Introduction to Artificial Intelligence

10: Fuzzy Logic

Luca Doria, KPH Mainz

Introduction

- Until now: Propositional Logic and Predicate Logic (PL1)
- Atomic sentences can be: True XOR False.
- Fuzzy logic: assume a “graded” transition between True and False
- Fuzzy sets and Fuzzy Logic
- Reasoning with Fuzzy Logic

Motivation

- What about sentences like: “Jack is very tall”?
- How can we feed this information into a computer without specifying the hight exactly?
- A real life sentence like: “It takes a lot of work to get a CS M.Sci” is fine.
- How do we translate for a computer “very tall” and “a lot of work”?

A new set theory: fuzzy sets

L. Zadeh (1965): new way of looking at the old notions of: set, containment, and subset. His goal was to describe more ”vague” concepts.

Same example as before:

Jack is very tall.

If we know that Jack is 1.65m tall, we could be not so sure about the truth of the sentence.

Classically, we have to decide if Jack is in the set of the tall people or not.

In the fuzzy set theory we can express to what degree 1.65m makes Jack tall.

Membership Functions

We can express the membership of an element x to a set A through the membership function

$$\chi_A(x) = \begin{cases} 1 & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases}$$

This function is called crisp or definite.

We further define a fuzzy membership function

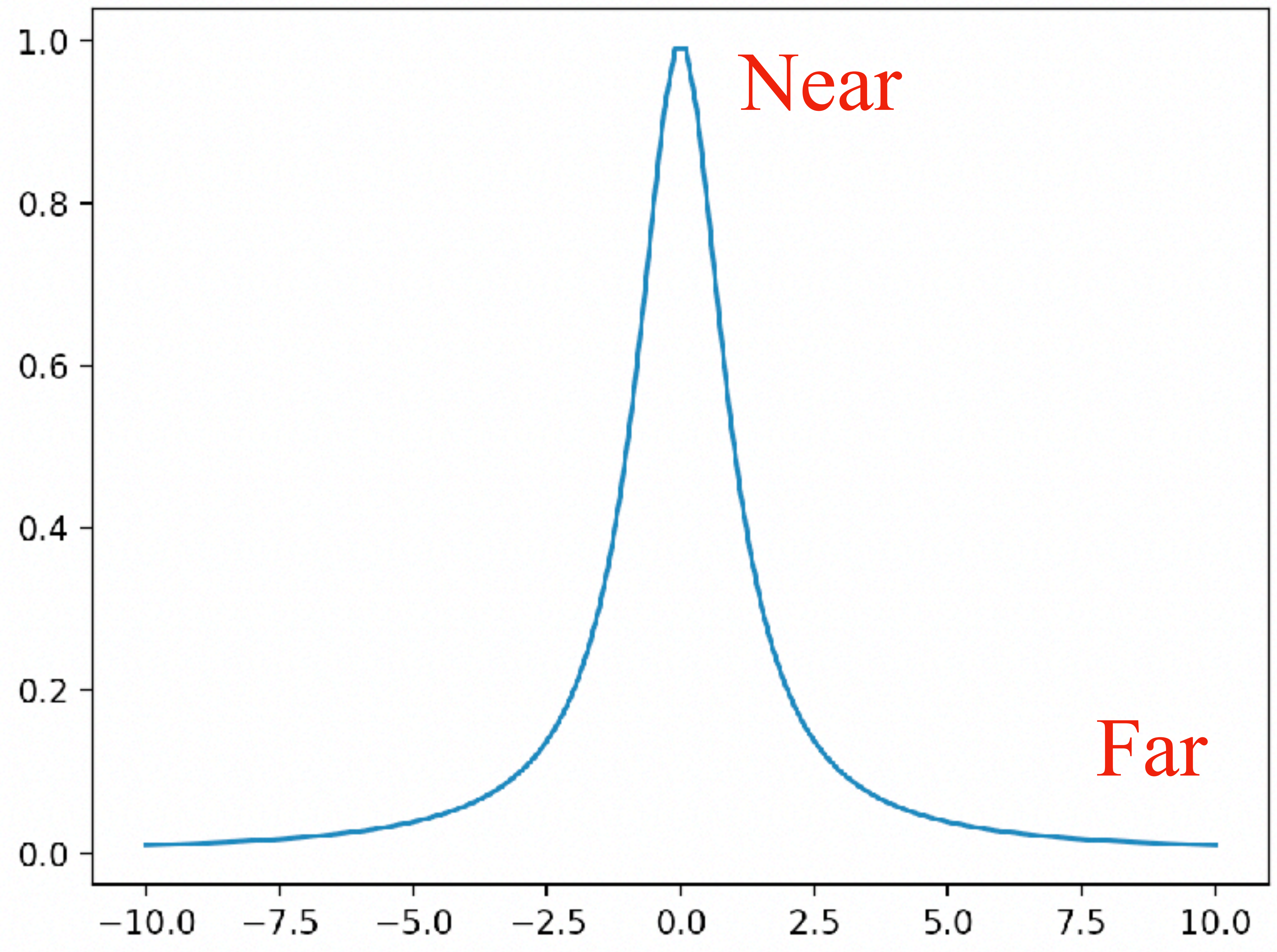
$$\mu_A : X \to [0, 1]$$

If $\exists$x : $0 < \mu_A(x) < 1$, then the set A is a fuzzy set.

Membership Functions

An example provided already from Zadeh is the following function which summarises the concept: “a real number near zero”:

