From QCD to Hadrons and Nuclei

Advanced Subatomic Physics Course

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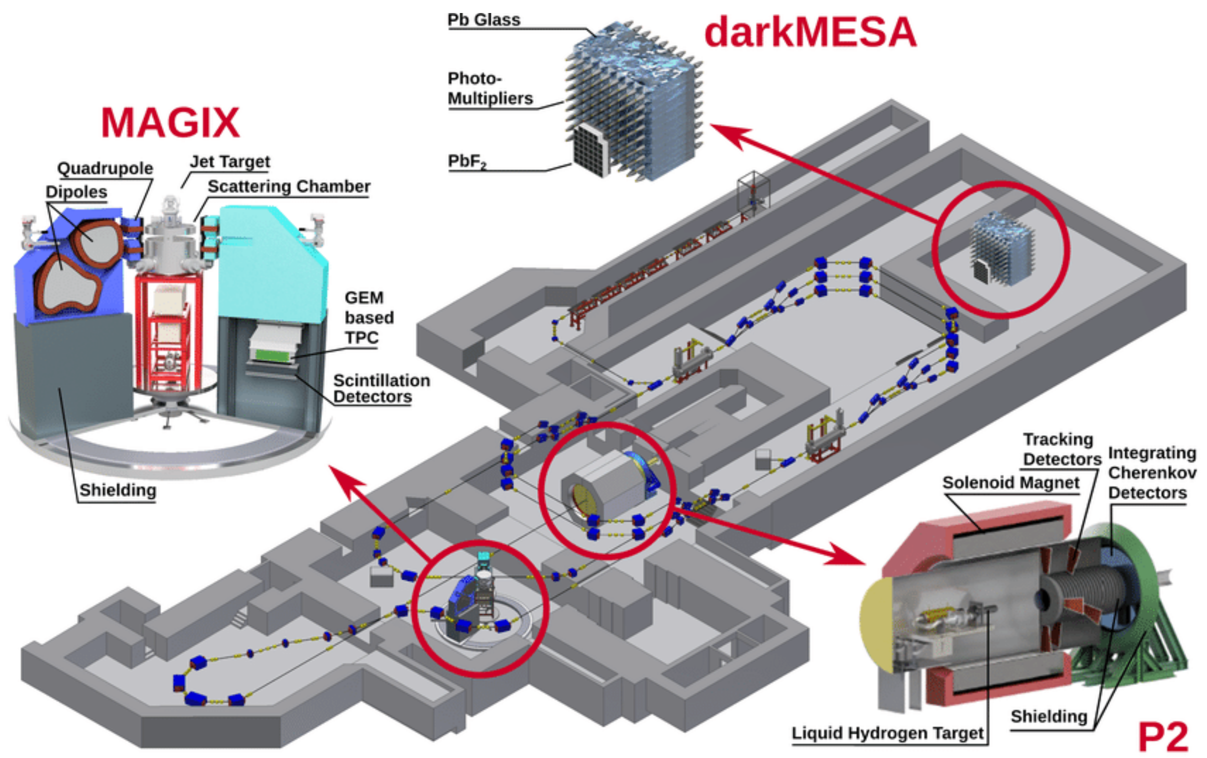
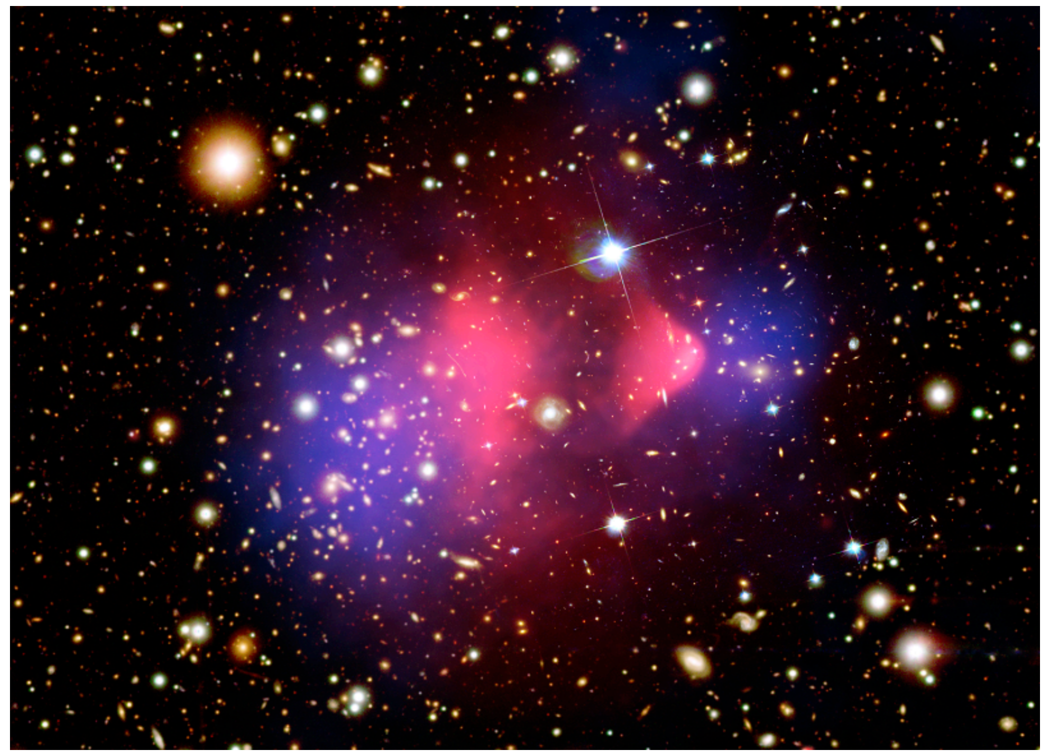
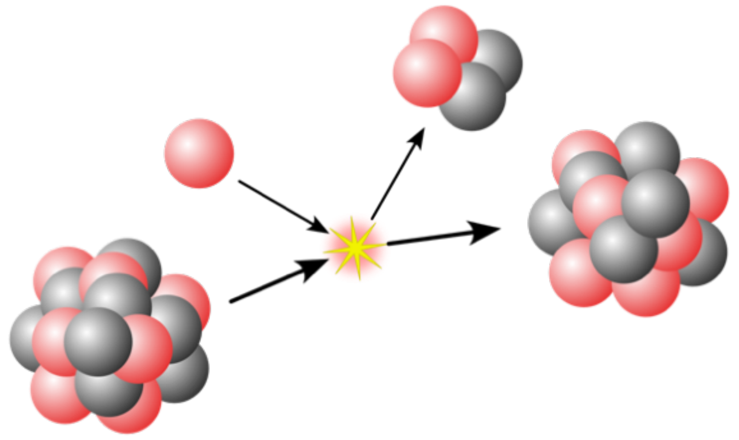
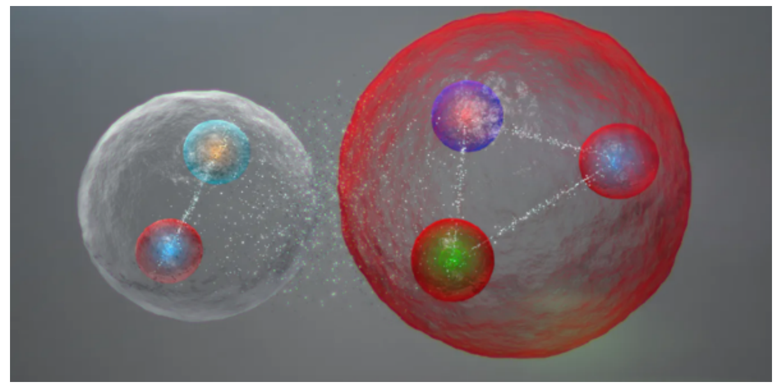








