A Virtual Environment for Interactive Assembly Simulation: From Rigid Bodies to Deformable Cables

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ABSTRACT

This paper presents the application of rigid body simulation for assembly tasks in a virtual environment and the extension of this system to the real-time simulation of deformable cables. Presently our virtual reality tools evaluate virtual prototypes based on CAD models of the design department. We motivate the need for deformable objects, especially cables, by practical examples and explain our physically based simulation approach.

The physical behaviour of a cable is simulated as a mass-spring model with generalized springs. The main objective is a physically plausible real-time simulation for cables of moderate complexity. In comparison to research done in the field of cloth simulation for computer animation, cables show a very stiff bending behaviour. In order to improve dynamic bending behaviour we use torsion springs. Their restoring forces are proportional in angle and not in elongation.

Special attention is given to the integration into the virtual environment for assembly simulation. Collision detection and response are necessary for the interaction with rigid bodies and with input devices used by the engineer.

Keywords: virtual environments, assembly simulation, physically based modeling, deformable objects, torsion springs,

1 Introduction

The automotive industry is one of the leading industries concerning the application of virtual reality. Styling and design reviews in the early stages of development are two major applications of VR tools in daily use. These applications take advantage of 3D visualization and immersion which are given by virtual environments.

The changing development processes in the automotive industry create a demand for new VR applications. *Physical Mock-Ups* will be replaced in part by *Digital Mock-Ups* in order to meet the demands of a higher quality and a reduction of costs and time for development. In order to verify the digital design virtual prototyping and physical rapid prototyping techniques complement each another.

The task of assembly simulations is a much more demanding than the immersive visualization of a virtual prototype. It involves complex interaction between man and machine and requires a real-time simulation of object behaviour. The benefit of a virtual environment for assembly simulation is given by the possibility of direct manipulation of objects in an intuitive way.

This paper is organized as follows: A short overview of the application field of virtual environments to assembly simulations is given in section 2. In section 3, we give a short summary of previous work. We motivate the extension to deformable bodies from practical examples in section 4, and explain our approach of physically based cable modeling in section 5. Section 6 discusses collision detection for deformable bodies and describes an approach based on distance calculation. Finally we give some conclusions and outline future work in section 7.

2 Virtual Assembly Simulations

The use of a virtual environment for the verification of assembly tasks and fitting simulations can be found in the literature since the mid 90's. Several software systems were developed with different approaches and main foci depending on their application background.

Because the assembly and fitting simulation is a computationally demanding task which involves several hardware and software components, several large software systems have been developed, which try to fulfill the wide range of requirements. As an example of such a system ”VADE” (Virtual Assembly Design Environment) from the Washington State University can be mentioned [10]. It consists of several modules and is capable of supporting a variety of VR devices. The connection of the CAD system with the virtual environment is based on a parametric description within the CAD system.

The integration into the process chain is only possible, if the development departments use parametric CAD systems.

Gomes de Sá and Zachmann [4] present the integration of the ”Virtual Design 2” system into an existing industrial development process at BMW. Based on the process chain they describe the requirements for such applications, the benefits as well as problems for the users.

The evaluation of virtual environments for these purposes is still a topic of ongoing research and must be seen as embedded in the *Digital Mock-Up* (DMU) strategies of the automotive industry.

Our work is mainly influenced by research done in the field of real-time collision detection and physically based object behaviour. These real-time simulations are one of the prerequisites for the simulation of assembly tasks. Our main objective was the simulation of assembly tasks and fitting tests in an interactive virtual environment to support the user with an intuitive handling.

For the integration into the process chain the CAD data must be transferred from the *Product Data Management* (PDM) system and converted to suitable formats for VR applications. Converters and tesselators automatically prepare the VR data sets from current CAD models.