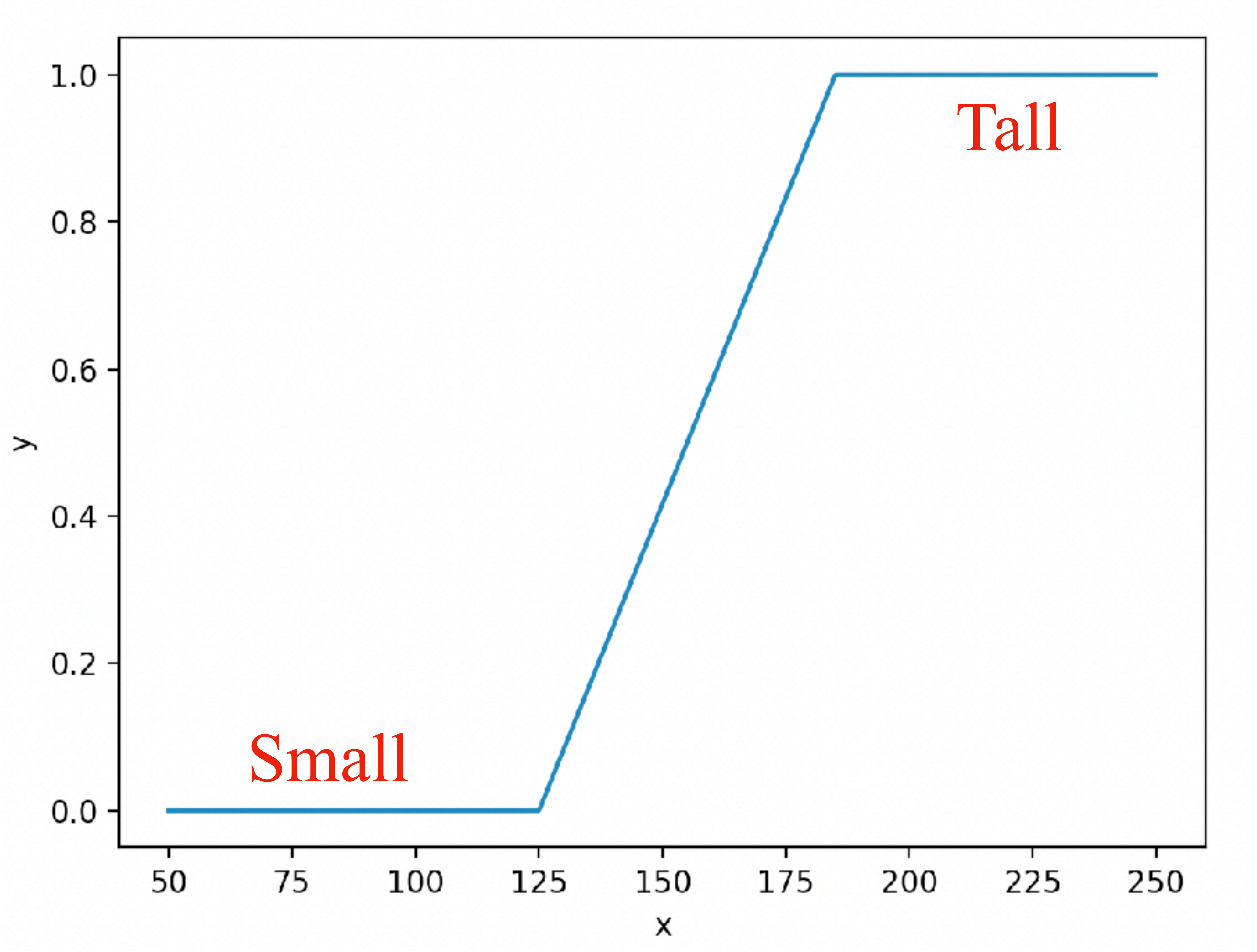
$$\mu(x) = \frac{1}{1+x^2}$$



Membership Functions

A possible fuzzy function for the previous "height problem".

