# Introduction to Artificial Intelligence

## 3: Solving by Searching

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# Problem-Solving Agents

- Agents can better solve problems by planning ahead.
- Planning might imply looking for possible solutions —> **Search Problem**.
- A search can be informed or uninformed.

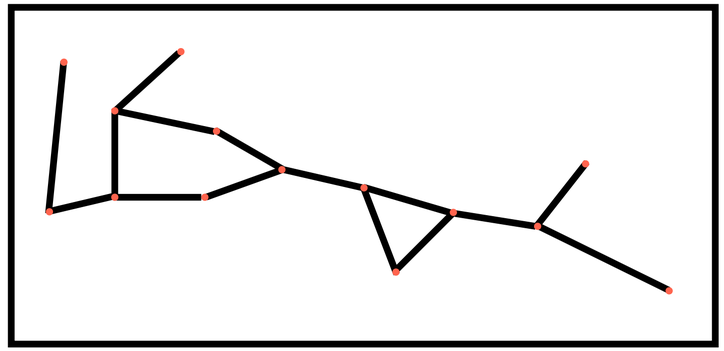
Goal-based agents:

- problem described as a state-space and goal(s).
- Goal can be reaching a specific state thought appropriate actions.
- Search a specific action sequence and execute the actions.

# A Search Problem

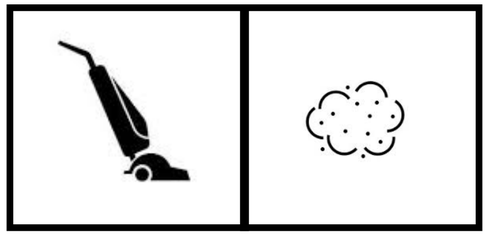
# Problem Specification

**Specifications for the Canada Trip case:**



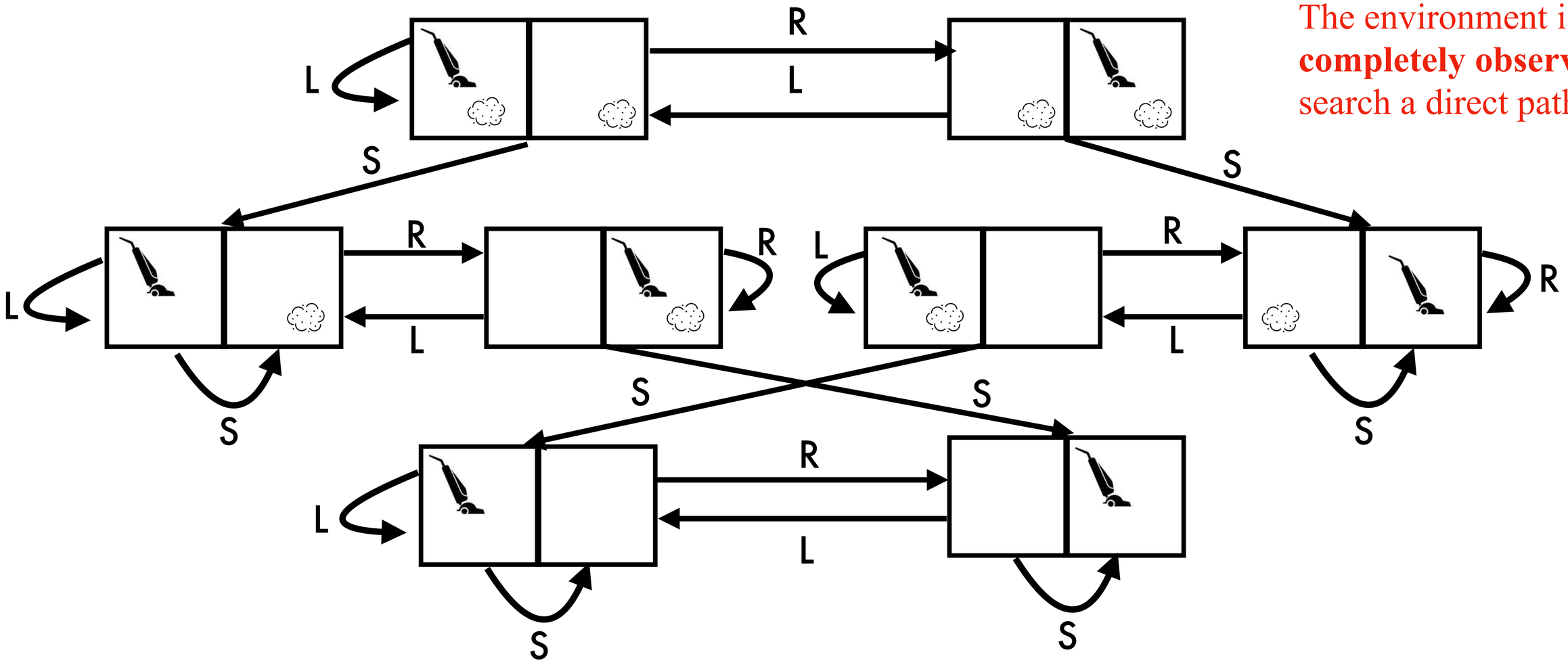
A set of states (the cities and their distances)
The initial state (initial city)
A set of goal states (final city, final city with shorter trip, …)
Actions: ACTION(Ottawa) = {toSudbury, toToronto, toMontreal}
A transition model: RESULT(state,action)=state —> e.g.: RESULT(Ottawa,toMontreal)=Montreal
An action-cost function ACTION-COST(state, action,state)—>
e.g. ACTION-COST(Ottawa,toMontreal,Montreal)=200

**Specifications for the "vacuum-cleaner problem":**



Set of states: Where is the VC, is(are) the square(s) dirty?
Initial state, Set of goal states: all the squares must be clean.
Actions: MoveDX, MoveDS, on, off
Action cost: e.g. 1.

# The State-Space Graph




The environment is **completely observable**: search a direct path.

8 states = (2 cells)x(L dirty?)x(R dirty?) = 2x2x2. For an n-cells problem: $nx2^n$ states.
The state space can become quickly very large!

# Another Example

Given by D. Knuth (1964)

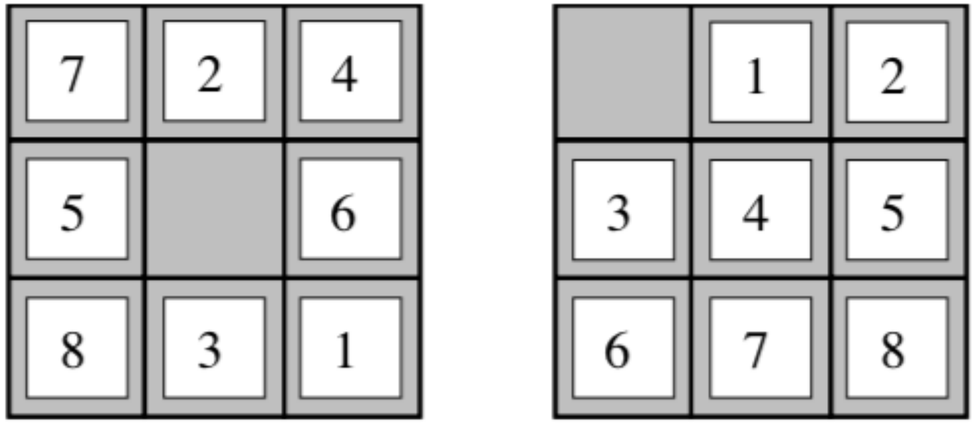
The state-space can be even infinite!

Problem: starting from the integer number 4, reach a certain integer x using the `factorial`, `floor`, and `square_root` operations only.