3 Previous Work

In the VR software platform ”DBView”, developed at the Virtual Reality Competence Center (VRCC) of DaimlerChrysler Research, all algorithmic prerequisites for the interactive simulation of assembly tasks with rigid bodies exist. The software includes a real-time collision detection module [7] (which is a basic algorithmic requirement for physically based simulation and intuitive interaction). In addition, it contains modules for real-time interactive contact simulation [2], real-time multibody dynamics [13] and allows an intuitive grasping of virtual objects with one or two datagloves [15]. The contact simulation system realizes a physically plausible compliant motion of objects if collisions occur. This allows a smooth gliding of interactively moved objects along the surface of other objects and parts. The dynamics simulation module determines how objects in contact behave according to Newtonian dynamics with a special emphasis on friction effects. An intuitive grasping of objects can be done with one or two datagloves. The algorithmic approach provides interaction methaphors which allows the grasping of virtual objects with both hands or the precise manipulation between fingers,

In contrast to the work of Fröhlich et al. [8], which connected the users’s hand with a set of springs to the virtual objects, we use a direct interaction metaphor.

Therefore our prior work focused on rigid bodies and allows interactive assembly simulations in a virtual environment in real-time (15-25 visualized frames per second).

4 Motivation for the Extension to Deformable Cables

The system was intended to be used within the DaimlerChrysler business units, especially in the passenger car and commercial vehicle design departments. During our work with data sets of the design department we first encountered the difficulty of assembly situations involving cables. In CAD systems cables are modeled as rigid bodies, with their geometrical configuration, given by their final spatial position at the end of the assembly task. This leads to situations in which it is impossible to find a collision free assemby path within the virtual environment.

Up to now it was necessary for the engineer to mark the flexible part of the geometry as ”collision insensitive” in our simulation and to redo the assembly task. But this allows only to check the rigid geometry. With the limitation to the simulation of rigid bodies a correct verification of objects including cables is not possible, because they do not have a fixed shape (see figure 3).

Furthermore the simulation of deformable objects is a prerequisite for the examination of cable installing itself within complex geometric environment (eg. inside the motor compartment). Our motivation was therefore to improve this situation by working towards the simulation of deformable cables with a physically based model and by giving them a plausible object behaviour for this kind of applications.

5 Physically Based Modeling of Cables

Several different approaches for the simulation of deformable bodies are used in engineering applications. They vary greatly in exactness and computational cost. Two of the most widely used are finite element methods and multibody systems, but these methods often have a different purpose. Because of the requirements of real-time simulation (in terms of VR) and of the integration into the assembly simulation of rigid bodies, our objectives are low computational cost and a visually satisfying object behaviour.

The modeling of deformable bodies in computer graphics became popular in the mid-80's. In the field of computer animation particle systems have found a broad range of applications. For animation purposes coupled particle systems, often called mass-spring systems, are widely used for cloth simulation. In this model the particles are mass points, which are connected by generalized springs. The positions and velocities of the particles are ruled by these springs which represent internal forces due to material properties. Additionally, external forces determine the object behaviour with respect to eg. gravity.

From the point of VR applications one of the most important contributions was from Barraff and Witkin [1]. Their mathematical formulation for mass-spring systems substantively increased the numerical performance in regard to run-time. The use of an implicit integration scheme allowed much larger steps sizes and the iterative solver exploited the sparse structure of the system of equations and can also incorporate constraints in a very efficient way. Desbrun [5] and later Kang [11] presented real-time methods for moderately complex objects by using approximated implicit methods. The disadvantage of these methods is the restriction to linear springs.

The use of torsion springs was proposed by Dai [3] and implemented for a kinematic simulation of cables with exact length preservation by Hergenröther [9].

Notation The mathematical description of these mass-spring systems involves a mesh of n particles within three-dimensional space. This means that a force acting on each particle is a generalized force vector $\mathbf{f} \in \mathbb{R}^{3n}$. The component acting on the ith particle is denoted by $\mathbf{f}_i$, and consequently is $\mathbf{f}_i \in \mathbb{R}^3$.

Similarly a matrix $\mathbf{M}$ acting on such a generalized force vector is a element of $\mathbb{R}^{3n\times 3n}$ and its component $\mathbf{M}_{ij} \in \mathbb{R}^{3\times 3}$.

