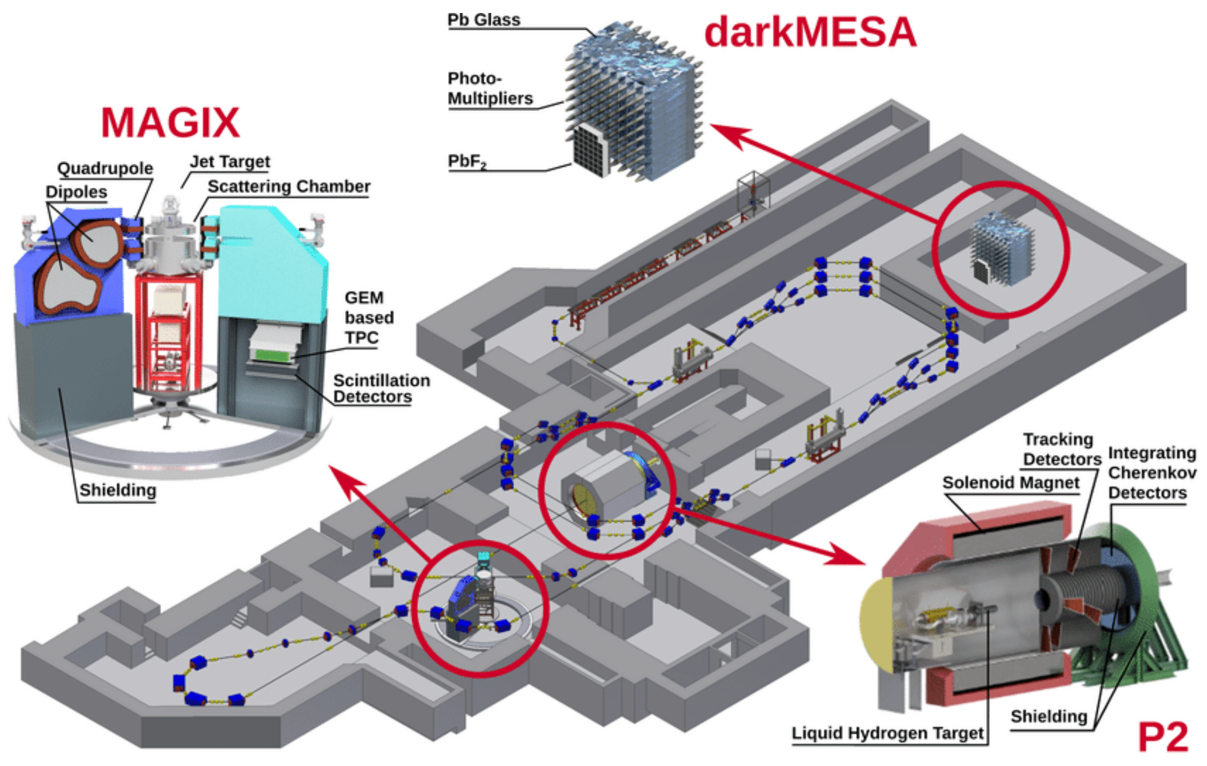
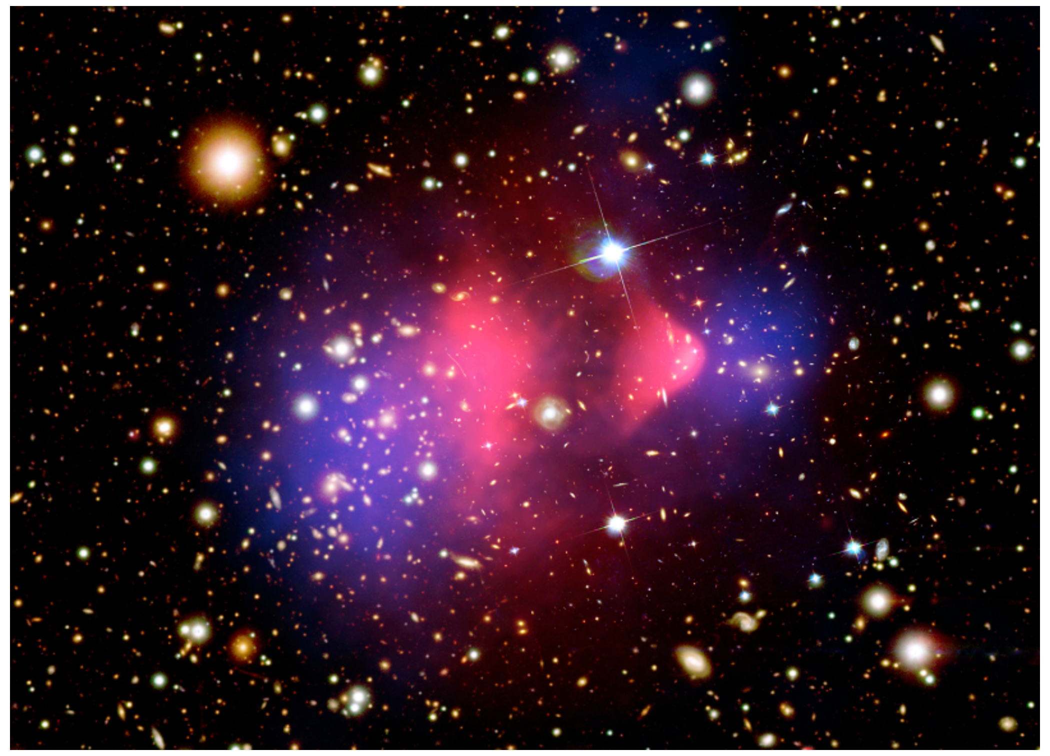
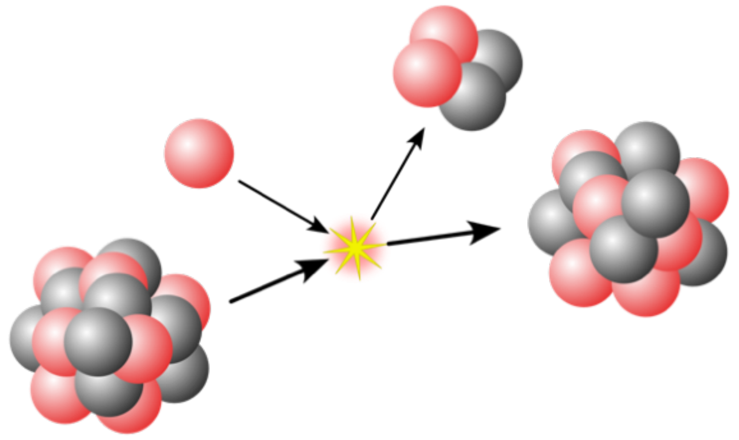
From QCD to Hadrons and Nuclei

