The general circulation of the atmosphere

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Lecture: 2 Stunden pro Woche

Recommended reading


1 Introduction: *Eddies versus zonal symmetry*

The introduction essentially follows chapter 8 of Marshall and Plumb (2008). In particular we distinguish between the tropics and the extratropics: the tropics (and subtropics) can be understood in terms of the zonally averages circulation alone, while in the extratropics the eddies play a dominant role. Midlatitude eddies arise from baroclinic instability, which converts zonal mean available potential energy into eddy kinetic energy.
2 The Hadley Cell: no eddies, but strongly nonlinear

As we shall see later in this course, the eddies play a major role in midlatitudes; in particular, the mean meridional circulation is essentially controlled by the eddies, and the heating is best considered as relaxational in nature responding to the imposed vertical motion. The situation is entirely different in the tropics. In the deep tropics the eddies can be neglected to a first approximation (although in the subtropics they play a subtle role). Observations indicate a thermally direct mean meridional circulation (MMC), called the Hadley cell. In this chapter we try to understand the essence of Hadley cell dynamics. We essentially present the paper of Held and Hou (1980), which draws on ideas formulated by Schneider and Lindzen (1977).

The flow is assumed to be strictly axisymmetric. Viscosity is quantified by $\nu$, and heating is relaxational in character, i.e. $Q = -\alpha(\theta - \theta_e)$ with a specified equilibrium temperature $\theta_e(\phi, z)$. For inviscid flow there is a simple solution, the so-called “thermal equilibrium” (TE) solution; it is characterized by $(v, w) = (0, 0)$ and $\theta = \theta_e$. Requiring thermal wind balance and $u = 0$ at the bottom boundary yields strong upper westerlies above the equator, corresponding to an absolute maximum of angular momentum $m$. This is fine as long as $\nu = 0$, but for $\nu \neq 0$ the TE solution is completely wrong even in the limit $\nu \to 0$. Apparently the limit $\nu \to 0$ is a singular limit. This can be shown through Hide’s theorem, which states that there can be no maximum of $m$ in the interior of the atmosphere for viscous axisymmetric flow (no matter how small $\nu$ may be). In addition, the TE solution is in stark contrast to the observed flow in the tropics.

It turns out that the limit of a viscous solution for $\nu \to 0$ is better represented by the so-called angular momentum conserving (AMC) solution. The latter is characterized by a nonzero meridional circulation $(v, w) \neq (0, 0)$, and by $m$ being conserved in the upper troposphere while air parcels move poleward away from the equator. Held and Hou (1980) found an elegant theory for the AMC solution which allows one to obtain an approximate quasi-analytical solution. Given the angular momentum conserving zonal wind $u_m$ at the top of the troposphere, as well as $u = 0$ at the bottom of the domain, one obtains the vertically averaged temperature $\bar{\theta}_m$ through the thermal wind equation. The meridional circulation is then determined by the difference between $\bar{\theta}_e$ (which is specified) and $\bar{\theta}_m$. In addition it is assumed that the Hadley cell is energetically closed. These assumptions allow one to find the strength of the meridional circulation as well as the width of the Hadley cell. The observations turn out to be much more consistent with the AMC solution than with the TE solution.

The MMC in midlatitudes is controlled by the eddies, and the heating just follows. If in a thought experiment one increases $\theta_e$ locally, this results in some transient behavior; eventually one obtains a new steady state that has the same strength MMC (assuming that the eddies have not changed) but with the temperature changed such that the heating remains unchanged, too. In the tropics the situation is very different. If, again, $\theta_e$ is changed in the thought experiment, now $\theta$ cannot adjust because it is constrained by angular momentum; as a result the MMC has to change. It follows that according to this simple theory the tropical MMC is controlled by the thermal heating.
3 The Hadley cell with off-equatorial heating: strong north-south-asymmetry

The model by Held and Hou (1980) represents equinox or annual mean conditions, corresponding to the maximum of the thermal equilibrium temperature right over the equator. Not surprisingly, the solution is symmetric about the equator, too. On the other hand, observations indicate that the Hadley cell is asymmetric about the equator throughout most of the year with a large and strong winter cell and a weak and small summer cell.

The model by Held and Hou (1980) can be modified in order to account for an off-equatorial maximum of the equilibrium temperature (Lindzen and Hou, 1988). This modified model predicts easterlies above the equator and an asymmetry between the winter and the summer cell with the winter cell both larger and stronger (as observed).

More recent research by Bordoni and Schneider (2008) shows that the Hadley cell is approximately angular momentum conserving during solstice conditions, but this is not true for equinox conditions. It turns out that the abrupt onset of the Asian monsoon can be interpreted as a regime transition between these two conditions; in this context the eddies play an important role in that they are associated with a feedback which leads to such an abrupt transition. This effect can be reproduced in a dry dynamical aquaplanet simulation, suggesting that the land-ocean distribution cannot play an important role for the onset of the monsoon.