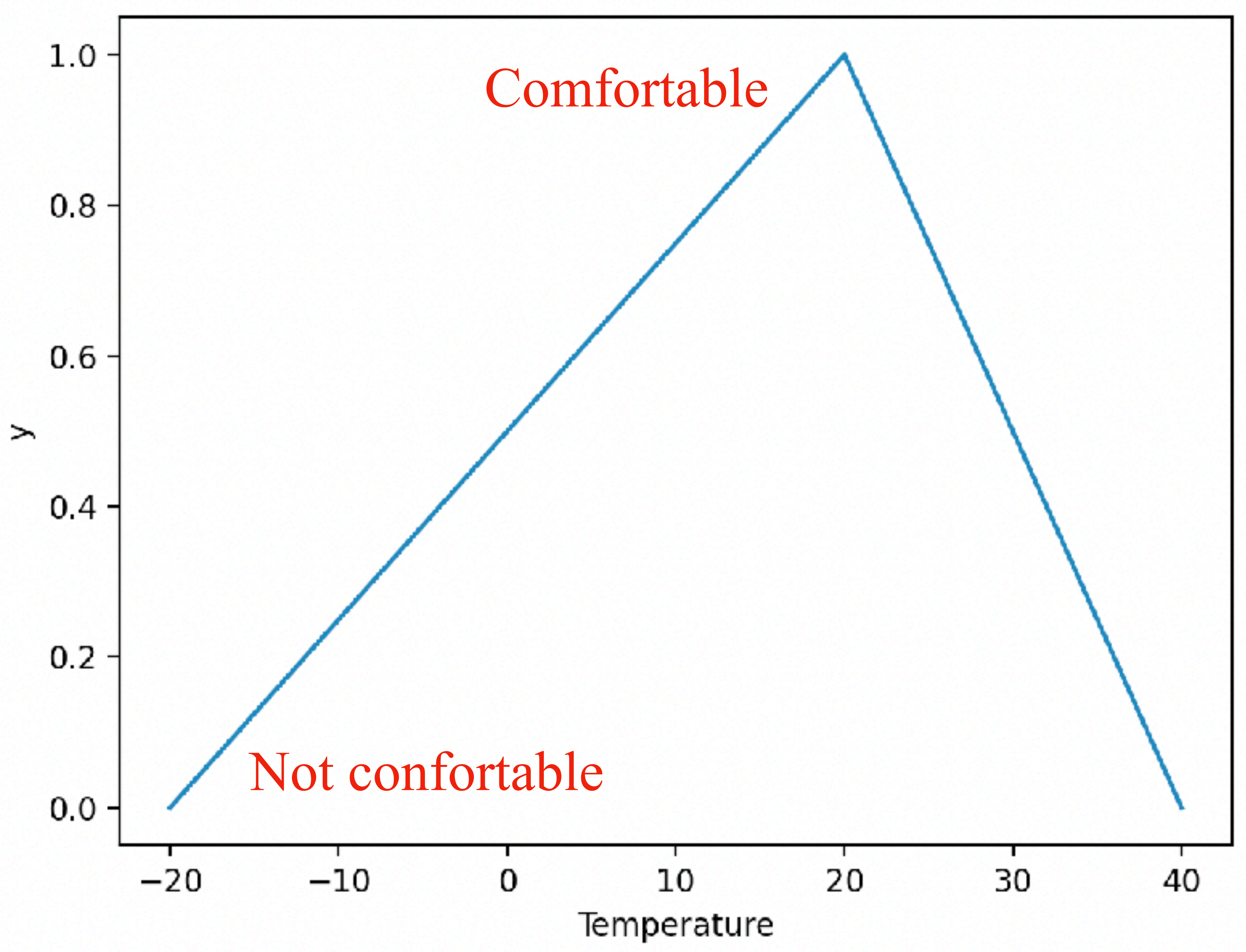
$$\mu(x) = \begin{cases} 0 & x \leq 125 \\ 1 & x \geq 185 \\ \frac{x-125}{185-125} & 125 \leq x \leq 185 \end{cases}$$