Sommersemester 2022

Syllabus

1. Introduction to strong interactions in the perturbative and non-perturbative regimes.
2. Hadrons and Nuclei
3. Electron and neutrino scattering experiments on hadrons and nuclei: form factors, elastic and inelastic scattering, resonances, deep inelastic physics.
4. Experimental methods and facilities with focus on MAMI and MESA at JGU Mainz.
5. Dark Matter
6. Search for dark matter with "intensity frontier" experiments, in particular, electron scattering experiments.
7. Search for dark matter with “direct detection” experiments with focus on argon.
8. Nuclear astrophysics and nuclear reactions of astrophysical relevance (in the Big Bang and stars).
9. Experiments for measuring astrophysical reactions with accelerators.
10. Discussion of a relevant published scientific paper on one of the topics discussed during the course.

Lecture Plan

1	Wed, 20. Apr. 2022	12:15	13:45	01 128 Galilei-Raum
2	Th, 21. Apr. 2022	10:15	11:45	01 128 Galilei-Raum
3	Wed, 27. Apr. 2022	12:15	13:45	01 128 Galilei-Raum
4	Th, 28. Apr. 2022	10:15	11:45	01 128 Galilei-Raum
5	Wed, 4. May 2022	12:15	13:45	01 128 Galilei-Raum
6	Th, 5. May 2022	10:15	11:45	01 128 Galilei-Raum
7	Wed, 11. May 2022	12:15	13:45	01 128 Galilei-Raum
8	Th, 12. May 2022	10:15	11:45	01 128 Galilei-Raum
9	Wed, 18. May 2022	12:15	13:45	01 128 Galilei-Raum
10	Th, 19. May 2022	10:15	11:45	01 128 Galilei-Raum
11	Wed, 25. May 2022	12:15	13:45	01 128 Galilei-Raum
12	Wed, 1. Jun. 2022	12:15	13:45	01 128 Galilei-Raum
13	Th, 2. Jun. 2022	10:15	11:45	01 128 Galilei-Raum
14	Wed, 8. Jun. 2022	12:15	13:45	01 128 Galilei-Raum
15	Th, 9. Jun. 2022	10:15	11:45	01 128 Galilei-Raum
16	Wed, 15. Jun. 2022	12:15	13:45	01 128 Galilei-Raum
17	Wed, 22. Jun. 2022	12:15	13:45	01 128 Galilei-Raum
18	Th, 23. Jun. 2022	10:15	11:45	01 128 Galilei-Raum
19	Wed, 29. Jun. 2022	12:15	13:45	01 128 Galilei-Raum
20	Th, 30. Jun. 2022	10:15	11:45	01 128 Galilei-Raum
21	Wed, 6. Jul. 2022	12:15	13:45	01 128 Galilei-Raum
22	Th, 7. Jul. 2022	10:15	11:45	01 128 Galilei-Raum
23	Wed, 13. Jul. 2022	12:15	13:45	01 128 Galilei-Raum
24	Th, 14. Jul. 2022	10:15	11:45	01 128 Galilei-Raum
25	Wed, 20. Jul. 2022	12:15	13:45	01 128 Galilei-Raum
26	Th, 21. Jul. 2022	10:15	11:45	01 128 Galilei-Raum

Exam format

1. General questions about the course topics
2. Discussion of one of the topics presented during the lecture
3. Discussion and presentation of a published physics paper (seminar+slides)

Preliminary Definitions

Natural Units

Consider the two fundamental constants from quantum mechanics and relativity:

Planck's constant: $\hbar$
Speed of light in vacuum: **c**
and a typical subatomic scale: **GeV** (rest-mass energy of the proton)

Expressing the SI units in terms of these constants:

Energy:	**GeV**
Mass:	**GeV/c^2**
Momentum:	**GeV/c**
Length:	**$(\mathrm{GeV}/\hbar c)^{-1}$**
Time:	**$(\mathrm{GeV}/\hbar)^{-1}$**