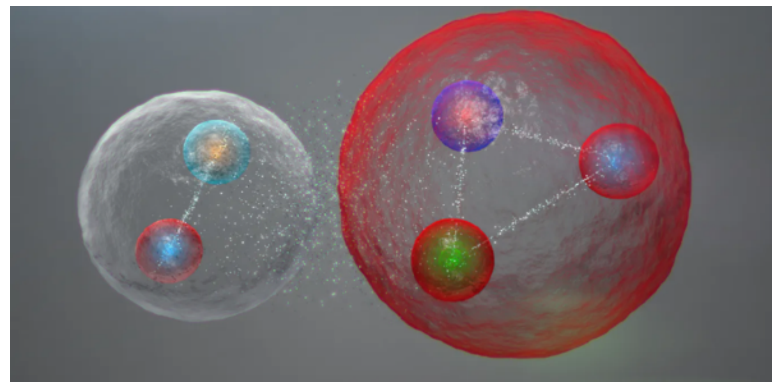
Advanced Subatomic Physics Course

Luca Doria (doria@uni-mainz.de)
PRISMA+ Cluster of Excellence and Institute for Nuclear Physics
Johannes Gutenberg University Mainz







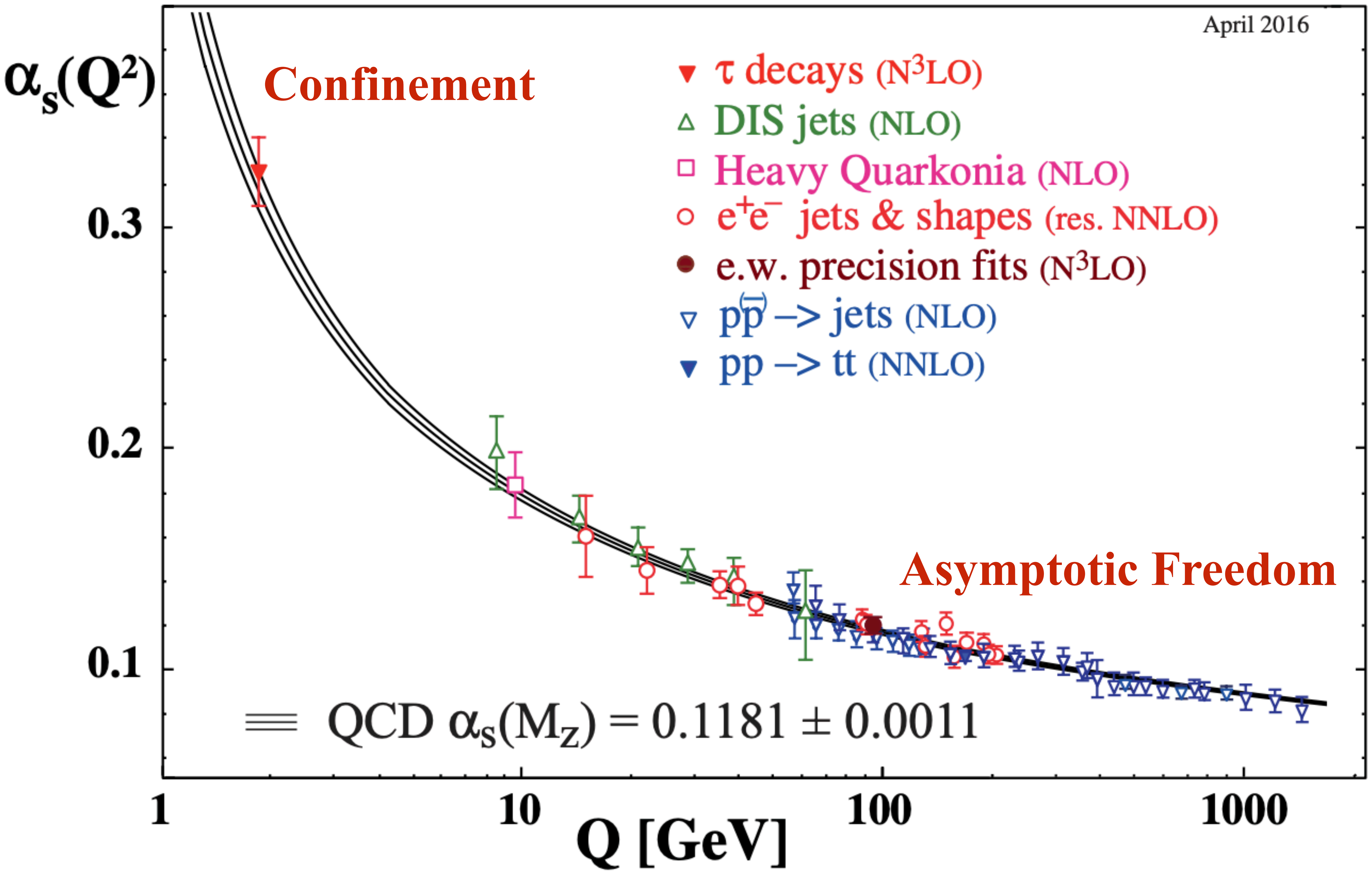


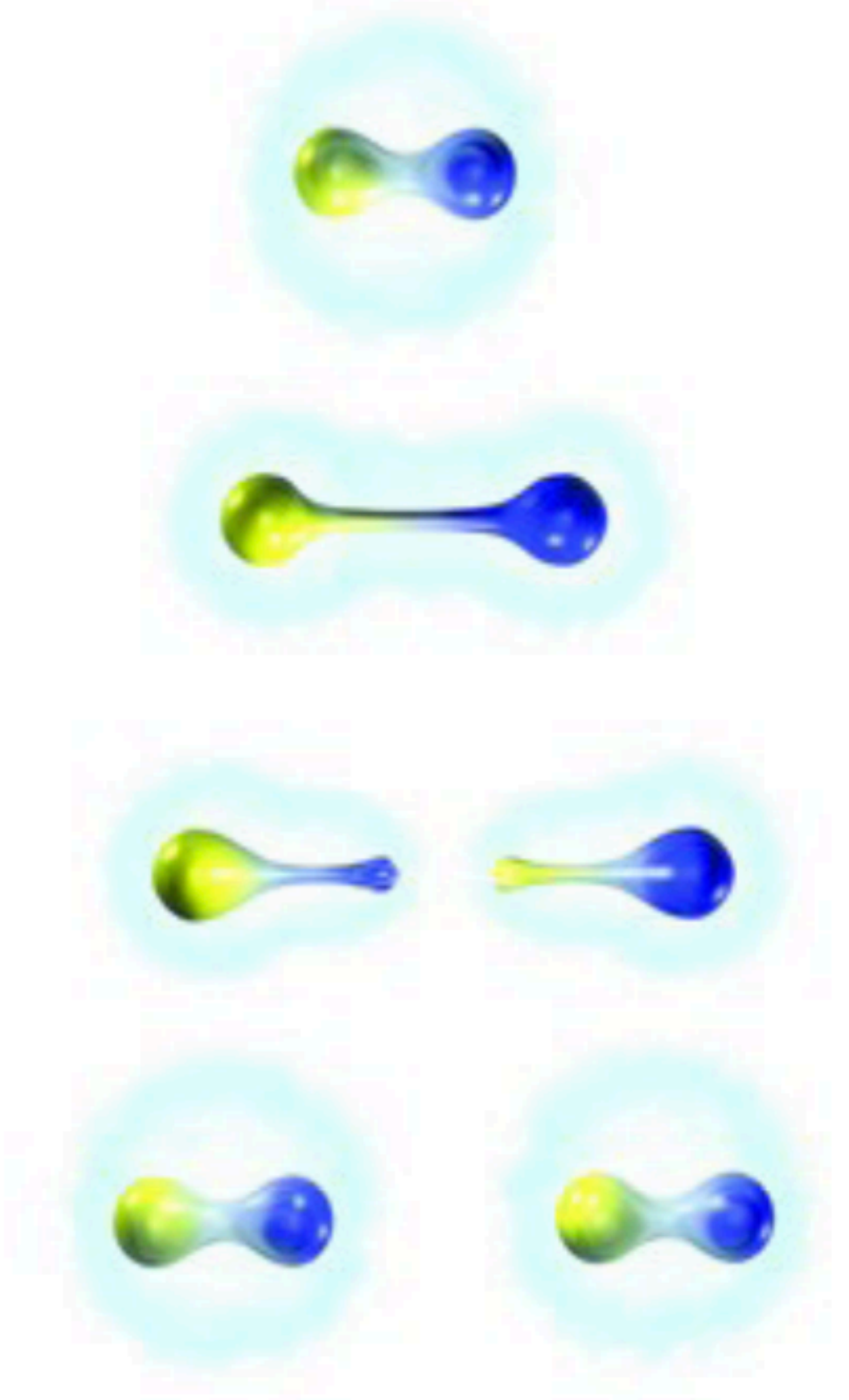
Sommersemester 2022

Syllabus

1. Introduction to strong interactions in the perturbative and non-perturbative regimes.
2. Hadrons and Nuclei
3. Electron and neutrino scattering experiments on hadrons and nuclei: form factors, elastic and inelastic scattering, resonances, deep inelastic physics.
4. Experimental methods and facilities with focus on MAMI and MESA at JGU Mainz.