Initial state: 4.
State space: all the positive real numbers.
Actions: Apply sqrt(), floor(), factorial().
Transition model: as defined by the allowed mathematical operations.
Goal: x.
Action cost: 1.

Knuth's example: starting from 4, the shortest path to 5 is: $\text{floor}\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}}\right) = 5$ (!)

# Another Example: the 8-Puzzle




- States: 8 tiles + blank square
- Initial state: initial configuration of the puzzle
- Actions: moving the blank N,S,E,W
- Goal test: does the present configuration match the “Goal State”?
- Cost function: 1 unit (equal to the move length)

# Notation

-Node expansion: generating all the successor nodes considering the available actions.

-Frontier: set of all nodes available for expansion.

-Search Strategy: defines which node is expanded next.

-Tree-based Search: some states can repeat, leading to infinite loops. To avoid this, graph searches keep a set of already visited nodes (the “explored set”).

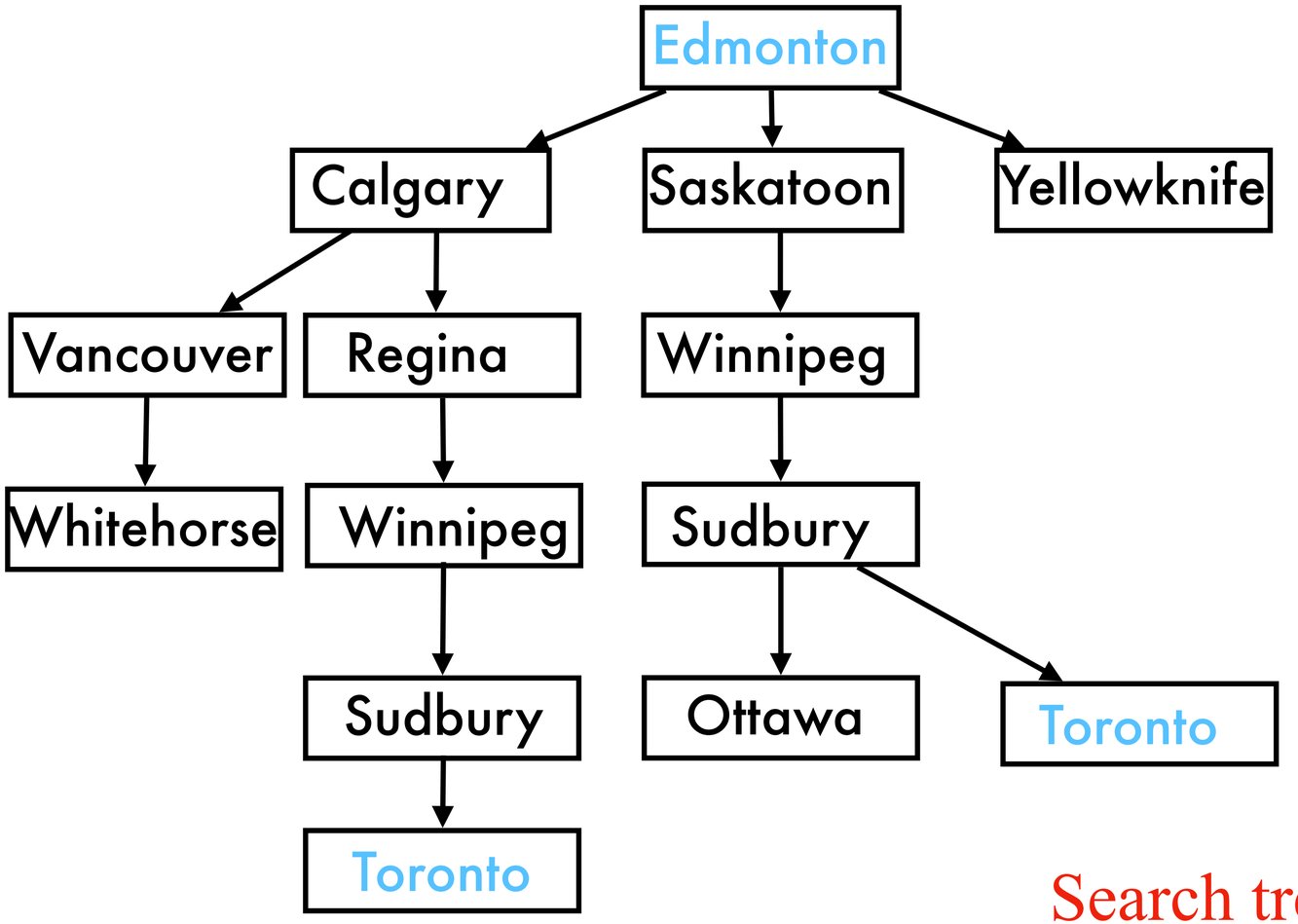
# Searches on trees and graphs

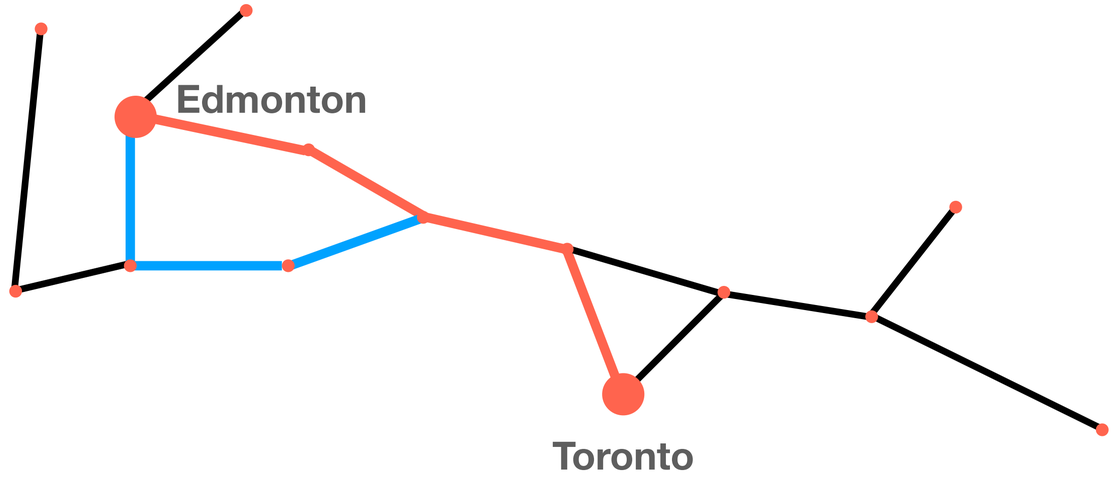
**function** TREE-SEARCH(problem) **returns** a solution or failure
initialise the frontier using the initial state of the problem
**loop do**:
**if** the frontier is empty, **then** failure
choose a leaf node and remove it from the frontier
**if** the node contains a goal state, **then return** corresponding solution.
expand the chosen node, adding the resulting nodes to the frontier.

**function** GRAPH-SEARCH(problem) **returns** a solution or failure
initialise the frontier using the initial state of the problem
initialise the explored set to be empty
**loop do**:
**if** the frontier is empty, **then** failure
choose a leaf node and remove it from the frontier
**if** the node contains a goal state, **then return** corresponding solution.
add the node to the explored set
expand the chosen node, adding the resulting nodes to the
frontier only if not in the frontier or explored set.

# Searches on trees and graphs

A search tree can be superimposed to the state space.
Example from the Canada trip: a journey from Edmonton to Toronto:




Search tree and state space are two different things!