5.1 A Discrete Model for Continous Cables

Our first step in the mathematical modeling is the formulation of a discrete model for the continous deformable object. In terms of physics we have a discretisation in time and in space. The physical simulation is based on a mass-spring system, consisting of mass points connected by generalized springs. The position of these mass points define the shape of the cable. They are the control points for rigid segments which form the geometric representation. The mass points are conected by stiff linear springs, which preserve the length of the cable. Torsion springs situated at the mass points describe the elastic bending behaviour. Their restoring forces are proportional in angle and not in elongation. Provot [12] proposed *flexion springs*, which are linear springs connecting the ith mass point with the $(i+2)$th mass point, but from our experiments a model using torsion springs corresponds better to the behaviour of cables. Figure (1) illustrates these two different spring models. The drawback of the *flexion springs* is that for small angles the length of this spring is almost $2l$ resulting in a too small restoring force.

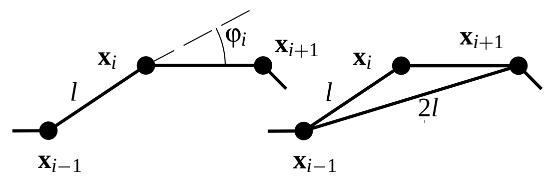

Figure 1. Bending forces resulting from torsion springs (left) or from linear springs of length $2l$ (right)

In mass-spring systems the motion of the mass points is governed by Newton's second law.

$$\ddot{\mathbf{x}} = \mathbf{M}^{-1} \underbrace{\left(-\frac{\partial E}{\partial \mathbf{x}} + \mathbf{F} \right)}_{f(\mathbf{x},\dot{\mathbf{x}})} \tag{1}$$

In this differential equation the values of the vector $\mathbf{x}$ represent the positions and the entries of the diagonal matrix $\mathbf{M}$ the mass of the points along the chain forming the cable. E is a scalar function describing the internal energies and $\mathbf{F}$ represent the external forces. They form a function of $\mathbf{x}$ and $\dot{\mathbf{x}}$. By introducing an additional variable $\mathbf{v} = \dot{\mathbf{x}}$ to represent the velocity, we transform the second order equation into a pair of coupled first order equations leading to the following system of equations

$$\frac{d}{dt}\begin{pmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{pmatrix} = \frac{d}{dt}\begin{pmatrix}\mathbf{x}\\ \mathbf{v}\end{pmatrix} = \begin{pmatrix}\mathbf{v}\\ \mathbf{M}^{-1} f(\mathbf{x},\mathbf{v})\end{pmatrix}. \tag{2}$$

This equations describing a mass-spring system with stiff linear and torsion springs form a stiff problem. For the numerical solution of such an ODE we need an integration scheme which allows us to compute a fast solution with a large timestep. The large size of the timestep is not only essential for reducing computational costs, but also because in a virtual environment the other components (user interaction e.g. tracking, collision detection, rendering) have a limited range of possible timestep sizes.

Even if the simulation runs as a separate process and uses a very small timestep, this simulation would get new data only after many timesteps, which results in increased decoupling from the user interaction. The application of a implicit integration method allows much larger timesteps than an explicit Euler or Runge-Kutta method.

In order to cope with numerical problems arising from the stiff linear and torsion springs we use a implicit Euler scheme. Such a method leads to a nonlinear system of equations for calculating the position and velocity changes for the next timestep (with step size h)

$$\begin{pmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{v} \end{pmatrix} = h \begin{pmatrix} \mathbf{v}_t + \Delta\mathbf{v} \\ \mathbf{M}^{-1}\mathbf{f}(\mathbf{x}_{t+h}, \mathbf{v}_{t+h}) \end{pmatrix}. \tag{3}$$

In order to simplify the solution of the system we linearize $\mathbf{f}$ using a first-order Taylor series expansion

$$\mathbf{f}(\mathbf{x}_{t+h}, \mathbf{v}_{t+h}) = \mathbf{f}(\mathbf{x}_t, \mathbf{v}_t) + \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Delta\mathbf{x} + \frac{\partial\mathbf{f}}{\partial\mathbf{v}}\Delta\mathbf{v}. \tag{4}$$

The derivatives $\partial\mathbf{f}/\partial\mathbf{x}$ and $\partial\mathbf{f}/\partial\mathbf{v}$ are evaluated at state $(\mathbf{x}_t, \mathbf{v}_t)$.