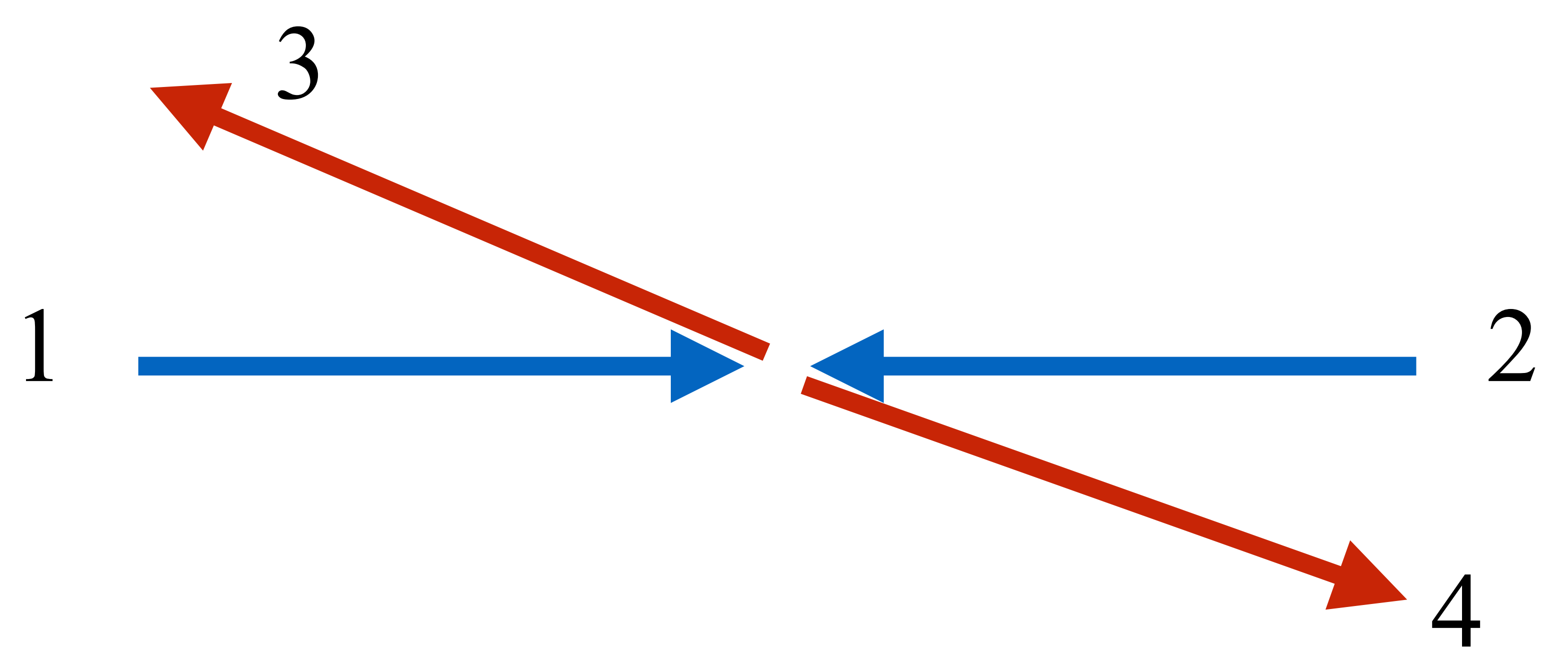
Simplify calculations setting $\hbar$=c=1:

Energy:	**GeV**
Mass:	**GeV**
Momentum:	**GeV**
Length:	**GeV^{-1}**
Time:	**GeV^{-1}**

Example: $E^2 - p^2 = m^2$ …

Mandelstam Variables

Particle scattering is a fundamental process to study in particle physics.
Since Lorentz invariance is a fundamental physical property, it is convenient to express particle scattering amplitudes with Lorentz-invariant quantities. A convenient choice is th use of the Mandelstam variables.
Considering the 2-2 scattering:



the Madelstam variables are:

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

which have the property: $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

Example: s can be seen as the CM energy, since: $p_1^* = (E_1^*, p^*) \quad p_2^* = (E_2^*, -p^*)$

and thus $s = (E_1^* + E_2^*)^2$

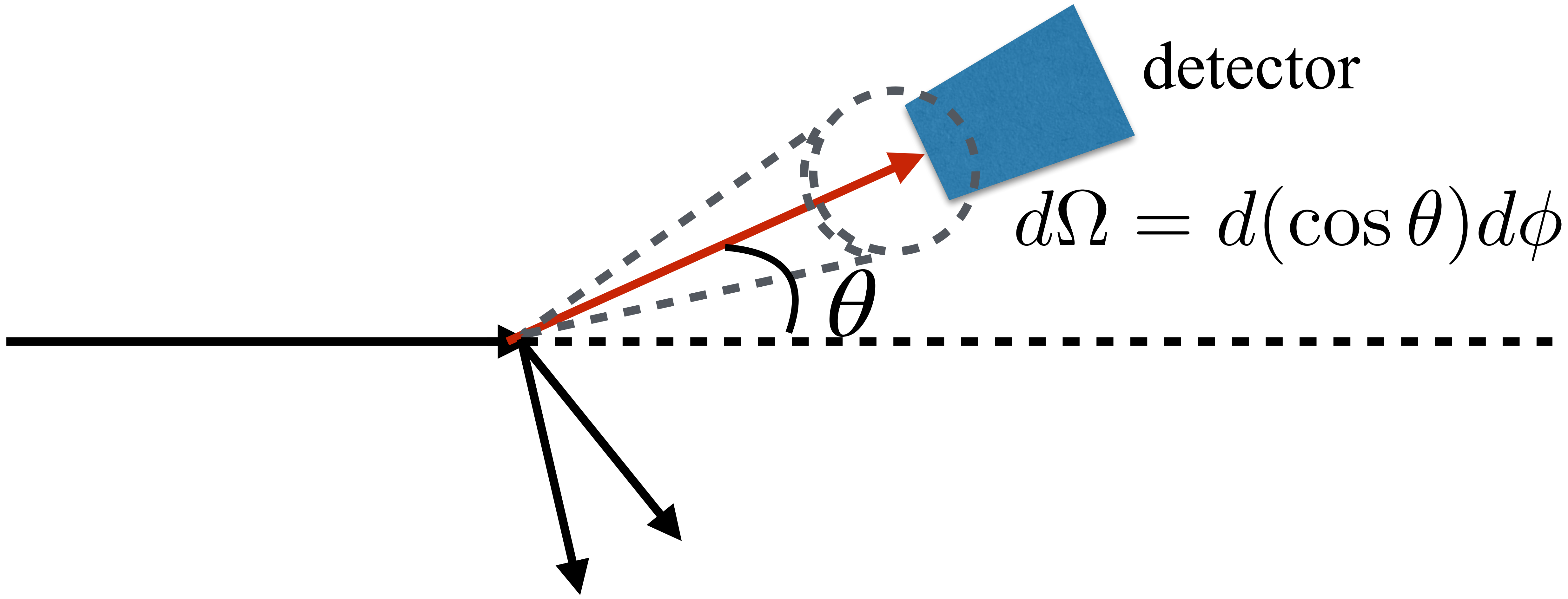
Since these variables are scalar products of 4-vectors, they are the same in every reference frame.