5. Dark Matter
6. Search for dark matter with "intensity frontier" experiments, in particular, electron scattering experiments.
7. Search for dark matter with "direct detection" experiments with focus on argon.
8. Nuclear astrophysics and nuclear reactions of astrophysical relevance (in the Big Bang and stars).
9. Experiments for measuring astrophysical reactions with accelerators.
10. Discussion of a relevant published scientific paper on one of the topics discussed during the course.

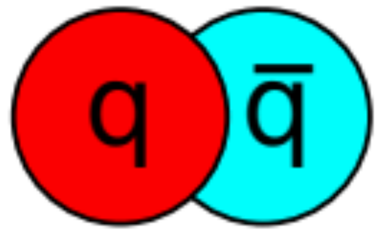
Consequences of a non-abelian group: Running Coupling



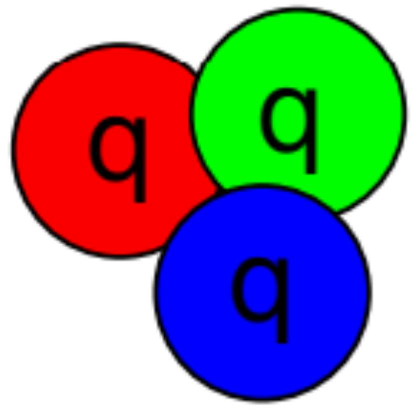


"String confinement"