# Characterization of Search Strategies

-Completeness: Is it guaranteed that the strategy will find a solution (if it exists)?
-Time Complexity: How long does it take for finding the solution?
-Space Complexity: How much memory does the search strategy require?
-Optimality: Does the strategy find the “best” solution (with lowest path cost)?

-Problem describing quantities:
- **b**: branching factor
- **d**: depth of the shallowest goal node
- **m**: maximum length of any path in the state space.

# Uninformed Search Strategies

How to navigate the state space and find the appropriate action?

The simples strategy is a through search.

Uninformed (or blind) search strategies have no idea about where the target might be and their only chance of finding it is to perform an exhaustive search.

Examples: breadth-first, depth-first, inform cost search, bidirectional search, iterative-deepening search,.. .

In contrast with informed or heuristic approaches.

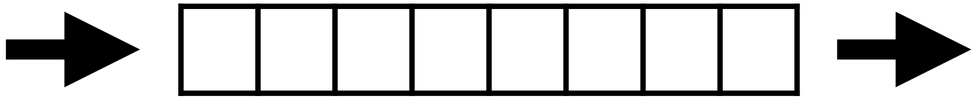
# Uninformed Search Strategies

Two basic uninformed algorithms for traversing (searching) trees and graphs:

- Breadth-first search: In BFS, start from a node and explore all of its neighbours at the current depth level before moving on to the nodes at the next depth level.
- Depth-first search: DFS explores the tree/graph as deep as possible before backtracking.

Based on two basic data structures:

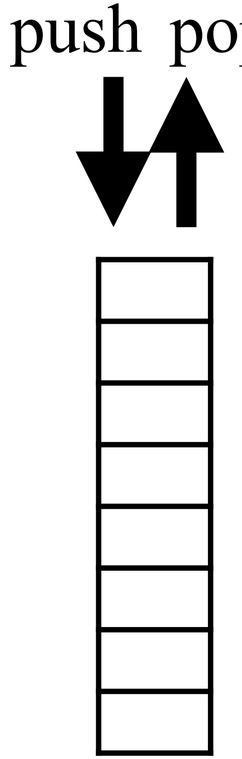
Queue (FIFO)



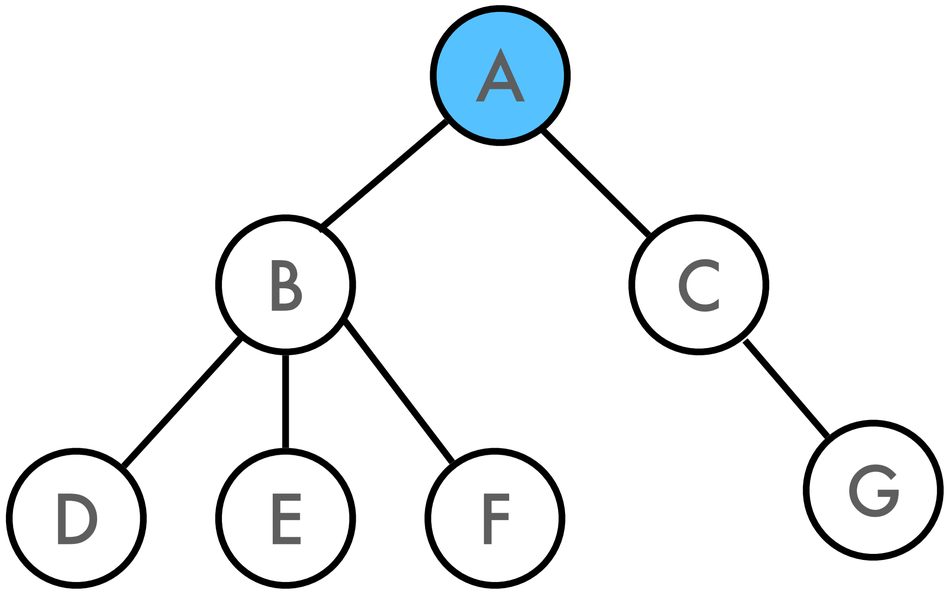
enqueue (push)

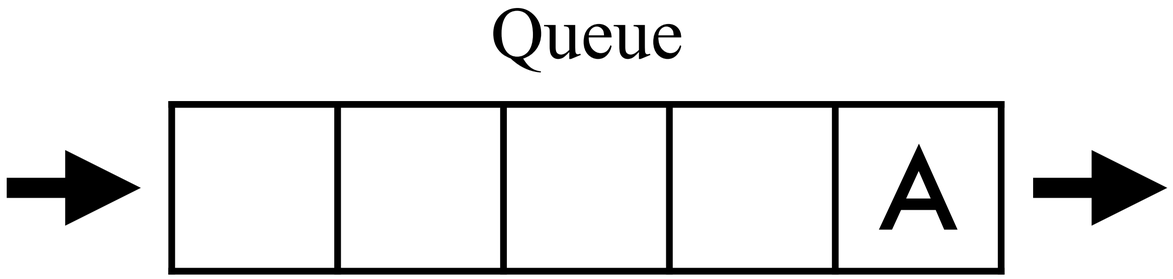
dequeue (pop)

Stack (FILO)