4 The Hadley cell with concentrated heating: subtropical radiation becoming the key player

Observations indicate, that the net heating and associated upwelling within the Hadley cell is limited to a narrow strip, while radiative cooling and associated downwelling takes place over large areas. The model by Held and Hou can be modified in order to accommodate for this observed “concentrated heating”. Hou and Lindzen (1992) introduced some ad-hoc method in order to redistribute the heating such as to make it concentrated on the equator. They find that this leads to a strengthening of the Hadley cell. Another, arguably more physical method, would be to use two different values for the relaxation constant in the Newtonian heating Ansatz: a large value $\alpha_c$ for the convective regions, and a smaller values $\alpha_r$ for the radiative regions; the relevant region is determined by the sign of the vertical wind ($w > 0$ for convective region, $w < 0$ for radiative region). This replaces the “equal area argument” from Held and Hou by a “weighted area argument”.

In the limit of $\alpha_c \gg \alpha_r$, the strength of the Hadley cell saturates at a finite value and is determined exclusively by $\alpha_r$, i.e. by radiative processes in the subtropics. Convection than simply “follows” and the rising branch of the Hadley cell contains simply all mass transport which is “driven” by subtropical radiative cooling.

The latter logic is implicitly contained in a very smart asymptotic model for the Hadley cell, which was designed by Fang and Tung (1996). In the spirit of convective quasi-equilibrium, the authors explicitly set $T = T_e$ at the location of the convection, which is assumed to be an infinitely thin strip. In contrast with Held and Hou’s model (and its variants) the model
by Fang and Tung does not average in the vertical direction, but rather accounts explicitly for vertical dependence. Angular momentum conservation inside the Hadley cell then implies that there cannot be any horizontal temperature gradient inside the Hadley cell. Downwelling in the subtropical part of the Hadley cell determines its mass flux, and the heating is made just large enough as to accomodate mass continuity.

5 The moist Hadley cell: Moisture renders the theory for the Hadley cell strength tricky

Transition from a dry to a moist theory of the Hadley cell requires to account for latent heat release in addition to TOA radiative energy fluxes. It turns out that the width of the Hadley cell is essentially constrained by the TOA radiative energy fluxes; on the other hand, the strength of the Hadley cell is much less constrained.

A vertically integrated budget of the dry static energy requires the knowledge of the vertically integrated heating; the latter includes the effect of latent heat release, which in turn depends on the strength of the circulation. An alternative option is to consider the vertically integrated budget of moist static energy (following Neelin and Held 1987). In this case the latent heat release drops out of the net heating on the right hand side (which is desirable). On the other hand, moist static energy is almost constant with altitude such that, again, the strength of the Hadley cell is not well constrained by this argument either.

6 Surface wind and vorticity mixing: Why are there surface westerlies in midlatitudes?

In this chapter we try to understand the zonal mean circulation on a zonally symmetric Earth-like planet. We analyze the budget of angular momentum $m$. The observed distribution of surface winds requires a net $m$-flux from the tropics to the mid-latitudes. It turns out that this flux is essentially performed by upper tropospheric eddies.

Consider a thin layer of a barotropic fluid (meant to represent the upper troposphere). Using Stokes theorem and Kelvin’s circulation theorem, it can be shown that a stirred region sucks angular momentum into itself, at the expense of angular momentum in the neighboring latitudes. In a (statistically) steady state the $m$-gain must be compensated by an equal amount of $m$-loss. The only way this can happen is transfer of $m$ to the surface (to be explained later) and $m$-loss at the surface. This requires surface westerlies! Adding westerly shear with altitude, this explains the westerly jet in the upper troposphere.

The stirring mechanism is baroclinic instability. The exact location where one gets eddy activity and $m$-flux convergence depends on the relation between the size of the eddies and the size of the planet. On big enough a planet (e.g. Jupiter) one obtains several regions of eddy activity (i.e. several jets). On Earth the relation just happens to be such as to obtain one extratropical jet.
7  A linear perspective on eddy momentum fluxes: what quantity measures “wavieness” and wave mean flow interaction?

The argument from the previous chapter was rather qualitative. In this chapter we aim to get more quantitative insight based on linear Rossby wave theory. Analysis of the Rossby wave dispersion relation shows that the meridional group velocity and the meridional momentum flux have opposite sign, i.e. a stirred region (from which Rossby waves propagate away meridianally) sucks in angular momentum. Signature of a meridional flux of angular momentum is a phase tilt with latitude. Diagnosing the real world eddy activity indicates that important properties of (linear) Rossby waves seem to hold, although real eddies are not necessarily nor exclusively linear Rossby waves. The Taylor identity makes a connection between the meridional vorticity flux and the convergence of momentum flux; these are the eddy forcing for the mean flow $\bar{u}$. A natural measure for the strength of a wave is “wave activity” or wave “pseudo momentum” $P$. $P$ is globally conserved, and there is a local relation $\bar{u} + P = const$. Wave energy, on the other hand, is not globally conserved, i.e. there may be a net transfer of energy from the wave to the mean flow even in a globally integrated sense.