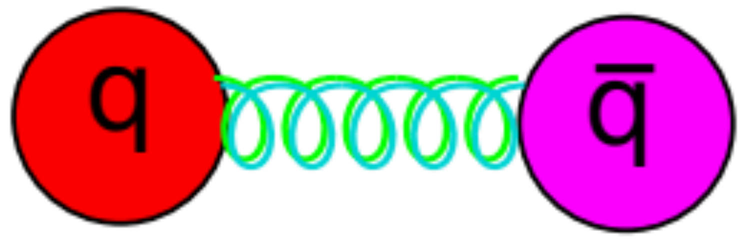
Quark Combinatorics



$q\bar{q}$ Meson



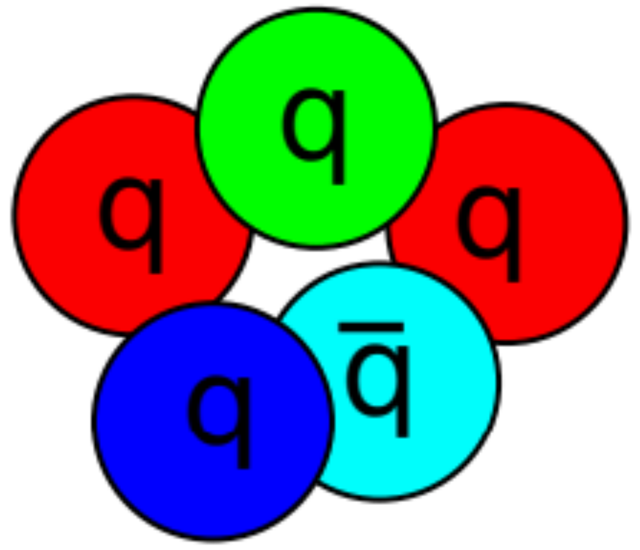
qqq Baryon



$q\bar{q}g$ Hybrid

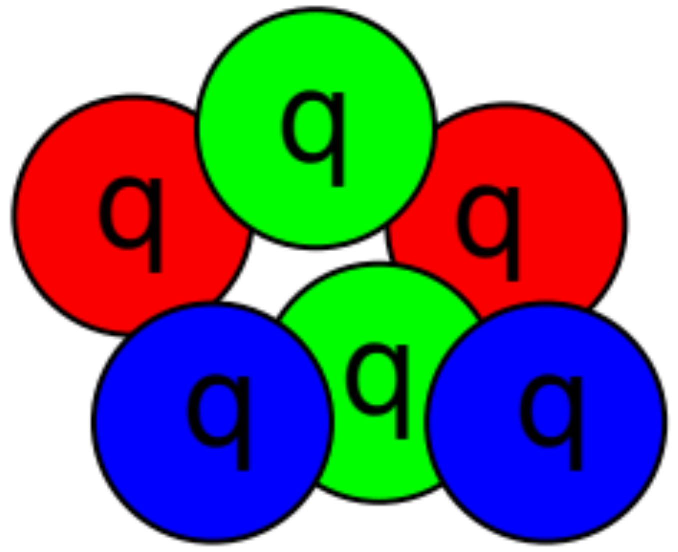
$q\bar{q}q\bar{q}$ Tetraquark

Observed!



$qqqq\bar{q}$ Pentaquark

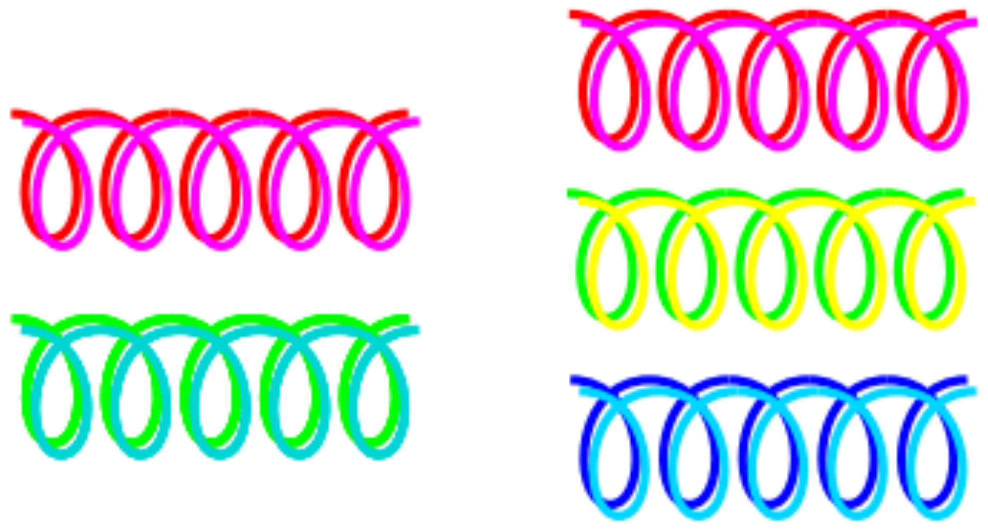
Observed!