So we obtain a linear system of equations

$$\underbrace{\left(\mathbf{M} - h\frac{\partial\mathbf{f}}{\partial\mathbf{v}} - h^2\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right)}_{\mathbf{A}} \Delta\mathbf{v} = \underbrace{h\left(\mathbf{f}_0 + h\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\mathbf{v}_0\right)}_{\mathbf{b}} \tag{5}$$

of the form $\mathbf{A}\Delta\mathbf{v} = \mathbf{b}$, which we solve for $\Delta\mathbf{v}$ using the conjugate gradient (cg) method. Afterwards we simply compute the first row of Eq.(3) $\Delta\mathbf{x} = h(\mathbf{v}_t + \Delta\mathbf{v})$.

5.2 Forces and their Derivatives

As mentioned in the previous section we modeled our cable as a chain of mass points connected by linear springs for length preservation and torsion springs for bending properties. This material behaviour is described by a scalar energy function $E(\mathbf{x})$. In order to have a simple expressions to build up the equation system Eq.(5) we decompose the energy function in the way that $E(\mathbf{x}) = \sum_{i=1}^{n} E_i(\mathbf{x})$ with i as the index of the mass point. The force affecting the ith mass point from this energy is $\mathbf{f}_i(\mathbf{x}) = -\partial E/\partial\mathbf{x}_i$. In order to be able to compute Eq.(5) we must calculate the vector $\mathbf{f}$ and the matrices $\partial\mathbf{f}/\partial\mathbf{x}$ and $\partial\mathbf{f}/\partial\mathbf{v}$.

Figure (2) shows some segments of the chain of mass points. The formulation of the forces for the length preservation and the bending properties is described in the following.

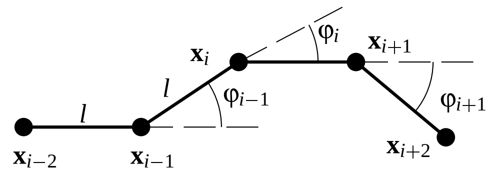

Figure 2. Segments of the chain of mass points

5.2.1 Length Preservation

The energy function for the linear spring connecting the ith and the $(i-1)$th particle is

$$E_i(\mathbf{x}) = \frac{k_L}{2}(|\mathbf{x}_i - \mathbf{x}_{i-1}| - l)(|\mathbf{x}_i - \mathbf{x}_{i-1}| - l). \tag{6}$$

where l is the rest length and k_L the elastic modulus of this spring.

The force acting on the ith particle is determined by the incident springs. So the force as the first derivative of the energy is

$$\begin{aligned} \mathbf{f}_i(\mathbf{x}) = & -\frac{\partial E}{\partial\mathbf{x}_i} = -\left(\frac{\partial E_i}{\partial\mathbf{x}_i} + \frac{\partial E_{i+1}}{\partial\mathbf{x}_i}\right) \\ = & \ k_L\left(\frac{l}{|\mathbf{x}_i - \mathbf{x}_{i-1}|} - 1\right)(\mathbf{x}_i - \mathbf{x}_{i-1}) \\ & -k_L\left(\frac{l}{|\mathbf{x}_{i+1} - \mathbf{x}_i|} - 1\right)(\mathbf{x}_{i+1} - \mathbf{x}_i). \end{aligned} \tag{7}$$

