8 min
-0.8 min
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20



+1
+0.5
+0.2
-0.2
-0.5
-1
4 min
0.8 min
-0.8 min
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Autocorrelations

one time series (X_t) : $X_1, X_2, X_3, \ldots, X_N$

Define a new time series (Y_t) by shifting (X_t) by a time lag τ:

$$(Y_i := X_{i+\tau}), \qquad 1 \leq i \leq N - \tau$$

Autocorrelation

$$\varrho_\tau = \frac{\langle X(0)X(\tau)\rangle - \langle X(0)\rangle\langle X(\tau)\rangle}{\sqrt{\mathrm{Var}(X(0))\mathrm{Var}(X(\tau))}}$$

with the restricted time series

$$(X(\tau)_i) : X_{1+\tau}, X_{2+\tau}, X_{3+\tau}, \ldots, X_{N-\tau_{\mathrm{max}}+\tau}$$

Autocorrelation is invariant against time reversal

Stationary Time Series

Weak condition:

Mean value and variance of a time series are constant.

Then formulas can be simplified:

$$\varrho_\tau = \frac{\langle X(0)X(\tau)\rangle - \langle X(0)\rangle^2}{\mathrm{Var}(X(0))}$$

But usually these time series are not stationary.

Theory of Autoregressive Processes

Autoregressive process or order p

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \varepsilon_t$$

with white noise ε_t

$$\begin{aligned}
\langle X(t) X(t-\tau) \rangle &= \alpha_1 \langle X(t-1) X(t-\tau) \rangle \quad + \ldots \\
&+ \alpha_p \langle X(t-p) X(t-\tau) \rangle \\
&+ \langle \varepsilon(t) X(t-\tau) \rangle \\
\langle X(t) \rangle \langle X(t-\tau) \rangle &= \alpha_1 \langle X(t-1) \rangle \langle X(t-\tau) \rangle \quad + \ldots \\
&+ \alpha_p \langle X(t-p) \rangle \langle X(t-\tau) \rangle \\
&+ \langle \varepsilon(t) \rangle \langle X(t-\tau) \rangle
\end{aligned}$$

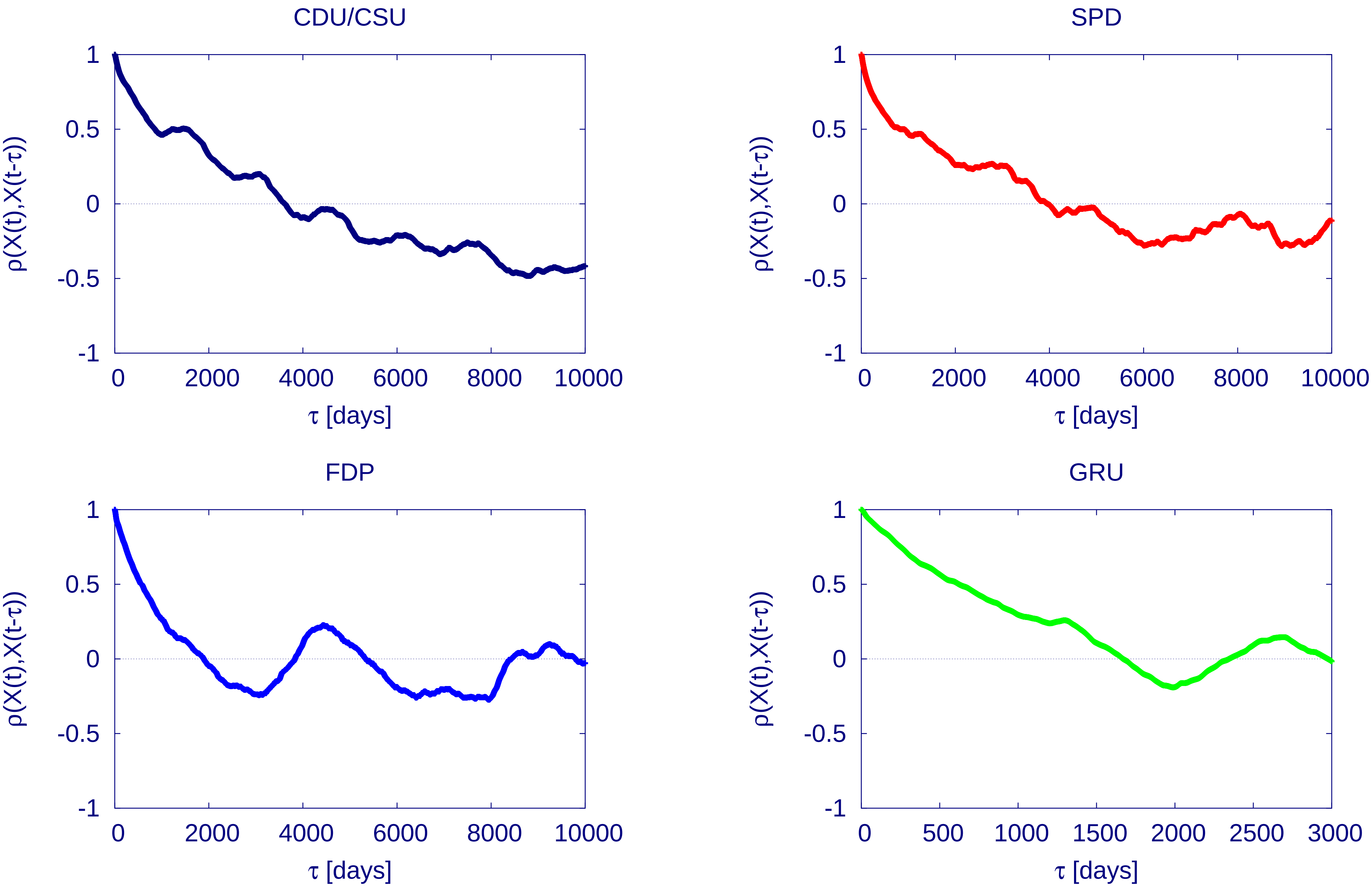
Assuming stationarity, one gets Yule-Walker-Equations:

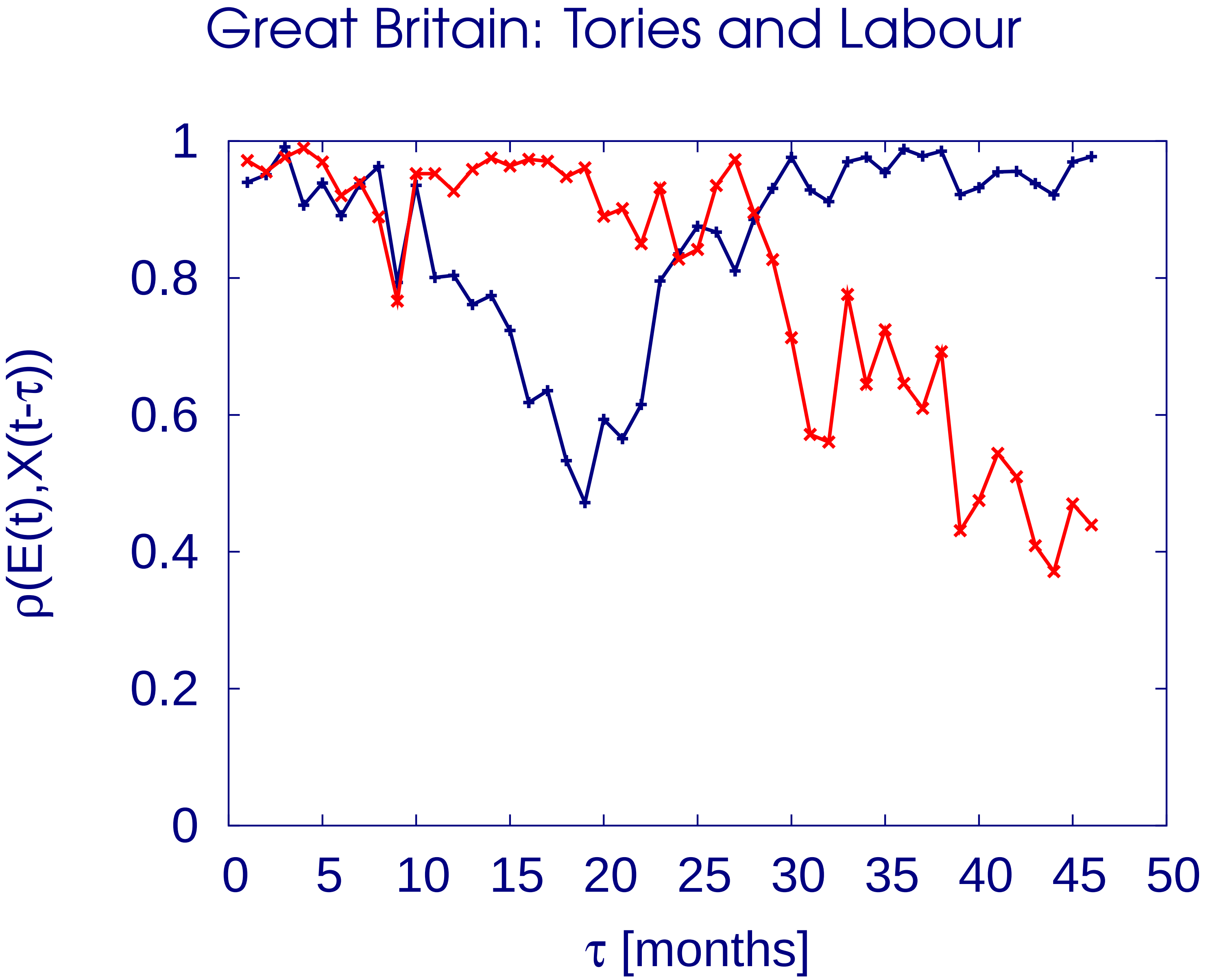
$$\begin{array}{ccccccccc}
\varrho_1 & = & \alpha_1\varrho_0 & + & \alpha_2\varrho_1 & + & \ldots & + & \alpha_p\varrho_{p-1} \\
\varrho_2 & = & \alpha_1\varrho_1 & + & \alpha_2\varrho_0 & + & \ldots & + & \alpha_p\varrho_{p-2} \\
\ldots & & & & & & & & \\
\varrho_p & = & \alpha_1\varrho_{p-1} & + & \alpha_2\varrho_{p-2} & + & \ldots & + & \alpha_p\varrho_0
\end{array}$$