$qqqqqq$ Dibaryon

$qqq\overline{qqq}$

Not observed yet

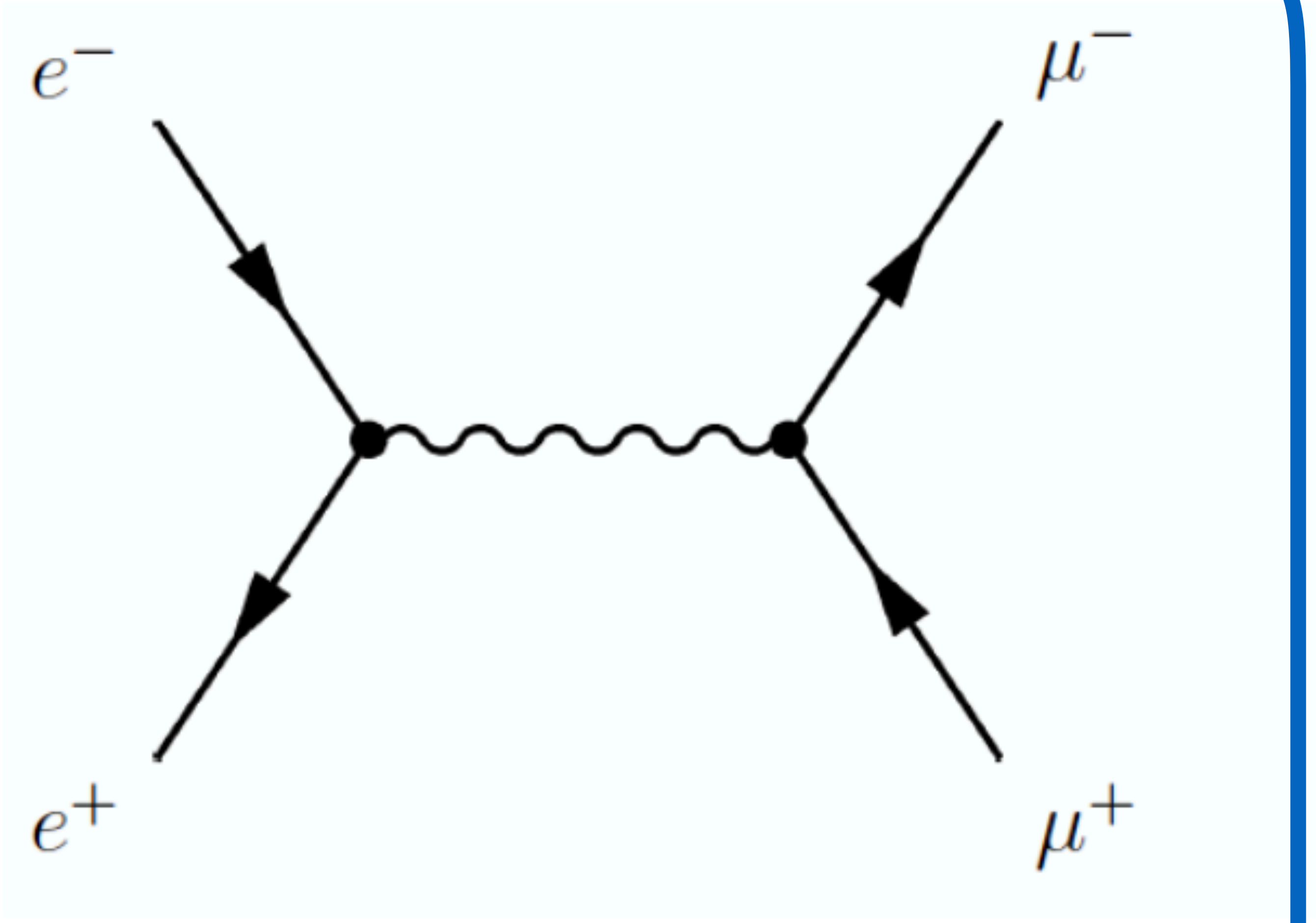


$g\bar{g}$, ggg Glueballs

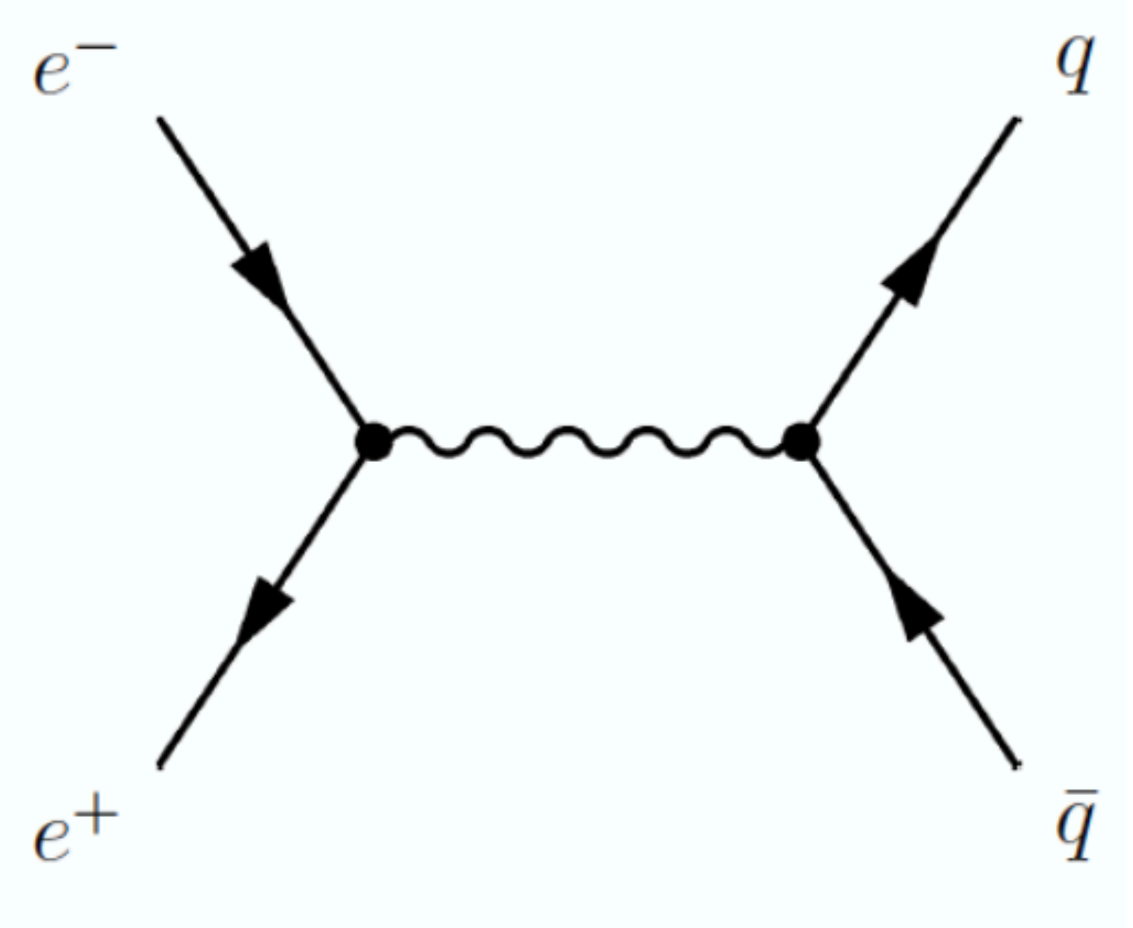
Not observed yet

Can be "observe" quarks?