Cross Section

$$\sigma = \frac{\#\text{ of interactions/time/target}}{\text{incident flux}}$$

flux = # of incident particles / area / time

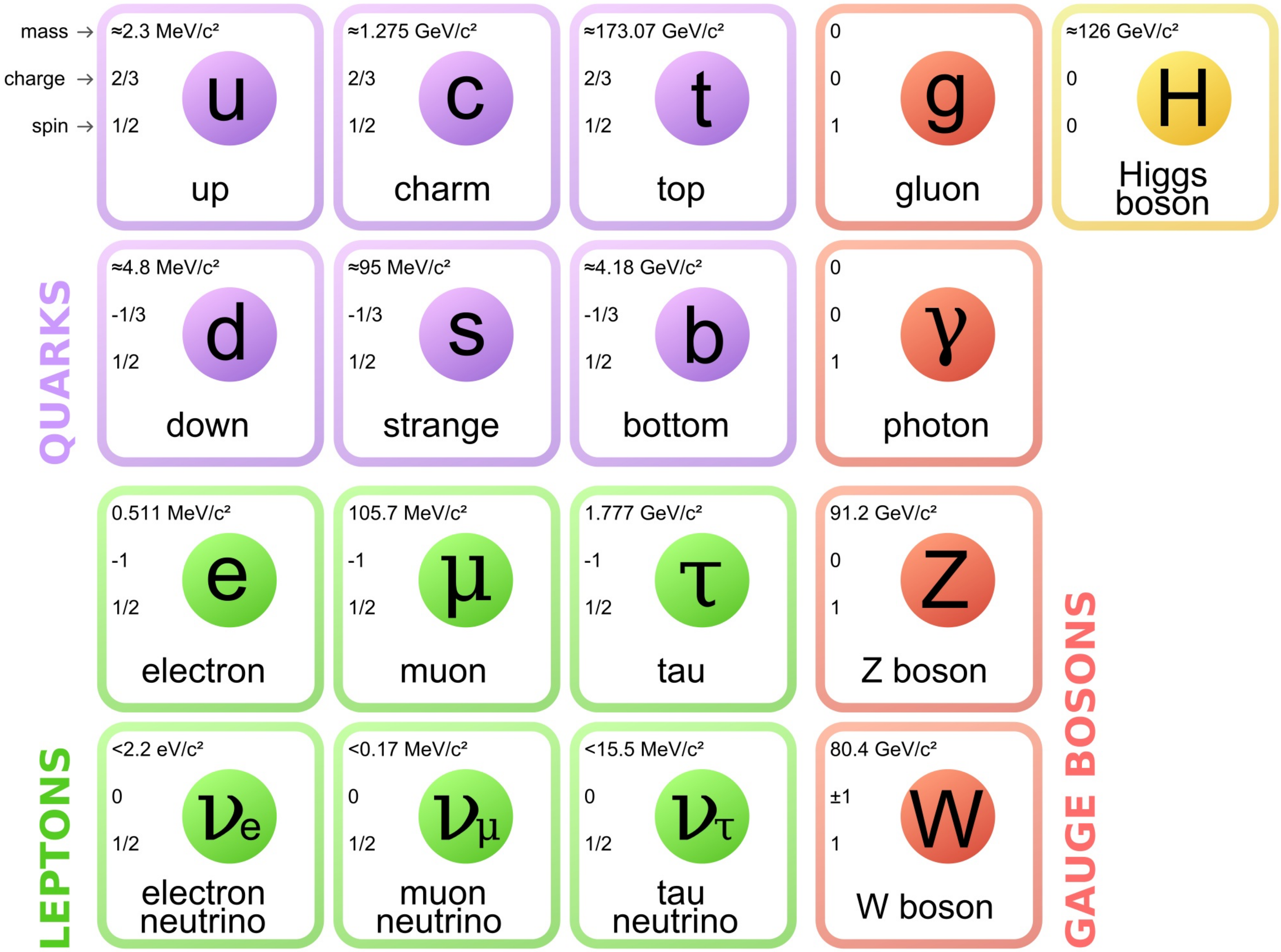
Differential cross section: $\dfrac{d\sigma}{d\Omega}$



$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

The cross section can be differential also in other variables like the energy, or even differential with respect to more variables: $\dfrac{d\sigma}{d\Omega dE \ldots}$

Standard Model



Quantum Chromo Dynamics (QCD)

QCD

Quantum field theory of the strong interaction

Based on a non-abelian symmetry group: SU(3)

Degrees of freedom: quarks and gluons (where are e.g. protons and neutrons?)