Membership Functions

A possible fuzzy function for the temperature comfort:

$$\mu(x) = \begin{cases} \frac{x-20}{40} + 1 & -20 \leq x \leq 20 \\ -2\frac{x-20}{40} + 1 & 20 \leq x \leq 40 \end{cases}$$



Height

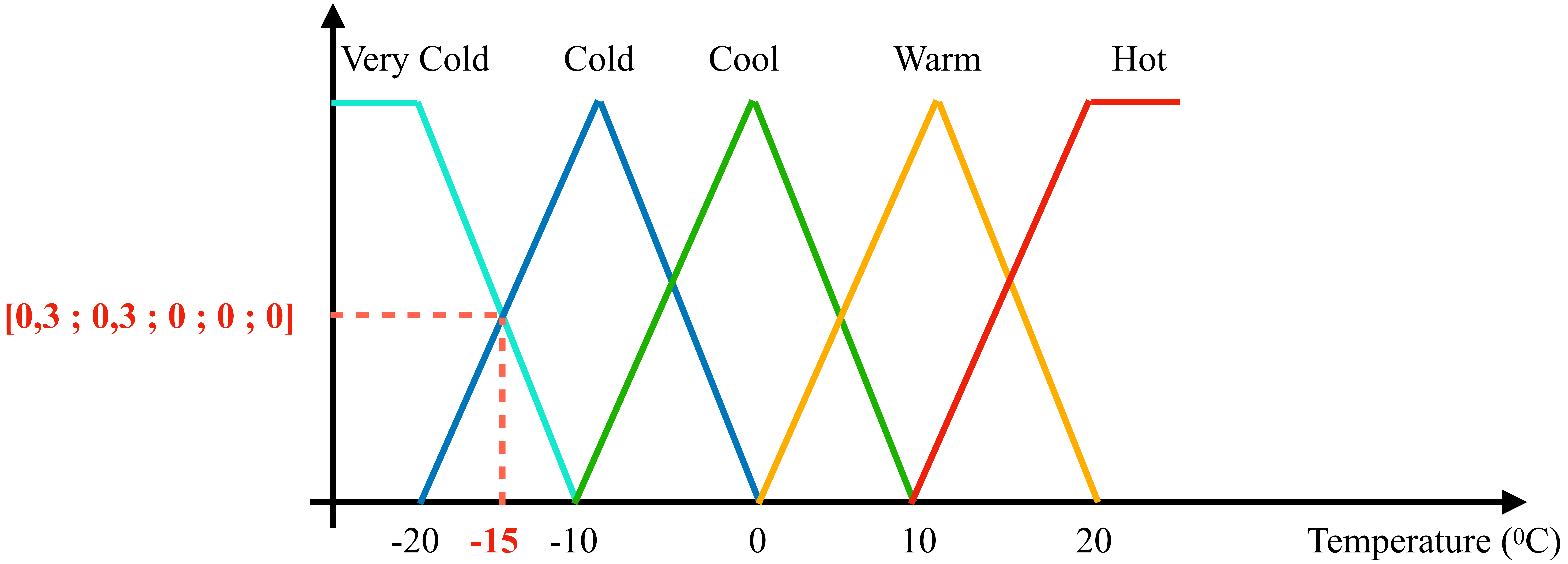
The height of a fuzzy set is defined as

$$H(F) = \max\{\mu(x) : x \in X\}$$

For uniformity with standard logic, we usually assume H=1, like in the previous examples.