Try to relate a known and "easy" to calculate QED process to a QCD process:



$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

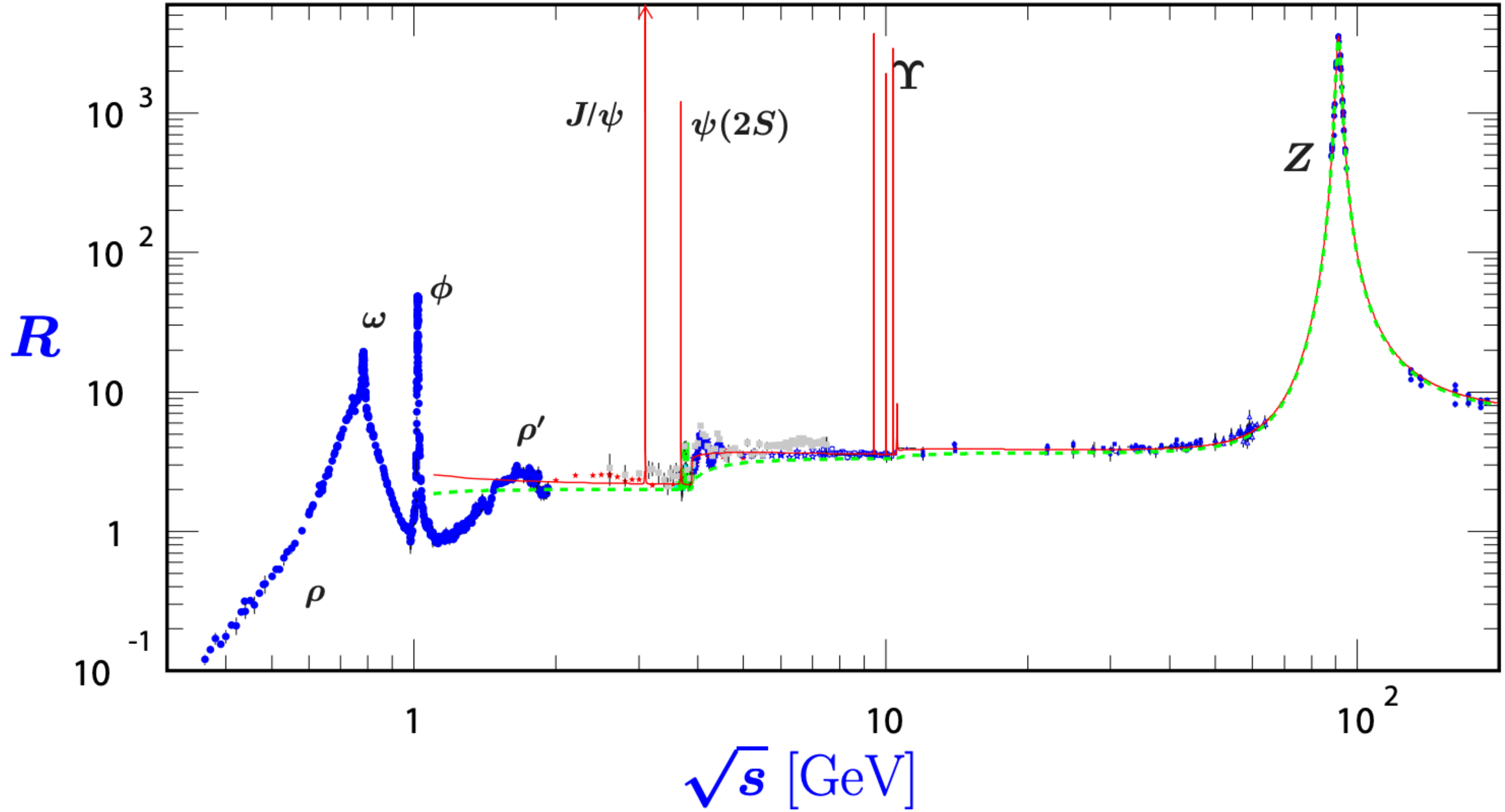


$$\sigma(e^+e^- \to q\bar{q}) = N_c e_q^2\, \sigma(e^+e^- \to \mu^+\mu^-)$$

$$e_q = \begin{cases} -\frac{1}{3} & \text{for } q = d,s,b \\ +\frac{2}{3} & \phantom{\text{for } q =} u,c,t \end{cases}$$

$$R = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q 3e_q^2$$

Can be "observe" quarks?