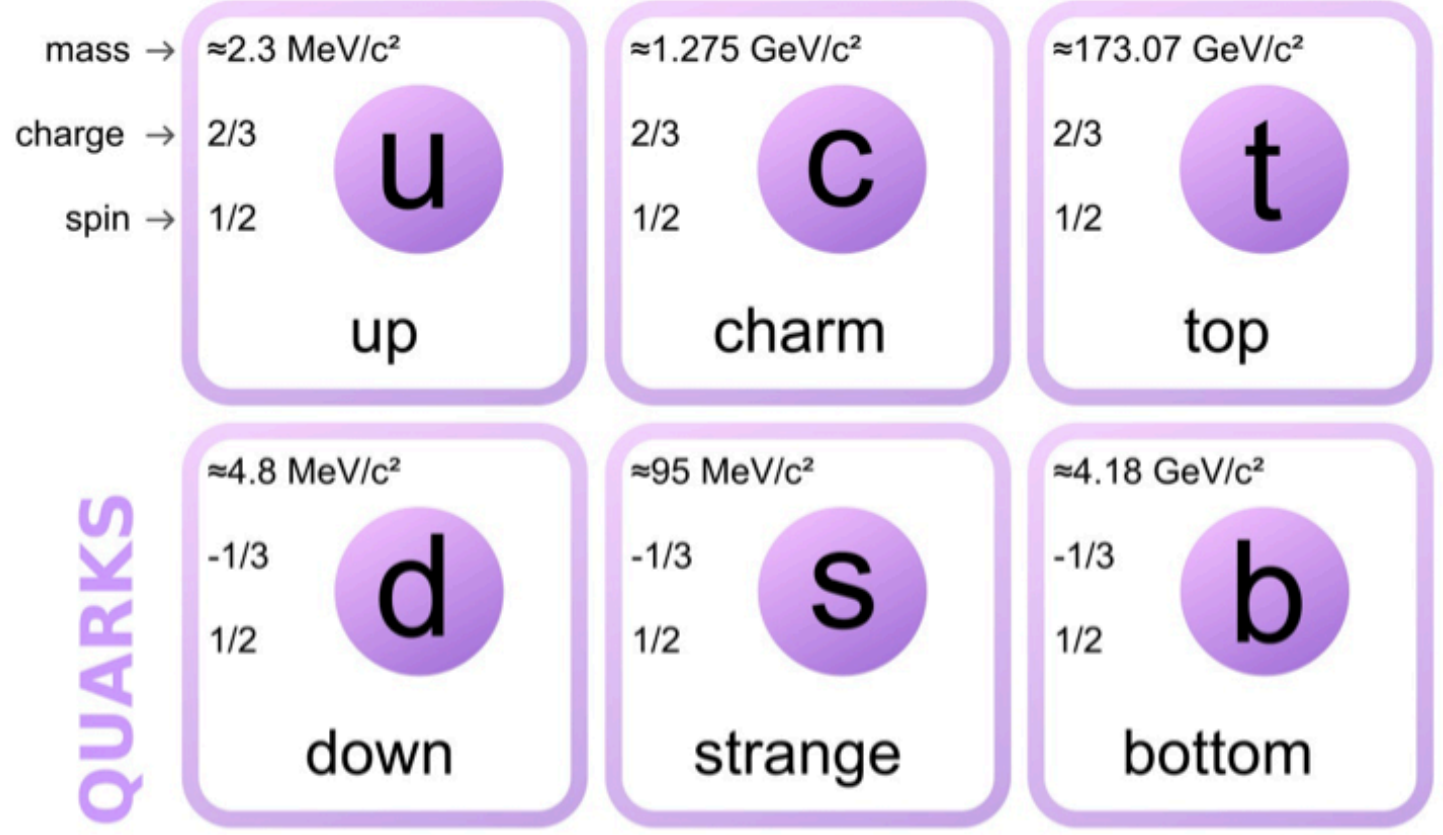
QCD: Free quark lagrangian

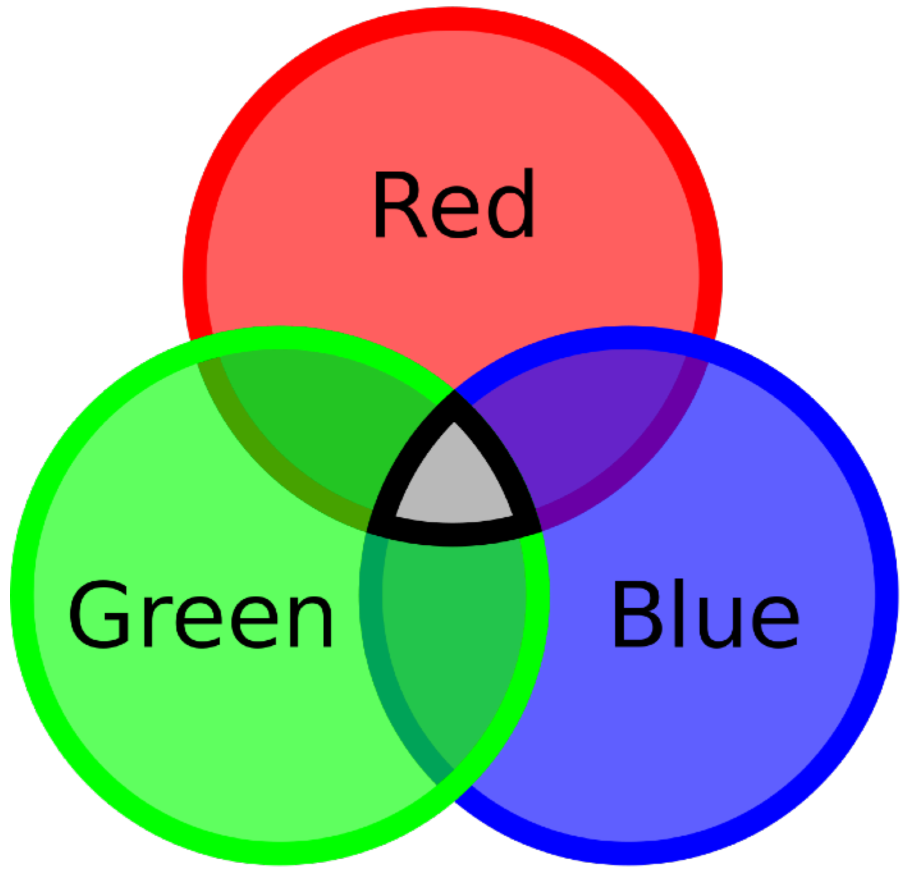
$$\mathcal{L}_0 = \sum_{f=1}^{N_f} \sum_{C=R,G,B} \bar{\psi}_C^f (i\gamma^\mu \partial_\mu - m_0^f)\psi_C^f$$

f: flavour

C: colour

Quark field





SU(3) symmetry

The quark fields (given a flavour f), can be arranged in triplets: $\boxed{\psi^f = \begin{pmatrix} \psi^f_R \\ \psi^f_G \\ \psi^f_B \end{pmatrix}}$
eliminating the colour sum in the previous lagrangian.

Quark fields transform under the SU(3) group: $\boxed{\psi'^f = e^{i\theta_a \lambda_a} \psi^f}$ "Local symmetry": $\boxed{\theta \to \theta(x)}$

where the λ^a are the Gell-Mann matrices

and $[\lambda_a, \lambda_b] = i f_{abc} \lambda_c$

"structure constants"

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

(Local) Gauge Symmetry

Step back: consider U(1) theory for simplicity (like QED).
Key observables invariant under a global phase transformation: $U = e^{ie\theta}$
Examples:

Prob. density or charge density: $\rho = \bar{\psi}\psi$

Electric current: $I = \bar{\psi}\gamma_\mu\psi$

are <u>invariant</u> if $\psi \to U\psi$

The same happens with the **Dirac lagrangian** $\mathcal{L} = \bar{\psi}(\gamma^\mu\partial_\mu - m)\psi$

Densities and currents are also invariant under LOCAL transformations $U(x) = e^{ie\theta(x)}$

…but the lagrangian is not!

(Local) Gauge Symmetry

Why the lagrangian is not invariant under **local** gauge transformations?

$$\mathcal{L} \to \bar{\psi} e^{-ie\theta}(i\gamma^\mu \partial_\mu - m) e^{-ie\theta}\psi = \bar{\psi}\left[i\gamma^\mu(\underbrace{\boxed{\partial_\mu + ie\partial_\mu\theta}}_{\neq \partial_\mu} - m)\right]\psi$$

The problem is that the derivative $\partial_\mu \psi$ is not locally gauge-invariant.
For fixing this problem, we have to change the derivative introducing a space-time dependent vector field:

$$\boxed{D_\mu = \partial_\mu - ieA_\mu}$$

which transforms as

$$\boxed{A_\mu \to A_\mu - \partial_\mu \theta(x)}$$

This reminds also the transformations allowed in classical electromagnetism for the 4-potential.