For the derivative $\partial\mathbf{f}/\partial\mathbf{x}$ we get a tridiagonal matrix because $\mathbf{f}_i$ depends only on $(\mathbf{x}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i+1})$. This leads to the formulation

$$\begin{aligned} \frac{\partial\mathbf{f}_i(\mathbf{x})}{\partial\mathbf{x}_i} = & \ k_L\Bigg(-\mathbf{I} + \frac{l}{|\mathbf{x}_i - \mathbf{x}_{i-1}|}\mathbf{I} - \frac{l}{|\mathbf{x}_i - \mathbf{x}_{i-1}|} \\ & \frac{(\mathbf{x}_i - \mathbf{x}_{i-1})(\mathbf{x}_i - \mathbf{x}_{i-1})^T}{|\mathbf{x}_i - \mathbf{x}_{i-1}|^2}\Bigg) \\ & +k_L\Bigg(-\mathbf{I} + \frac{l}{|\mathbf{x}_{i+1} - \mathbf{x}_i|}\mathbf{I} - \frac{l}{|\mathbf{x}_{i+1} - \mathbf{x}_i|} \\ & \frac{(\mathbf{x}_{i+1} - \mathbf{x}_i)(\mathbf{x}_{i+1} - \mathbf{x}_i)^T}{|\mathbf{x}_{i+1} - \mathbf{x}_i|^2}\Bigg) \end{aligned} \tag{8}$$

and similar terms for $\partial\mathbf{f}_i/\partial\mathbf{x}_{i-1}$ and $\partial\mathbf{f}_i/\partial\mathbf{x}_{i+1}$.

The forces modeling the length preservation do not depend on velocities and consequently the derivative $\partial\mathbf{f}/\partial\mathbf{v}$ is identical to zero.

In order to model the internal damping of the cable we need energy dissipation. Baraff pointed out in his work that damping terms cannot be derived from an energy expression, because this produced nonsensical results

[1]. The physical reason behind his argumentation is that damping forces are nonconservative forces and therefore cannot derived from a potential function (cf. eg. [14]). A reasonable formulation of a damping force term for the linear springs $\mathbf{f}_D$ has to be introduced as the time derivative of the linear spring force and not the energy. Eq. 9 gives a suitable formulation.

$$\begin{aligned}
&\mathbf{f}_{Di}(\mathbf{x}) = \\
&k_D\left(-(\mathbf{x}_i-\mathbf{x}_{i-1})\frac{(\mathbf{x}_i-\mathbf{x}_{i-1})^T(\mathbf{v}_i-\mathbf{v}_{i-1})}{|\mathbf{x}_i-\mathbf{x}_{i-1}|^2}\right) \\
&+k_D\left((\mathbf{x}_{i+1}-\mathbf{x}_i)\frac{(\mathbf{x}_{i+1}-\mathbf{x}_i)^T(\mathbf{v}_{i+1}-\mathbf{v}_i)}{|\mathbf{x}_{i+1}-\mathbf{x}_i|^2}\right)
\end{aligned} \tag{9}$$

5.2.2 Bending Properties

In a similar way it is also possible to formulate the energy and force terms for the torsion springs, but the expressions are more complicated in comparison to the linear springs. The energy of the ith torsion spring with the elongation φ_i in terms of the positions of the mass points is

$$\begin{aligned}
E_i(\mathbf{x}) &= \frac{k_B}{2}\varphi_i{}^2 \\
&= \frac{k_B}{2}\left[\arctan\left(\frac{|(\mathbf{x}_{i+1}-\mathbf{x}_i)\times(\mathbf{x}_i-\mathbf{x}_{i-1})|}{(\mathbf{x}_{i+1}-\mathbf{x}_i)^T(\mathbf{x}_i-\mathbf{x}_{i-1})}\right)\right]^2
\end{aligned} \tag{10}$$

and consequently the force acting on point i can be expressed as

$$\mathbf{f}_i(\mathbf{x}) = -\frac{\partial E}{\partial \mathbf{x}_i} = -\left(\frac{\partial E_{i-1}}{\partial \mathbf{x}_i}+\frac{\partial E_i}{\partial \mathbf{x}_i}+\frac{\partial E_{i+1}}{\partial \mathbf{x}_i}\right) \tag{11}$$

Because of the arctan function and the cross product the derivation of this force from the energy formulation is much more complicated. The calculation of the derivatives using mathematical standard software, which is capable of symbolic calculus, leads to very complex terms because it computes all terms with simple derivation rules and cannot find an optimized form.