Attaching words to functions

We can attach significance (words) to different functions (5 in this case)



Example of Inference

How to translate the modus ponens in fuzzy logic? $\frac{P \Rightarrow Q, P}{Q}$

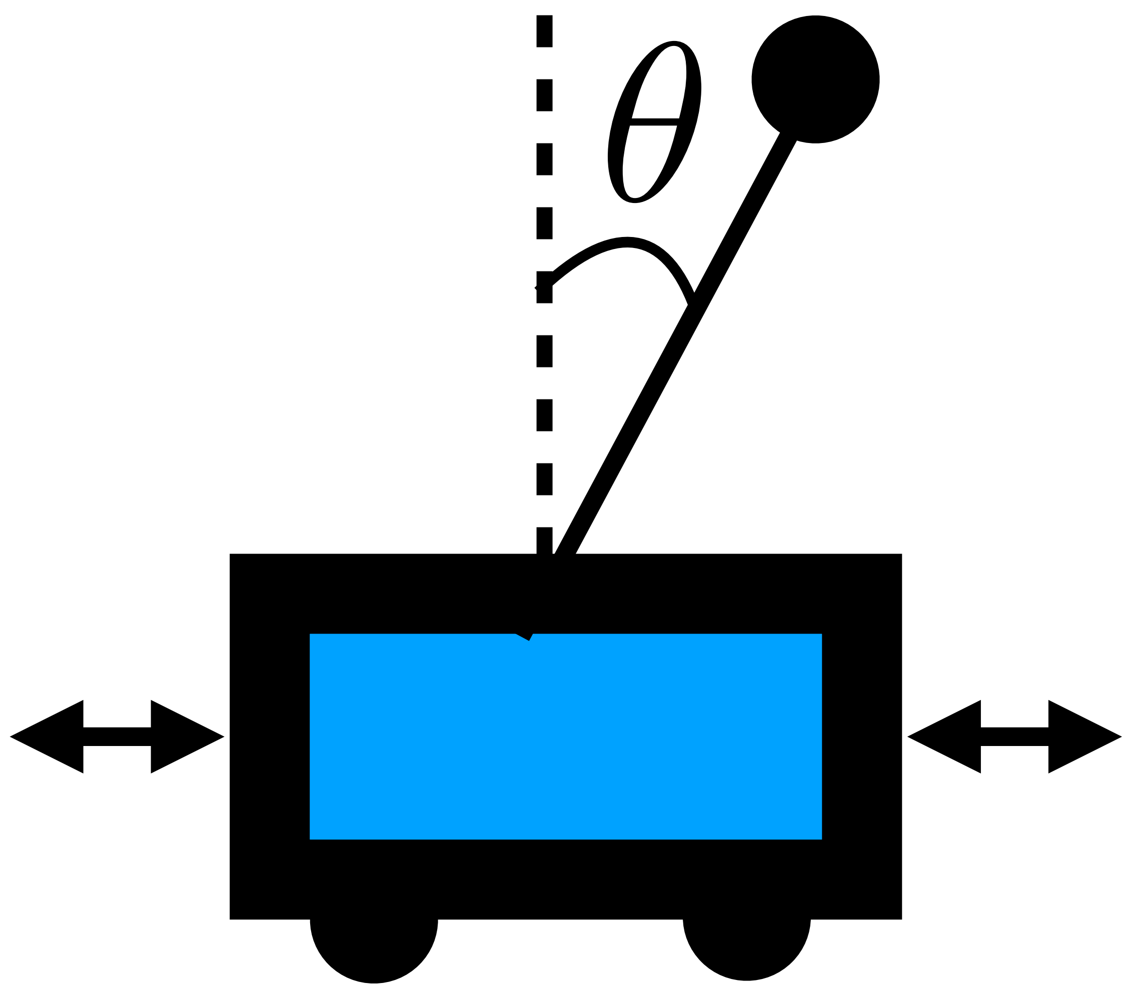
Let's take as example: "*If the temperature is high, then the pressure is high*".
Supposing that T=25

1) Fuzzification: $\mu_T(25) = 0.8$
2) Apply the implication: $\mu_P(X) = 0.8$
3) De-fuzzification: find a value for the pressure (note that μ_P is not in general invertible)

De-fuzzification can be done in many ways. One of the most used:

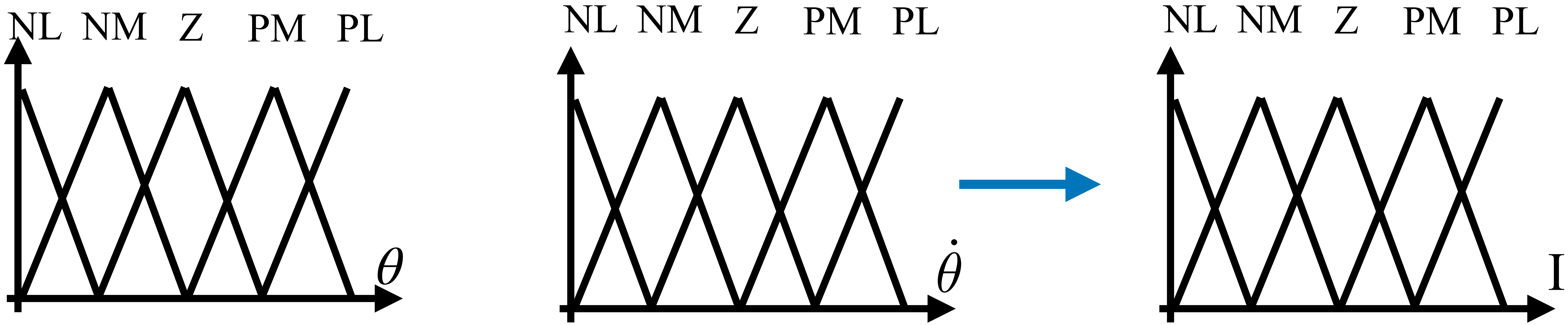
Centroid method: $P = \frac{\int \mu(x) x dx}{\int \mu(x) dx}$

Application: Systems Control



An unstable pendulum can be equilibrated with a moving cart which is controlled with an electric current.

Two detected variables: θ and $\omega = \dot{\theta}$
Control variable: current I.



NL: negative large , NM: negative medium, Z: zero, PM: positive medium, PL: positive large

Application: Systems Control. Rules Definition

$\dot{\theta}$ \ θ	NL	NM	Z	PM	PL
NL			NL		
NM			NM	NM	
Z	PL	PM	Z	NM	NL
PM		PM	PM		
PL			PL		

Table of rules designed by expert/experience.

Example:
If θ is NM and $\dot{\theta}$ is Z then the current is PM

Step by Step Procedure

1. Control variables θ, ω measurement: $\bar{\theta}, \bar{\omega}$
2. Fuzzyfication: $\mu_\theta(\bar{\theta}) = a \;\; ; \mu_\omega(\bar{\omega}) = b$

   The function is multidimensional and we obtain different results, e.g.:

   $\text{NL}(\bar{\theta}) = 0$

   $\text{NM}(\bar{\theta}) = 0$

   $\text{Z}(\bar{\theta}) = 0$

   $\text{PM}(\bar{\theta}) = 0.8$

   $\text{PL}(\bar{\theta}) = 0.1.$ and similarly for ω .

"min" represents the AND operation between the two variables. Other choices are possible.

Boolean	Fuzzy
AND(x,y)	MIN(x,y)
OR(x,y)	MAX(x,y)
NOT(x)	1 – x

3. Evaluate the rule table (see slide before) calculating: min(F1, F2)
4. Aggregation: take all the non-zero values of 3.
5. De-fuzzyfy with the current function I and e.g. the centroid method.

Numerical Example (1)

Inputs ("triangular" functions)

Angle θ

- Negative Large (NL): Peak at -30, zero at -15 and -45
- Negative Small (NS): Peak at -15, zero at -30 and 0
- Zero (Z): Peak at 0, zero at -15 and 15
- Positive Small (PS): Peak at 15, zero at 0 and 30
- Positive Large (PL): Peak at 30, zero at 15 and 45

Angular Velocity $\dot{\theta}$

- Negative Large (NL): Peak at -10, zero at -5 and -15
- Negative Small (NS): Peak at -5, zero at -10 and 0
- Zero (Z): Peak at 0, zero at -5 and 5
- Positive Small (PS): Peak at 5, zero at 0 and 10
- Positive Large (PL): Peak at 10, zero at 5 and 15

Outputs

Current to be applied to the electric motor

- Negative Large (NL): Peak at -20, zero at -10 and -30
- Negative Small (NS): Peak at -10, zero at -20 and 0
- Zero (Z): Peak at 0, zero at -10 and 10
- Positive Small (PS): Peak at 10, zero at 0 and 20
- Positive Large (PL): Peak at 20, zero at 10 and 30