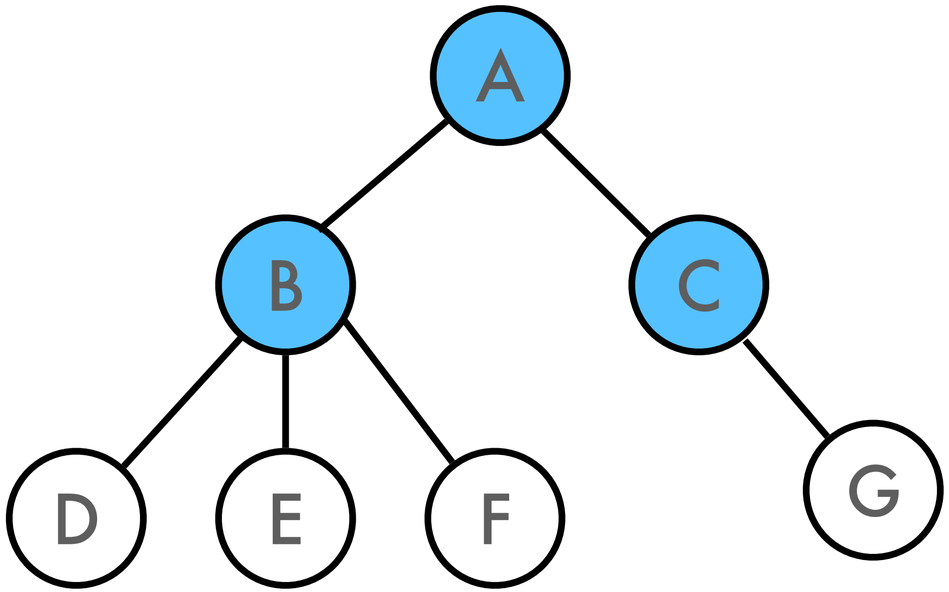
# Breadth-First Search

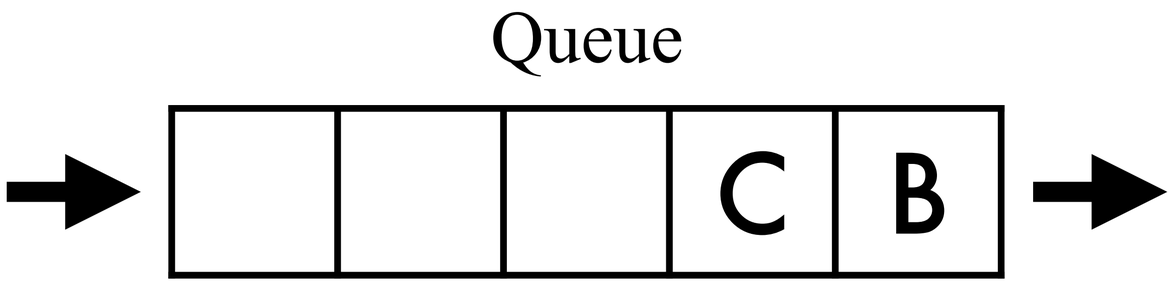


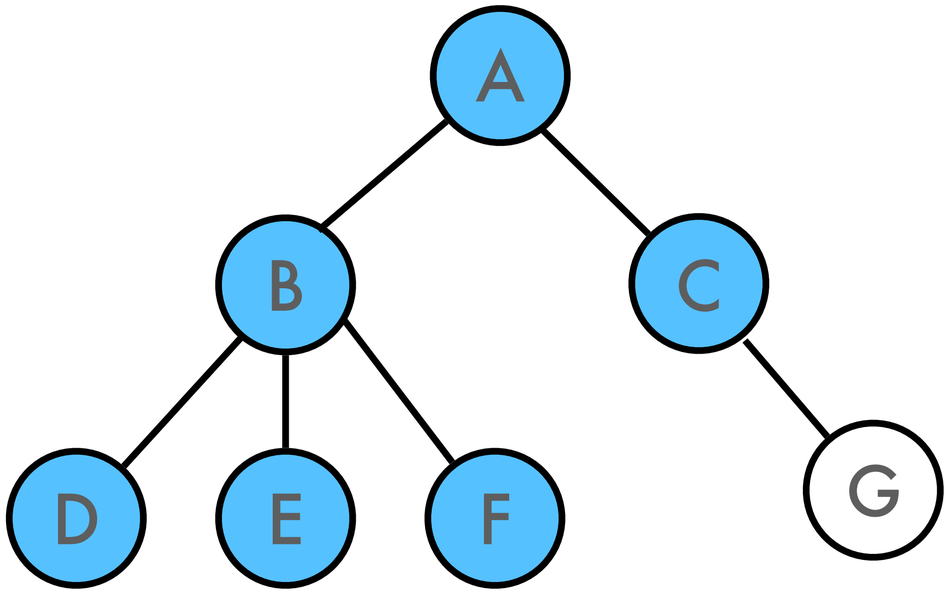

-A queue contains the list of the nodes to visit.
-The root node is initially introduced in a queue.
-The children nodes are introduced in the queue.
-The visited nodes are popped from the queue.

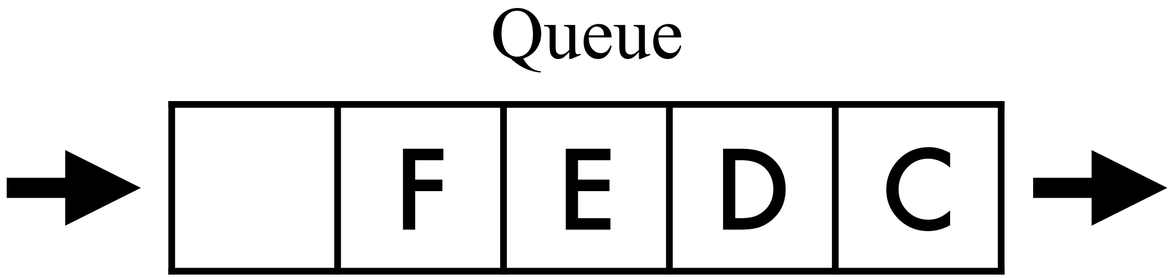
# Breadth-First Search



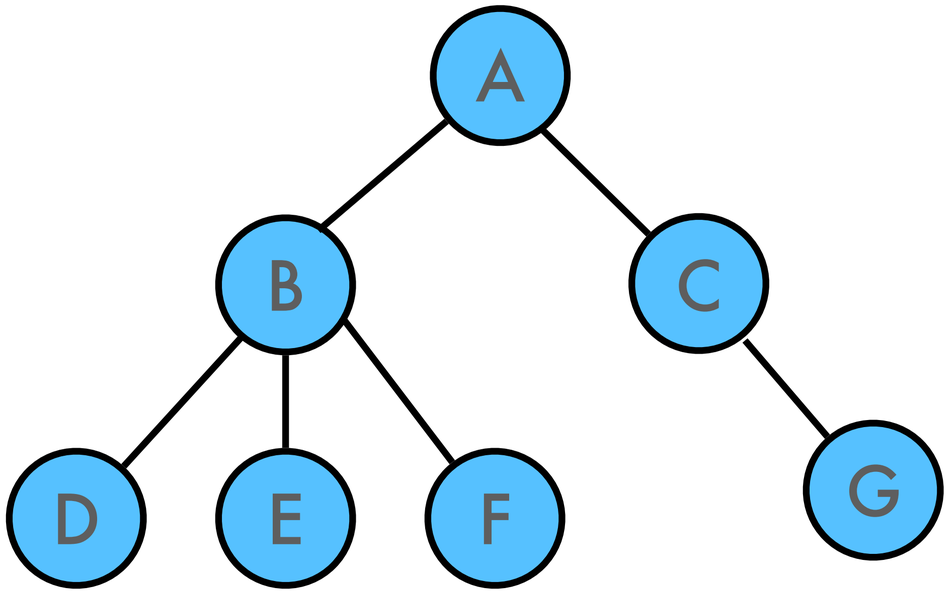



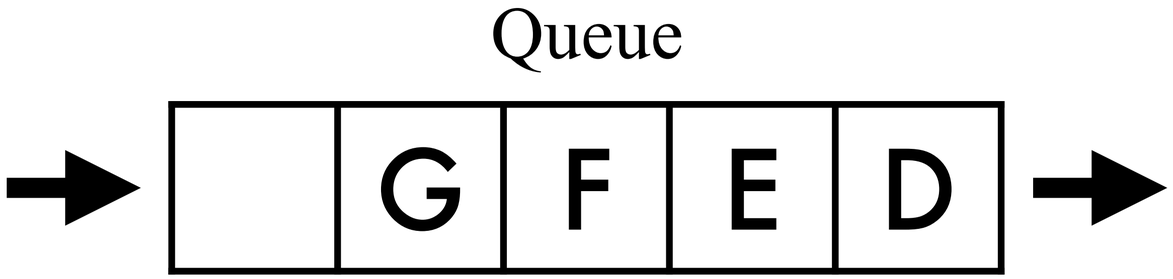
# Breadth-First Search



# Breadth-First Search



# Breadth-First Search

-The search is complete

-The search is optimal if the cost is constant and positive

-Time complexity:

-b is the maximal branching factor

-maximal # of expanded nodes: $T \sim b+b^2+b^3+\ldots+b^d \sim O(b^d)$

-Space complexity:

-every generated node is kept in memory

-Space needed for the frontier: $O(b^d)$.

# Example

- Suppose we have a branching factor of 10.
- We can visit $10^6$ nodes/second.
- Every node occupies 1kB of memory.