In order to simplify the expressions we define two vectors $\mathbf{a} = (\mathbf{x}_{i+1}-\mathbf{x}_i)$ and $\mathbf{b} = (\mathbf{x}_i-\mathbf{x}_{i-1})$ so that we can shorthand write

$$\varphi_i = \arctan\left(\frac{|\mathbf{a}\times\mathbf{b}|}{\mathbf{a}^T\mathbf{b}}\right) \tag{12}$$

and express Eq.(11) as

$$\mathbf{f}_i(\mathbf{x}) = -\frac{\partial E}{\partial \mathbf{x}_i} = -\frac{\partial E_{i-1}}{\partial \mathbf{a}_{i-1}}+\frac{\partial E_i}{\partial \mathbf{a}}-\frac{\partial E_i}{\partial \mathbf{b}}+\frac{\partial E_{i+1}}{\partial \mathbf{b}_{i+1}} \tag{13}$$

Baraff mentioned in [1] that they used a approximated derivative under the assumption that the length of the segments can be treated as constant due to the strong springs which prevent stretching. But the derivations can also be exactly computed with a moderate cost. The partial derivations of the energy expressions with respect to $\mathbf{a}$ and $\mathbf{b}$ are

$$\frac{\partial E_i}{\partial \mathbf{a}} = \varphi_i\frac{\partial \varphi_i}{\partial \mathbf{a}} \qquad \frac{\partial E_i}{\partial \mathbf{b}} = \varphi_i\frac{\partial \varphi_i}{\partial \mathbf{b}} \tag{14}$$

and with the formulation of φ_i as in Eq.(12) the derivations[1] with respect to $\mathbf{a}$ and $\mathbf{b}$ are

$$\frac{\partial \varphi_i}{\partial \mathbf{a}} = \frac{\mathbf{a}\times(\mathbf{a}\times\mathbf{b})}{\mathbf{a}^2|\mathbf{a}\times\mathbf{b}|} \qquad \frac{\partial \varphi_i}{\partial \mathbf{b}} = \frac{\mathbf{b}\times(\mathbf{b}\times\mathbf{a})}{\mathbf{b}^2|\mathbf{a}\times\mathbf{b}|} \tag{15}$$

and similar expressions can be found for the derivations of φ_{i-1} and φ_{i+1}.

For the derivative $\partial\mathbf{f}/\partial\mathbf{x}$ we get also a banded matrix because $\mathbf{f}_i$ depends only on $(\mathbf{x}_{i-2},\ldots,\mathbf{x}_{i+2})$.

In order to be able to compute the second derivative of the energy expression also terms of the form $\partial^2\varphi/\partial\mathbf{a}^2$, $\partial^2\varphi/\partial\mathbf{ab}$ and $\partial^2\varphi/\partial\mathbf{b}^2$ must be calculated.

The derivation $\partial^2\varphi/\partial\mathbf{a}^2$ can be calculated as

$$\begin{aligned}
\frac{\partial^2\varphi_i}{\partial\mathbf{a}^2} = &\ \frac{\partial}{\partial\mathbf{a}}\left(\frac{\mathbf{a}}{\mathbf{a}^2}\times\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}\right) \\
= &\ \frac{(\mathbf{a}^T\mathbf{b})(\mathbf{a}\times\mathbf{b})(\mathbf{a}\times\mathbf{b})^T}{\mathbf{a}^2|\mathbf{a}\times\mathbf{b}|^3} \\
&+\frac{\mathbf{a}^2(\mathbf{a}\mathbf{b}^T+\mathbf{b}\mathbf{a}^T)-2\mathbf{a}^T\mathbf{b}(\mathbf{a}\mathbf{a}^T)}{\mathbf{a}^4|\mathbf{a}\times\mathbf{b}|}
\end{aligned} \tag{16}$$

and in a similar way also the other partial derivatives. With these expressions we can calculate all the terms which are needed for the computation of $\partial\mathbf{f}/\partial\mathbf{x}$.