8  A shallow water model: the secret of the Ferrell cell

From a conventional Eulerian perspective the Ferrell cell is a thermally indirect cell, i.e. some mechanical forcing must set up this cell against the forcing from buoyancy (the latter would suggest a cell with reverse direction). The mechanical forcing is given by the momentum flux convergence discussed in the previous two chapters. The momentum which is converged in the upper tropospheric jet region is brought down to the surface through the mean meridional circulation $\bar{v}$. However, it turns out the the mass flux in the Ferrell cell is the opposite to the mean meridional flow $\bar{v}$. How can this be possible?

A simple shallow water model helps to understand this strange fact. The important new ingredient is the fact that the layer depth $H$ may vary, and now the meridional mass flux is given by $Hv$. Shallow water PV and its geostrophic counter part are materially conserved (for conservative flow). In the shallow water model, the zonal mean forcing includes both the convergence of the momentum flux and the Coriolis term; the latter may now be nonzero because the physical constraint is $\bar{v}H = 0$, which still allows $\bar{v} \neq 0$. In this simple model the Coriolis terms is equal to a term proportional to the eddy mass flux; the latter can be shown to be equivalent to the form drag which the fluid exerts on the orography. The steady state (upper tropospheric) momentum balance is thus a balance between acceleration from eddy momentum flux convergence and deceleration from form drag. Eddy PV flux is a sum of eddy vorticity flux and eddy mass flux. In steady state $q'v' = 0$ implies $\bar{v} < 0$ above the orography. That’s the secret of the Ferrell cell; at the same time the mass flux $\bar{v}H = 0$. A residual circulation $\bar{v}^*$ is defined which has units of $m/s$ but at the same time is proportional to the mass flux. The equation for mean momentum can be formulated using either $\bar{v}$ or $\bar{v}^*$. 5
9 A two layer model: *communication between upper and lower troposphere*

A simple two-layer model is introduced. It essentially represents two isentropic layers in midlatitudes, one for the upper troposphere and one for the lower troposphere. Important new constraints involve the form drag that one layer exerts on the other layer, as well as a generalization regarding the meridional integral of the eddy PV-fluxes (involving now the vertical integral, too).

Baroclinic eddies are an important ingredient in the causal chain. Growing conservative eddies must be downgradient the mean PV gradient, locally for linear eddies and at least in a meridionally integrated sense for nonlinear eddies. This plus the constraint on PV fluxes requires a change in sign of mean PV gradient (Charney-Stern). When this sign change stems from thickness gradients, the associated instability is called baroclinic instability. The resulting sign of the eddy PV fluxes implies a residual circulation which is opposite in direction to the classical Eulerian mean Ferrell cell. Key to obtaining the residual circulation from data is to consider the mass flux (rather than the volume flux, i.e. one has to include mass weighting when performing the zonal averages). If this analysis is done in isentropic coordinates one obtains two well separated direct cells for the mass circulation, one in tropics and one in midlatitudes.

The argument for surface westerlies is revisited in the two-layer model. Now the meridional mass flux must be zero only in a vertically integrated sense (physical constraint). This implies a relation saying that the local surface winds are given by the sum of mass-weighted eddy-PV-fluxes above. This, in turn, implies that the eddy PV fluxes must be more spread out in the upper troposphere than in the lower troposphere. This is consistent with the fact that eddies in the upper troposphere are more linear and can propagate more readily away from the source region. The larger amount of linearity of the eddies in the upper layer is related to the fact that the steering level is located in the lower troposphere and \((\bar{u} - c)\) is larger in the upper layer than in the lower layer (note that that \(u'/\bar{u} - c\) is a measure for the linearity of the waves). This interpretation is corroborated by idealized simulations showing an almost linear behavior of the waves in the jet core and wave breaking only south and north of the jet core.

In summary, baroclinic instability can be thought of as a two-step process: First the instability itself transfers momentum from the upper to the lower troposphere. Upper tropospheric momentum is replenished from the subtropics through Rossby waves. In terms of heat budget, the (weak) radiative heating and cooling increases the instability until eddy motions are triggered, and the result of the ensuing dynamics is that the radiative effect is compensated (through a direct residual circulation).

Obviously the two-layer model is but a rough approximation to the continuously stratified real atmosphere. A better approximation would be a model with a large number of isentropic layer stacked on top of each other. Those layers which do not intersect the bottom boundary can be dealt with like the upper layer in the 2-layer model, resulting in equatorward eddy PV flux and poleward mass flux. On the other hand, those layer which intersect the bottom boundary (so-called “interrupted layers”) require special attention, because PV cannot be defined where the layer is infinitesimally thin; therefore, the analogy to the 2-layer model becomes moot. It turns out that the lower layer in the 2-layer model effectively represents
those interrupted layers in the continuous model, and mixing of surface potential temperature generates an equatorward mass flux.

Literature