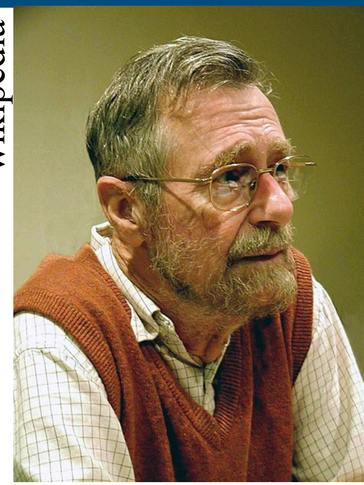
| Depth | Nodes | Time | Memory |
|---|---|---|---|
| 2 | 110 | 11ms | 107kB |
| 4 | 11110 | 11ms | 10.6MB |
| 6 | $10^6$ | 1.1s | 1GB |
| 8 | $10^8$ | 2m | 103GB |
| 10 | $10^{10}$ | 3h | 10TB |
| 12 | $10^{12}$ | 13d | 1PB |
| 14 | $10^{14}$ | 3.5y | 99PB |
| 21 | $10^{21}$ | >age universe | $10^6$EB |

# Uniform-Cost Search(*) (Dijkstra's Algorithm)

If the cost for the possible actions are equal, breadth-first search finds the path with the optimal costs.

If action costs are different, then uniform-cost can find the optimal solution.

Uniform-cost search expands the node with the lowest path cost g(n).
The data structure used for realising this search is the priority queue (heap).


wikipedia

Edsger W. Dijkstra
(1930-2002)

Another way to describe the algorithm is as "finding the shortest path between two points".
In the graph terminology, we are looking for the shortest path on a weighted graph.

(*) "Uniform cost search" is the name used in the AI community for the general Dijkstra's Algorithm

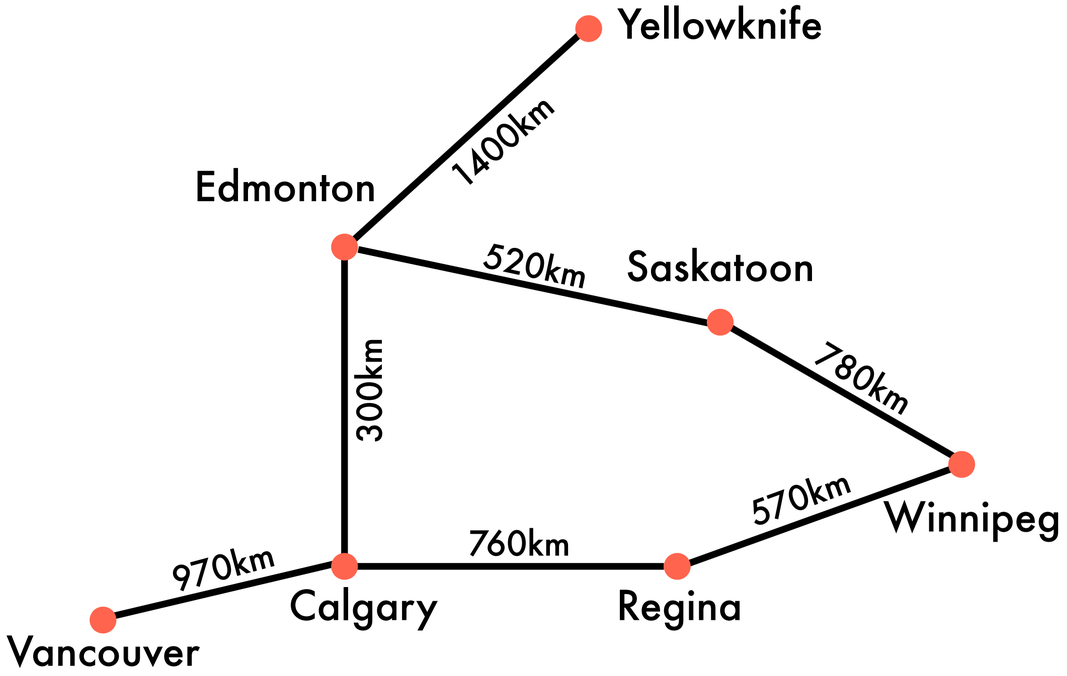
# Uniform-Cost Search (Dijkstra's Algorithm)

Problem: shortest path from Regina to Yellowknife

Initialization:

g(Regina) = 0

g(other cities) = +∞




Expand "Regina" and put in the priority queue:

g(Winnipeg) = 570

g(Calgary) = 760

Go for the minimum cost: "Winnipeg"

# Uniform-Cost Search (Dijkstra's Algorithm)

Problem: shortest path from Regina to Yellowknife

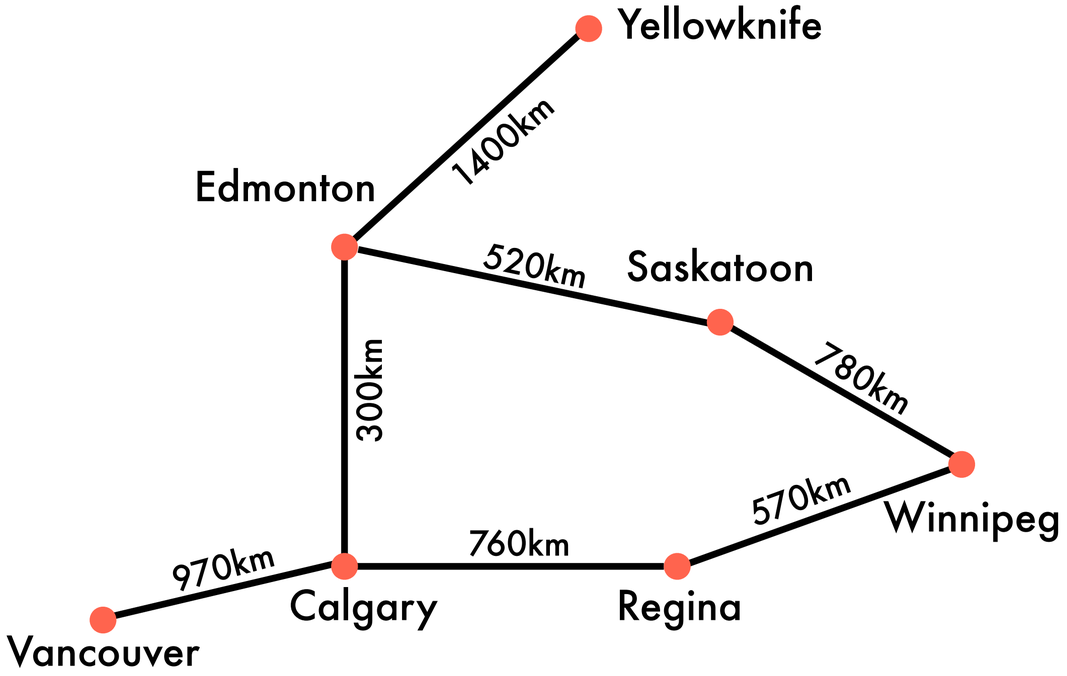
Expand Winnipeg:

g(Saskatoon) = 570 + 780 = 1350

<u>Priority queue:</u>

g(Calgary) = 760

g(Saskatoon) = 1350 (from Winnipeg)