5.3 Computation of the system of equations

Even if these expressions look rather complicated, one has to keep in mind that the formulation of these forces leads to a sparse linear system. Additionally the principle of superposition allows to compute each force separately and add the components thereafter. The analysis of

[1] for the transformations of the equations we used the formulation of the cross product $\mathbf{a}\times\mathbf{b} = \mathbf{a}^\times\mathbf{b}$ with $\mathbf{a}^\times$ an unsymmetric matrix and some identities for vector and matrix products

the force equations (Eq.(6)-(16)) shows that they consist of various subexpressions (eg. $\mathbf{a} \times \mathbf{b} = (\mathbf{x}_{i+1} - \mathbf{x}_i) \times (\mathbf{x}_i - \mathbf{x}_{i-1})$), which have to be calculated only once and can then be stored in auxiliary variables. Because of the chain structure of the mass points these subexpressions arising from $\mathbf{f}_i$ and $\partial \mathbf{f}_i / \partial \mathbf{x}$ can be partly reused for the calculation of $\mathbf{f}_{i+1}$ and $\partial \mathbf{f}_{i+1} / \partial \mathbf{x}$

The problems of stiffness in cloth simulation mainly arises from the stiff linear springs which prevent in-plane stretching, but cables also have a very stiff bending behaviour. In order to overcome the problems of the strong coupling of the particles the system of equations has to be rebuilt after some cg iterations. That means that the matrix $\mathbf{A}$ and the vector $\mathbf{b}$ is recomputed based on the new temporary positions and velocities. Afterwards the iterative cg solver is used to find the new solution for $\Delta \mathbf{v}$. The rebuild of the system is necessary several times in each time step and is a consequence of the high stiffness of the linear and torsion springs. The advantage is the faster propagation of forces along the particle chain.

In figure 4 we show the static end position of three cables with different material parameters. They also differ in their dynamic bending behaviour. Figure 5 gives an overview over the used parameters and performance.

6 Collision Detection for Cable

A lot of research has been done in the field of collision detection. In order to reduce complexity and achieve real-time performance for rigid objects hierarchical bounding boxes can be used [7]. The precalculation of such hierarchies is not possible for a complete cable, because of the possible large deformations. The best possible approximation is to calculate an oriented bounding box for each segment.

In order to avoid too much tests with the dynamically changing object we propose a different approach which exploits the geometric property of cables. They are rotationally symmetric along their axis. In order to avoid many collision tests between the polygons of the visual representation and the rigid environment, we use a distance based collision detection. Per segment we only have one distance calculation (vertex - environment) and (edge - environment) independent of the triangulation of the cylinder and sphere, which serves as the visual representation of this segment. If the calculated distance is smaller than the radius of the cable a collision occurs and the collision response has to be calculated. For the fast distance calculations we used the method of Eberly [6], which is based on the minimization of a quadratic function.

7 Conclusion and Future Work

We have presented a virtual environment system for interactive assembly simulation. Our previous work focused on the simulation of rigid bodies including collision detection, contact simulation and intuitive user interaction with data gloves. The application of this system within the business units motivated the extension of our system towards the real-time simulation of deformable cables.

Our approach models the cable as a chain of rigid segments. The physical simulation is based on a mass-spring system. The length preservation is modeled by stiff linear springs connecting the adjacent particles and torsion springs, which model the dynamic bending behaviour. We use a mathematical formulation, which allows the real-time simulation of moderate complex cables.

Based on the work of Baraff we also use an implicit method to calculate the next numerical timestep of the ODE and compute the solution of the resulting linear system of equations using an iterative solver (cg method). In order to simulate the stiff bending behaviour of cables we use torsion springs and show how to find appropriate formulations in order to compute the restoring forces and their derivatives in a real-time application. The implicit integration scheme allows larger timesteps, but the numerical stability can still not be guaranteed for all parameter configurations.

Nevertheless the model has a lot of advantages: The computational costs are suitable for real-time applications. The interaction mechanism can be incorporated easily into the model and it allows collision detection and response. Further on it allows the modeling of different material properties, which is important for our applications.