Numerical Example (2)

Fuzzy Rules definitions (similar to the matrix seen before)

R1: if θ is PL and $\dot{\theta}$ is PL then I is NL

R2: if θ is PL and $\dot{\theta}$ is Z then I is NL

R3: if θ is PL and $\dot{\theta}$ is NS then I is NS

R4: if θ is Z and $\dot{\theta}$ is PL then I is NL

R5: if θ is Z and $\dot{\theta}$ is Z then I is Z

R6: if θ is Z and $\dot{\theta}$ is NL then I is PL

Numerical Example (3)

Fuzzyfication

The sensors measure: $\theta = 20$ (deg) and $\dot{\theta} = -3$ (deg/s)

Inserting the values in the previous functions, the non-zero values are:

$\mu_{PL,\theta}(20) = 0.3$

$\mu_{PS,\theta}(20) = 0.7$

$\mu_{NS,\dot{\theta}}(-3) = 0.6$

$\mu_{Z,\dot{\theta}}(-3) = 0.4$

Numerical Example (4)

Rule Evaluation

Evaluate each rule with the minimum operator (AND) and aggregate using the maximum operator (OR):

R1: $\min(\mu_{PL}(20), \mu_{PL}(-3)) = 0$

R2: $\min(\mu_{PL}(20), \mu_{Z}(-3)) = \min(0.3, 0.4) = 0.3$ ← LN current input

R3: $\min(\mu_{PL}(20), \mu_{NS}(-3)) = \min(0.3, 0.6) = 0.3$ ← NS current input

R4: $\min(\mu_{Z}(20), \mu_{PL}(-3)) = 0$

R5: $\min(\mu_{Z}(20), \mu_{Z}(-3)) = 0$

R6: $\min(\mu_{Z}(20), \mu_{NL}(-3)) = 0$

Numerical Example (5)

Defuzzyfuction (with the Centroid Rule)

For LN (current) the centroid is -30
For LS (current) the centroid is -10

Current to be applied:

$$I = \frac{(0.3 \times -30) + (0.3 \times -10)}{0.3 + 0.3} \approx -20$$

Some real applications

Automotive Systems:

- **Automatic Transmission Control:** Fuzzy logic is used in automatic transmissions to determine the optimal gear shift points based on inputs like throttle position, and speed.
- **Anti-lock Braking Systems (ABS):** Fuzzy logic controllers help in regulating the braking force.

Home Appliances:

- **Washing Machines:** Fuzzy logic is used to optimize washing cycles based on the load size, fabric type, and dirt level.
- **Air Conditioners:** Fuzzy logic controllers in air conditioners adjust the cooling or heating output based on factors such as room temperature and humidity.
- **Microwave Ovens:** Fuzzy logic is used to determine cooking times and power levels based on the type and quantity of food.
- **Refrigerators:** Fuzzy logic helps in maintaining optimal temperature and humidity levels inside the refrigerator by adjusting the compressor and fan speeds.

Industrial Automation:

- **Process Control:** Fuzzy logic controllers are used in various industrial processes such as chemical manufacturing, water treatment, and oil refining to maintain desired levels of temperature, pressure, flow rate, etc.
- **Robotics:** Fuzzy logic is used in robotic systems for tasks like path planning, obstacle avoidance, and decision making.

Healthcare:

- **Medical Diagnosis:** Fuzzy logic is applied in diagnostic systems to interpret complex medical data and assist in diagnosing diseases.
- **Prosthetics Control:** Fuzzy logic is used to control prosthetic limbs, allowing for smoother and more natural movements.

Transportation:

- **Train Control Systems:** Fuzzy logic is used in automatic train operation systems to control acceleration and braking.
- **Traffic Management:** Fuzzy logic is applied in traffic signal control systems to optimize the flow of traffic based on real-time data.

Financial Services:

- **Credit Scoring:** Fuzzy logic is used in credit scoring systems to evaluate the creditworthiness of individuals and businesses based on various financial indicators.
- **Investment Decision Making:** Fuzzy logic is applied in investment management to assess market conditions and make portfolio allocation decisions.

Summary

- Fuzzy logic successfully applied to many problems (in particular controlling).
- Try to model uncertainty of the real world.
- It is not probabilistic.
- Characterised by the membership function.
- Examples seen:
    - Inference.
    - Control problem.