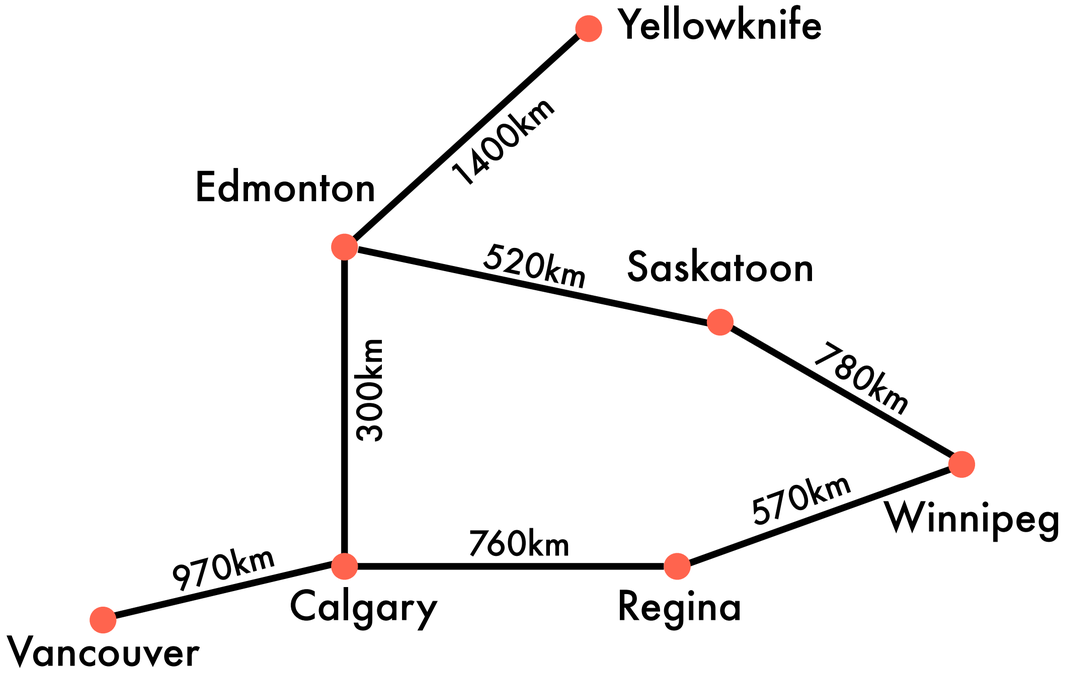


# Uniform-Cost Search (Dijkstra's Algorithm)

Problem: shortest path from Regina to Yellowknife

Expand Calgary:

g(Vancouver) = 760 + 970 = 1730
g(Edmonton) = 760 + 300 = 1060




Priority queue:
g(Edmonton) = 1060
g(Saskatoon) = 1350 (from Winnipeg)

….and so on (try to complete the search).

**Important note**: the algorithm checks for the goal only when it **expands** a node.

This means that when you reach Yellowknife from Edmonton the first time, you have still to check your queue.

# Uniform-Cost Search (Dijkstra's Algorithm)

```
function Dijkstra(graph, start):
    Initialize distances to all nodes as infinity
    Distance from start node to itself is 0
    priority_queue = [(0, start)]

    while priority_queue is not empty:
      current_distance, current_node = pop(priority_queue)

      if current_distance > distances[current_node]:
          continue

      for neighbour, weight in graph.current_node:
          distance = current_distance + weight
          if distance < distances[neighbour]:
              distances[neighbour] = distance
              push(priority_queue, (distance, neighbour))

    return distances
```

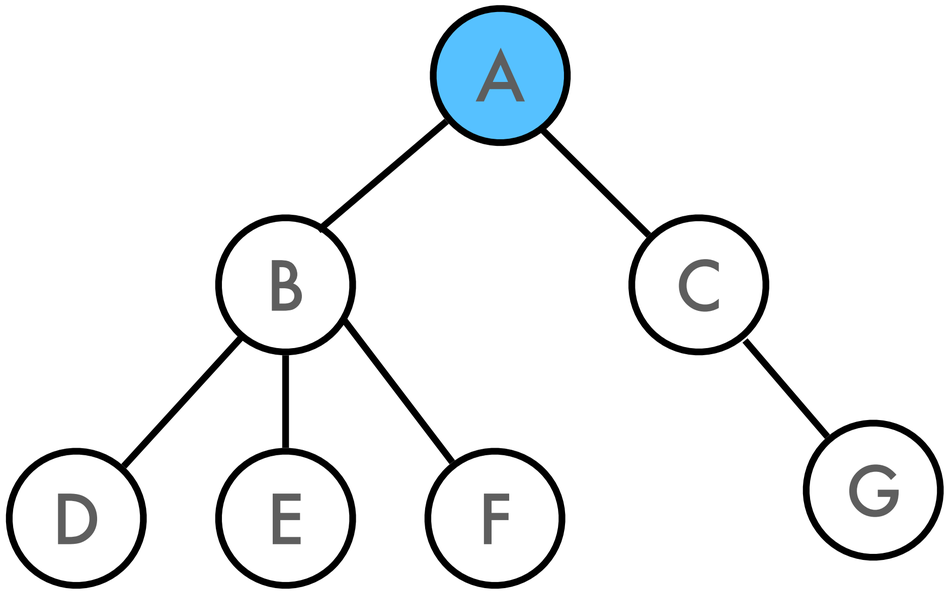
Pop node with smaller distance

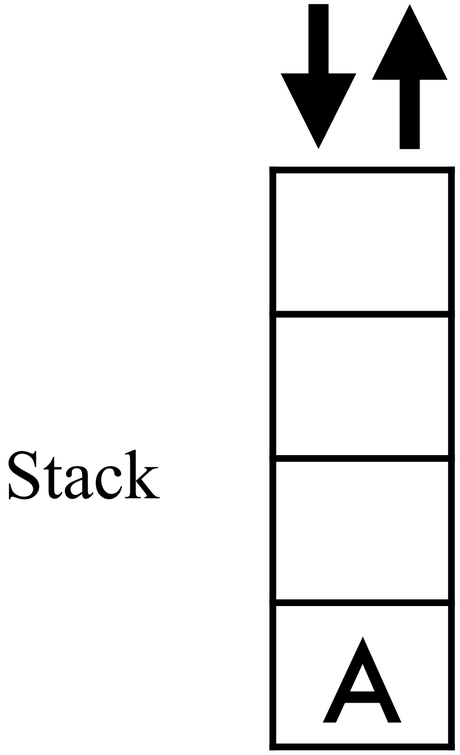
If current distance is larger, ignore

Check all neighbours of the current node

If shorter path found, update distance

# Depth-First Search

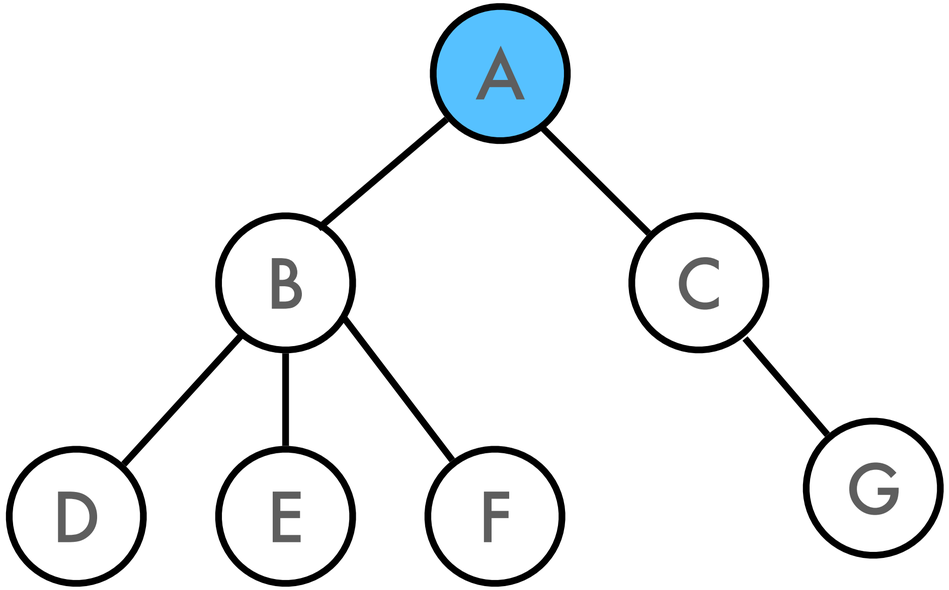


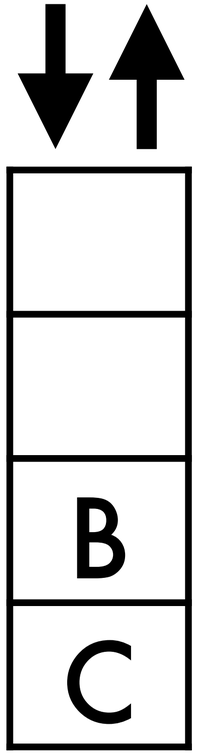

-A stack(*) contains the list of the nodes to visit.
-The root node is initially introduced in a stack.
-The children nodes are introduced in the stack.
-The visited nodes are popped from the stack.