The main focus of further work will be the improvement of the formulation in order to enhance realism of the simulation without loosing real-time performance. The integration of the collision response into the equation system has been sucessfully implemented for simple cases but has to be extended for complex geometric configurations. An important improvement will be inelastic deformations, which are typical for cable. Next steps also include the full integration into our ”DBView” framework and the verification of these methods for the assembly tasks in the design departments.

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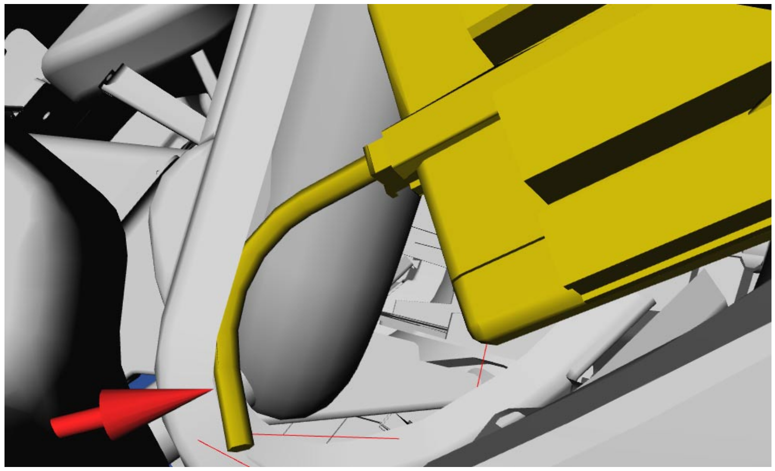

Figure 3. Cable modeled as a rigid geometry collides with the cross girder

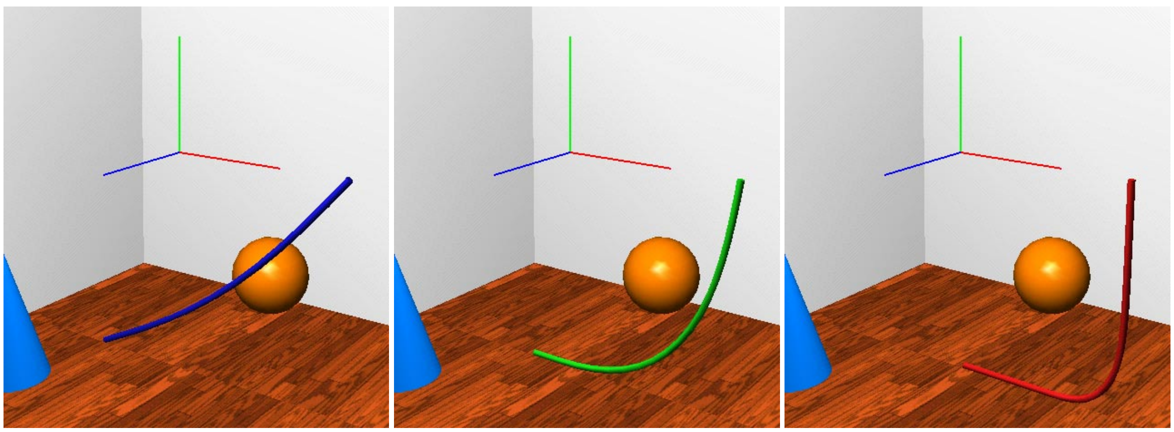

Figure 4. Simulation of cables with different material parameters. The right end is fixed, the left end has been released under gravity.

#segments	h	$\frac{k_L}{m}$	$\frac{k_D}{m}$	$\frac{k_B}{m}$	#rebuilds	simulation
12	0.01s	$6 \cdot 10^3$	6	$3 \cdot 10^3$	10	3ms
20	0.01s	10^4	12	$1.25 \cdot 10^3$	10	5.5ms
42	0.01s	$2.1 \cdot 10^4$	21	$2.625 \cdot 10^2$	10	17ms

Figure 5. Parameters for figure 4, rebuilds means the number of complete recalculation of the equation system within one timestep, performance of simulation on a MIPS R10000/150MHz.