Example: $p = 1$ (Markov process)

$$\varrho_1 = \alpha_1, \quad \varrho_\tau = \alpha_1^\tau \quad \forall\tau > 0$$

Do we see this exponential behavior?


CDU/CSU
SPD
FDP
GRU
ρ(X(t),X(t-τ))
τ [days]
1
0.5
0
-0.5
-1
0
2000
4000
6000
8000
10000
500
1000
1500
2500
3000


Great Britain: Tories and Labour
ρ(E(t),X(t-τ))
τ [months]
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35
40
45
50

Theory of Autoregressive Processes

Which value does the order p take?

Let us assume, the order is k:

$$\varrho_\tau = \Phi_{k1}\varrho_{\tau-1} + \ldots + \Phi_{kk}\varrho_{\tau-k}$$

$$Y_k\vec{\Phi}_k = \vec{\varrho}$$

Resolve for Φ_{kk}:

$$\Phi_{kk} = \frac{\begin{vmatrix} 1 & \varrho_1 & \cdots & \varrho_{k-2} & \varrho_1 \\ \varrho_1 & 1 & \cdots & \varrho_{k-3} & \varrho_2 \\ \varrho_2 & \varrho_1 & \cdots & \varrho_{k-4} & \varrho_3 \\ \vdots & & \ddots & & \vdots \\ \varrho_{k-1} & \varrho_{k-2} & \cdots & \varrho_1 & \varrho_k \end{vmatrix}}{\begin{vmatrix} 1 & \varrho_1 & \cdots & \varrho_{k-2} & \varrho_{k-1} \\ \varrho_1 & 1 & \cdots & \varrho_{k-3} & \varrho_{k-2} \\ \varrho_2 & \varrho_1 & \ddots & \varrho_{k-4} & \varrho_{k-3} \\ \vdots & & \ddots & & \vdots \\ \varrho_{k-1} & \varrho_{k-2} & \cdots & \varrho_1 & 1 \end{vmatrix}}$$

If $\Phi_{kk} = 0$ then $k > p$.