(*) Note: given the use of a stack, it is common to realise this search with recursion.

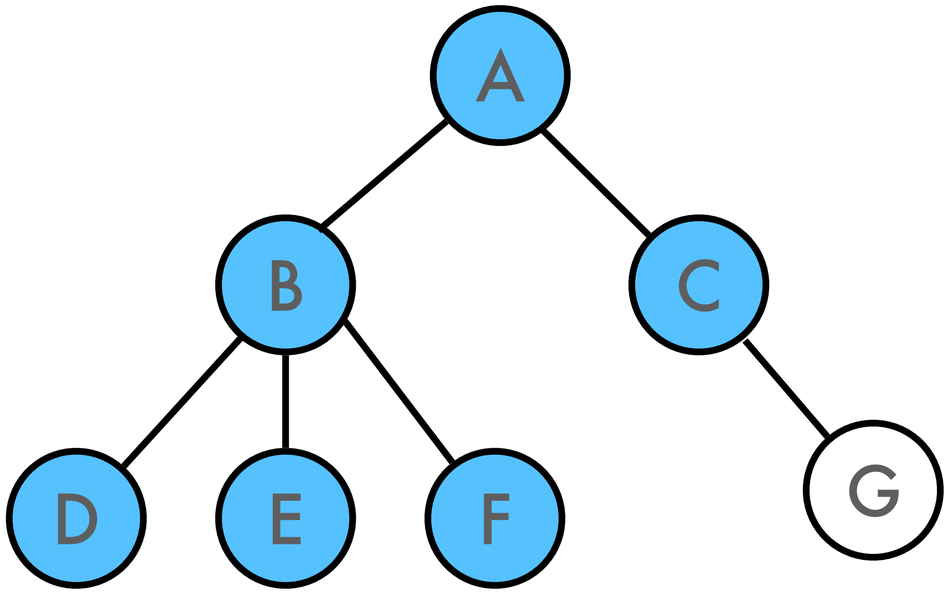
# Depth-First Search



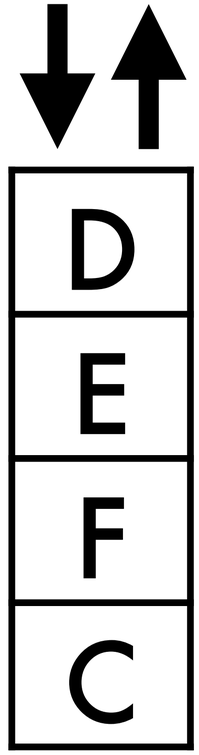

Stack



# Depth-First Search




Stack



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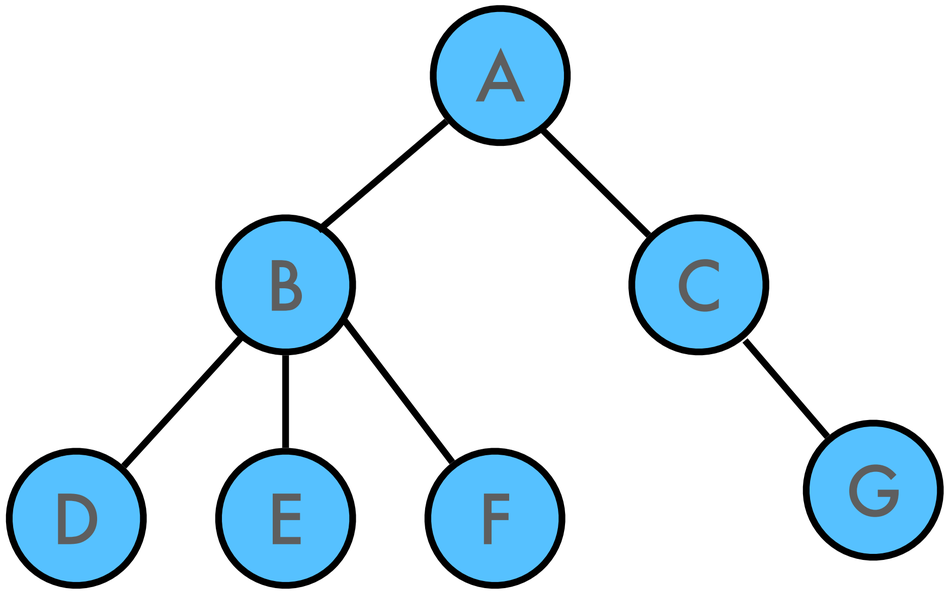
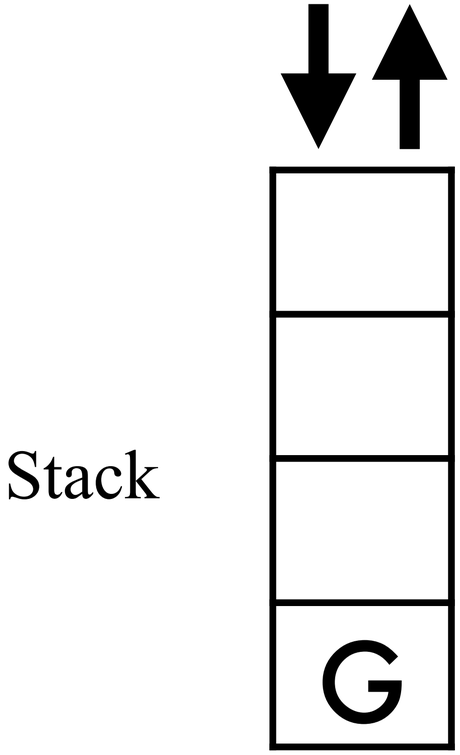


# Depth-First Search

- The solution is not, in general, optimal.
- Completeness is guaranteed only for graph-based searches and a finite state space.
- Time complexity:
    - In a graph-based search, bounded by the size of the state space (can be infinite).
    - In tree-based search, the algorithm might generate $O(b^m)$ nodes in the search tree which can be larger that the state space size (m is the max. length of a path in the state space).
- Space complexity:
    - Tree-based search: needs to store only the nodes along the path from the root to the leaf node. Once a node has been expanded, it can be removed from the memory as soon as all its descendants have been fully explored. Therefore, the memory requirement is only O(bm). This is the reason, why it is practically so relevant despite all the other shortcomings.
    - Graph-based search: in the worst case, all states need to be stored in the explored set (no advantage over breadth-first).

# Depth-Limited Search

Cut-off on the maximum reachable depth.

How to choose the cut-off:

- Choose maximum depth L = #nodes-1
- Choose the diameter d of the graph: maximum number of steps for reaching any node from any other one (in the “Canada graph”, d=9).