(Local) Gauge Symmetry

The new operator D_μ is also called "covariant", since it does not change under local gauge transformations.
This language is rooted in differential geometry, but here we will not discuss the geometric aspects of gauge transformations.
Let's prove explicitly the invariance:

$$\bar{\psi} D_\mu \psi = \bar{\psi}(\partial_\mu - ieA_\mu)\psi \rightarrow$$

$$\bar{\psi} e^{-ie\theta}(\partial_\mu - ie(A_\mu + \partial_\mu \theta))e^{ie\theta}\psi =$$

$$= \bar{\psi}(\partial_\mu - ieA_\mu - ie\partial_\mu\theta + ie\partial_\mu\theta)\psi =$$

$$= \bar{\psi}(\partial_\mu - ieA_\mu)\psi = \bar{\psi} D_\mu \psi$$

(Local) Gauge Symmetry

The introduced transformations leave the form of the lagrangian invariant, but after a gauge transformation and re-introducing the non-covariant partial derivative ∂_μ we obtain

$$\mathcal{L} = \bar{\psi}(ie\partial_\mu - ieA_\mu - m)\psi = \bar{\psi}(ie\partial_\mu - m)\psi + \boxed{e\bar{\psi}\gamma_\mu\psi A_\mu}$$

This is remarkable: the requirement of local gauge invariance automatically generated the correct interaction term between the field ψ and the gauge field A_μ . In the language of QED, the new term describes how photons couple with electrons.

$$\mathcal{L}_{int} = J_\mu A_\mu$$

$$J_\mu = e\bar{\psi}\gamma_\mu\psi$$

(Local) Gauge Symmetry

The free photon lagrangian (the "kinetic" term for the field A_μ) is

$$\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

with the field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Note that a mass term for the photon, like $\frac{1}{2} m^2 A_\mu A^\mu$ is excluded by local gauge invariance!

The final lagrangian is

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + J^\mu A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

(Local) Gauge Symmetry

Using the obtained lagrangian with the Euler-Lagrange equations, one can obtain the "equations of motion" for the theory:

$$\boxed{\begin{aligned} &(i\gamma^\mu D_\mu - m)\psi = 0 \\ &\partial_\nu F^{\mu\nu} = -e\bar{\psi}\gamma^\mu\psi \end{aligned}}$$

Do you recognise these equations?

Back to QCD

Quantum Chromodynamics is based instead on the SU(3) colour group and the local gauge transformation is (see before)

$$\boxed{\psi'^f = e^{i\theta_a \lambda_a} \psi^f}$$

The t matrices are non-diagonal!

$t_a \cdot t_b \neq t_b \cdot t_a$

and the lagrangian is

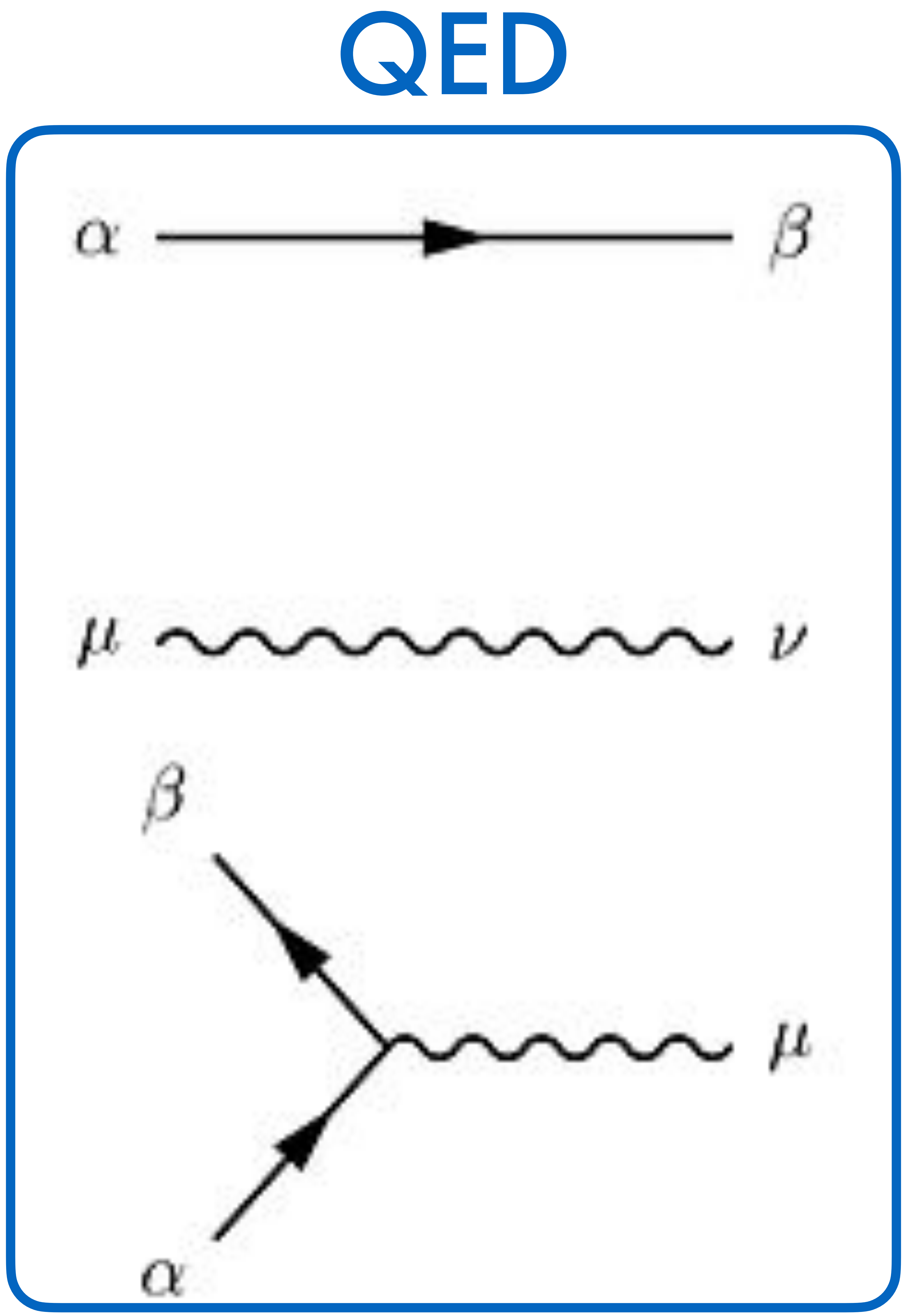
$$\mathcal{L}_{QCD} = \bar{\psi}^f (i\gamma^\mu \partial_\mu - m^f)\psi^f + g\bar{\psi}^f \gamma^\mu t_a \psi^f A_{a\mu} - \frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu}$$