# Iterative-deepening Search

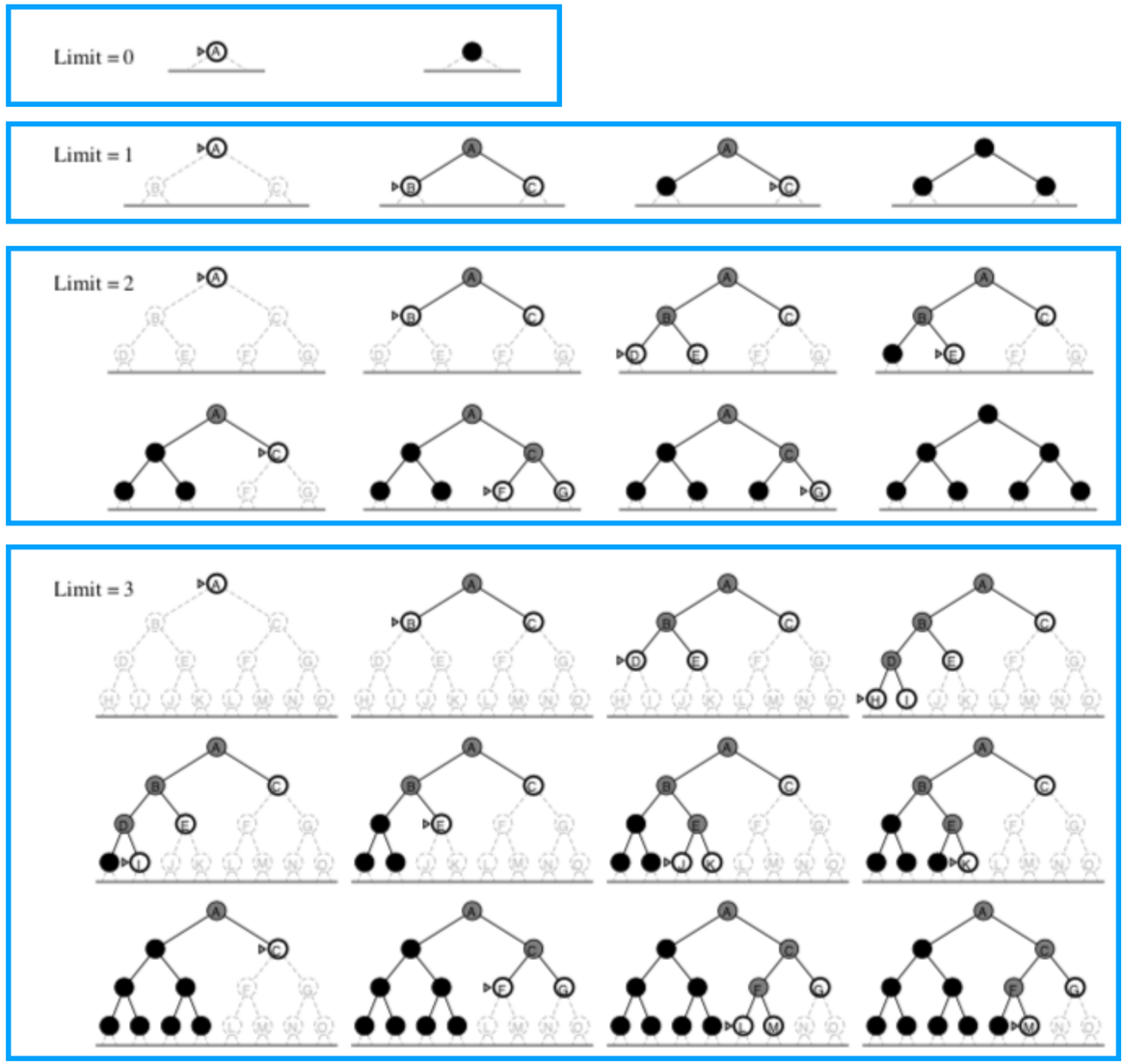
- Increase L by 1 starting from L=0 until a solution is found or $L_{max}$ is reached.

- Combines depth- and breadth-first searches benefits.

- Optimal and complete like BFS but less memory requirement O(bd).

- Time complexity slightly worse than BFS.

# Iterative-deepening Search



# Iterative-deepening Search

Like BFS, IDS is optimal for problems where with same-cost actions and is complete for acyclical state spaces or finite state spaces with checks for cycles.

IDS seems expensive, but repeating the expansion is cheap initially and grows with time. So the first node is repeated d times, the second nodes (d-1) times and so on:

Number of node expansions:

Iterative Deepening Search: $(d)b+(d-1)b^2+(d-2)b^3+\ldots+b^d \sim O(b^d)$
Breadth-First Search: $b+b^2+\ldots+b^{d-1}+b^d \sim O(b^d)$

Example (b=10, d=5):

N(BFS) = 10 + 100 + 1000 + 10000 + 100000 = 111110
N(IDS) = 50 + 400 + 3000 + 20000 + 100000 = 123450 ~ 11% higher

# Iterative-deepening Search

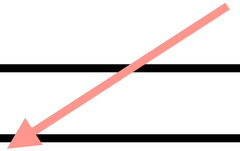
- IDS is the preferred uninformed search strategy when the entire search space cannot fit into the memory.

- If memory is enough, BFS can be used.

- Another option: hybrid approach:
  - Use BFS until memory allows.
  - Switch to IDS from the last frontier found by BFS.

# IDS and DLS Algorithms

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution or failure
    for depth=0 to infinity do
        result <—- DEPTH-LIMITED-SEARCH(problem, depth)
        if result != cutoff then return result
```



```
function DEPTH-LIMITED-SEARCH(problem, L) returns a node or failure or cutoff
    frontier<—- a stack with NODE(problem, INITIAL) as element
    result <—- failure
    while not IS-EMPTY(frontier) do
        node <—- POP(frontier)
        if problem.IS-GOAL(node, state) then return node
        if DEPTH(node) > L then
            result <—- cutoff
        else if not IS-CYCLE(node) do
            for each child in EXPAND(problem, node) do
                add child to frontier
    return result
```

# Bidirectional Search

- Simultaneously searches forward from the starting node and backwards from the goal node.
- Assume that the two paths will meet.
- Motivation: $b^{d/2}+b^{d/2} << b^d$

Example: b=10, d=10. Gain factor: $\frac{10^{10}}{2 \cdot 10^5} = 50.000$

The algorithm needs to keep track of two frontiers and two lists of reached nodes.
The two paths can be followed by different search strategies.

# Comparison of the Search Strategies

For tree structures without repetition check

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening | Bidirectional (if applicable) |
|---|---|---|---|---|---|---|
| Complete? | Yes[a] | Yes[a,b] | No | No | Yes[a] | Yes[a,d] |
| Time | $O(b^d)$ | $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ | $O(b^m)$ | $O(b^l)$ | $O(b^d)$ | $O(b^{d/2})$ |
| Space | $O(b^d)$ | $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ | $O(bm)$ | $O(bl)$ | $O(bd)$ | $O(b^{d/2})$ |
| Optimal? | Yes[c] | Yes | No | No | Yes[c] | Yes[c,d] |

$b$ branching factor
$d$ depth of solution
$m$ maximum depth of the search tree
$l$ depth limit
$C^*$ cost of the optimal solution
$\epsilon$ minimal cost of an action

Superscripts:
[a] $b$ is finite
[b] if step costs not less than $\epsilon$
[c] if step costs are all identical
[d] if both directions use breadth-first search

# Summary

- Before an agent can start the search for a solution, a goal must be defined. The goal must be used for formulating the problem.
- A problem consists in 5 parts: The state space, initial condition, actions, goal test and, path costs.
- A search algorithm can in principle be used for solving any problem.
- Search algorithms are described on the basis of completeness, optimality, time complexity, and space complexity.