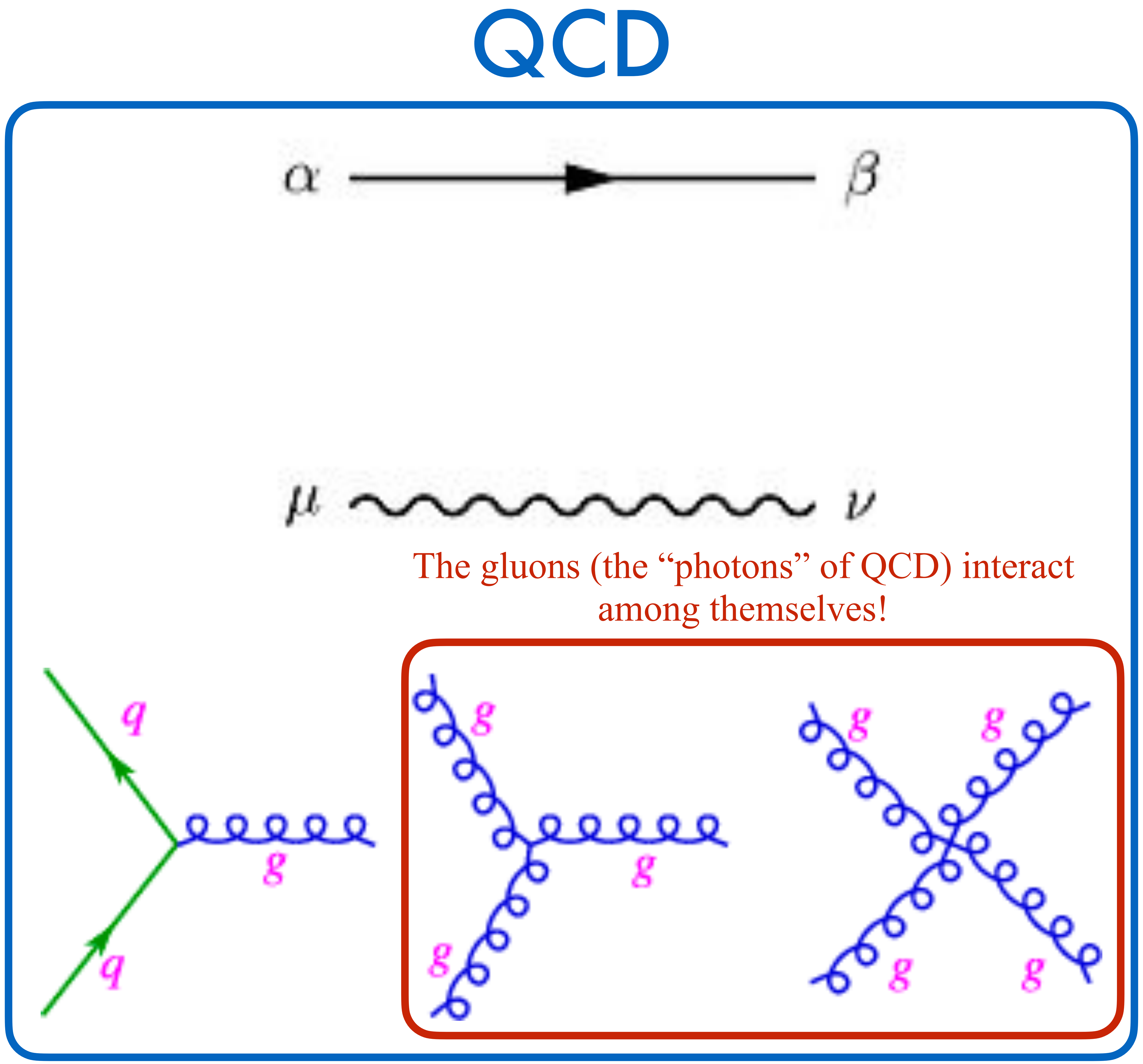
Note here the index "a" (a=1..8) which labels the 8 generators of the SU(3) group. The field tensor is now

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + \boxed{g f_{abc} A_b^\mu A_c^\nu}$$

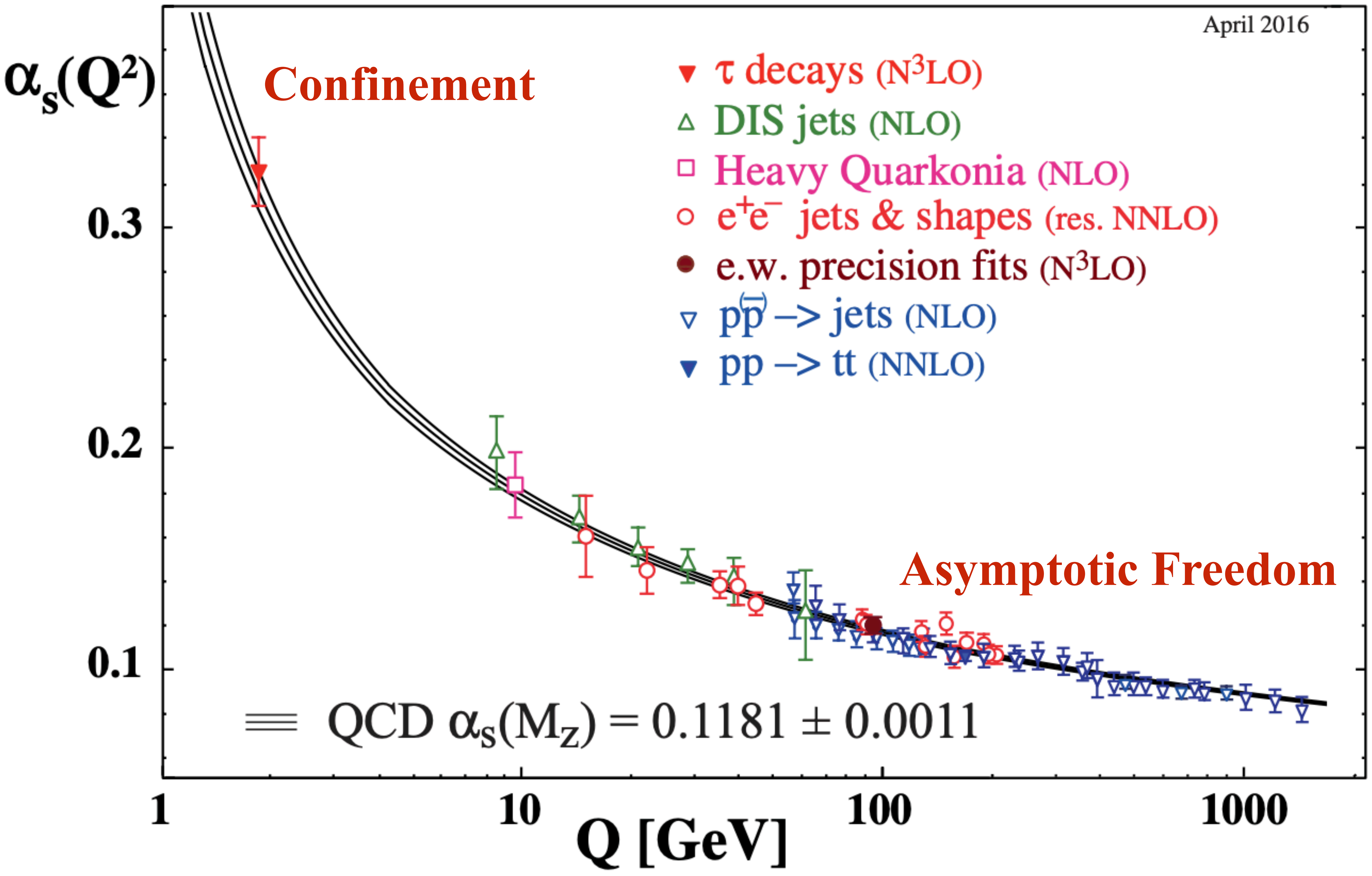
Term arising from the non-abelian property of the gauge group

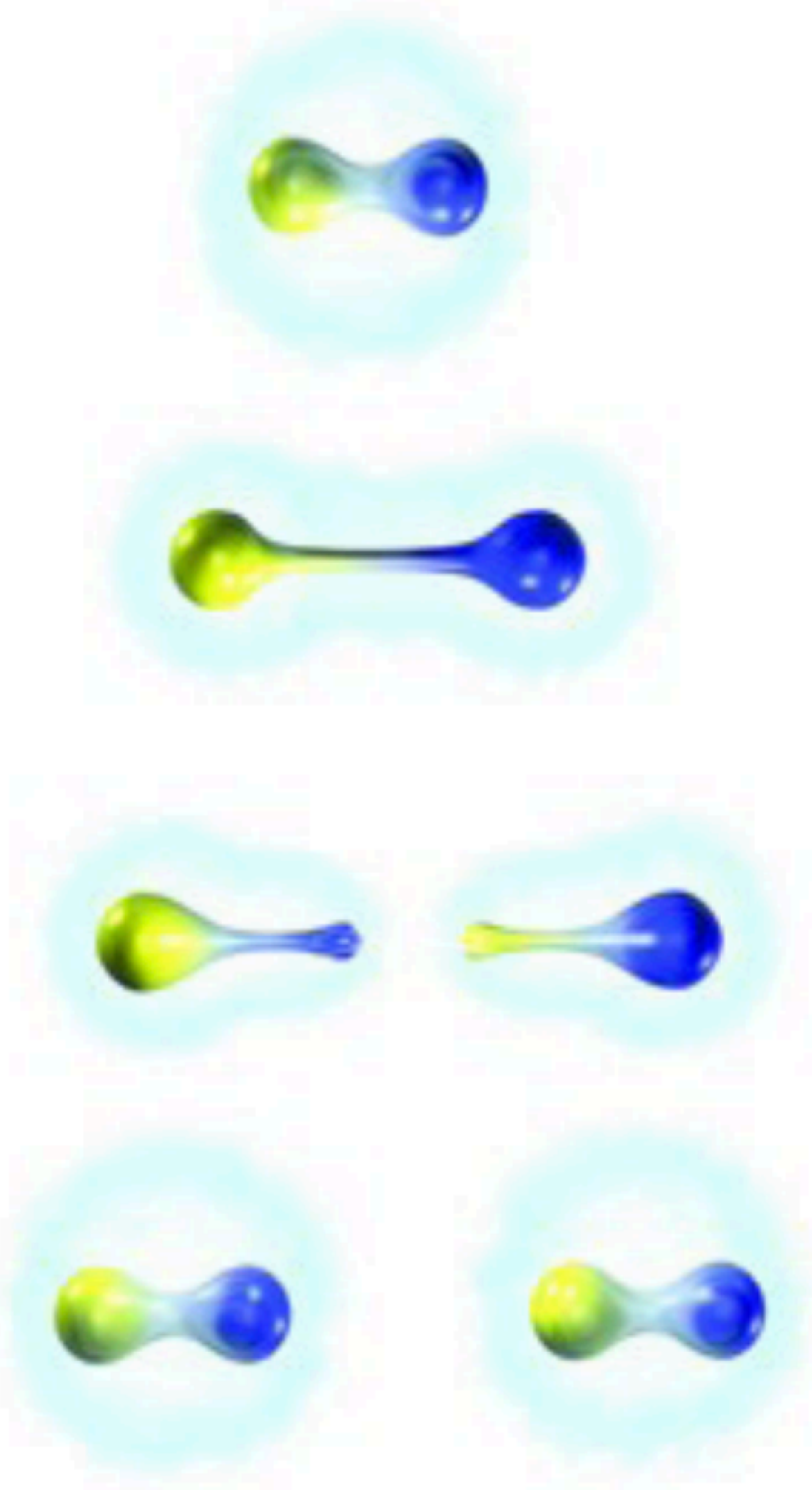
What is different from QED?





Consequences of a non-abelian group: Running Coupling





"String confinement"

Summary

* Quantum chromodynamics (QCD) is based on the non-abelian group SU(3).
* Local gauge invariance is a fundamental property of QFTs describing elementary particles.
* The basic degrees of freedom of QCD are quarks and gluons.
* Quarks have 6 flavours (up, down, strange, charme, bottom, top) and are fermions with spin 1/2 .
* Gluons are 8 (like the generators of SU(3)) and carry color, therefore they interact also with themselves.
* In QED, the photon does not carry electric charge, so it does not interact with himself.

* The non-abelian structure of QCD implies the charge of its gauge bosons and further consequences are:
* Asymptotic freedom: the force becomes small for high momenta/energies and a perturbative treatment is possible.
* Confinement: the force becomes large at small momenta/energies. The theory becomes non-perturbative.

* Confinement implies that we cannot observe free coloured particles: quarks are confined in uncoloured objects like hadrons (made by three quarks) and mesons (quark-antiquark states). Other more exotic combinations can exist.