Can be "observe" quarks?

u,d,s

$$R = \sum_q 3e_q^2 = 3\left(\left(\frac{2}{3}\right)^2 + 1\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right) = 2$$

u,d,s,c

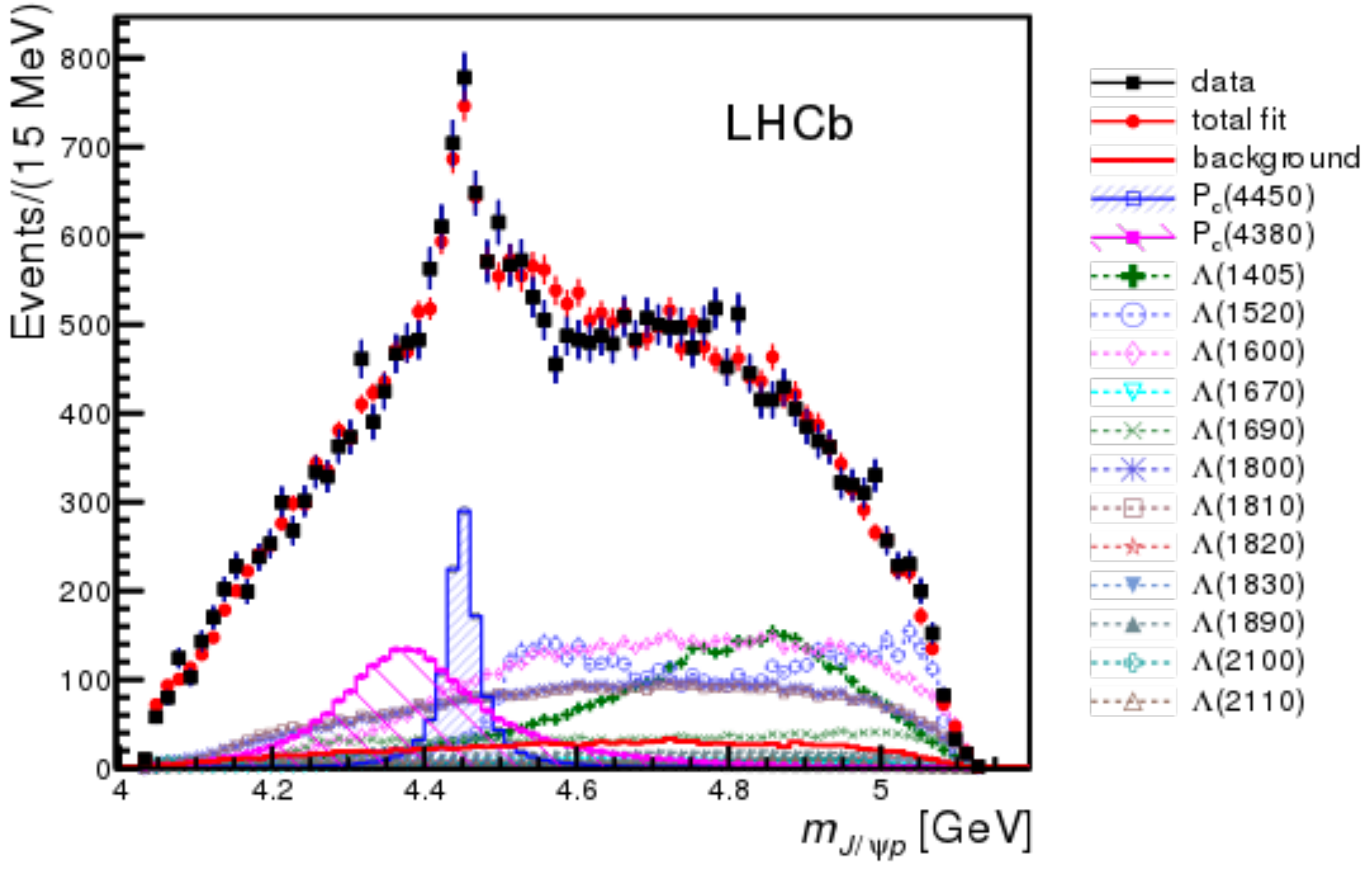
$$R = \sum_q 3e_q^2 = 3\left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right) = \frac{10}{3}$$

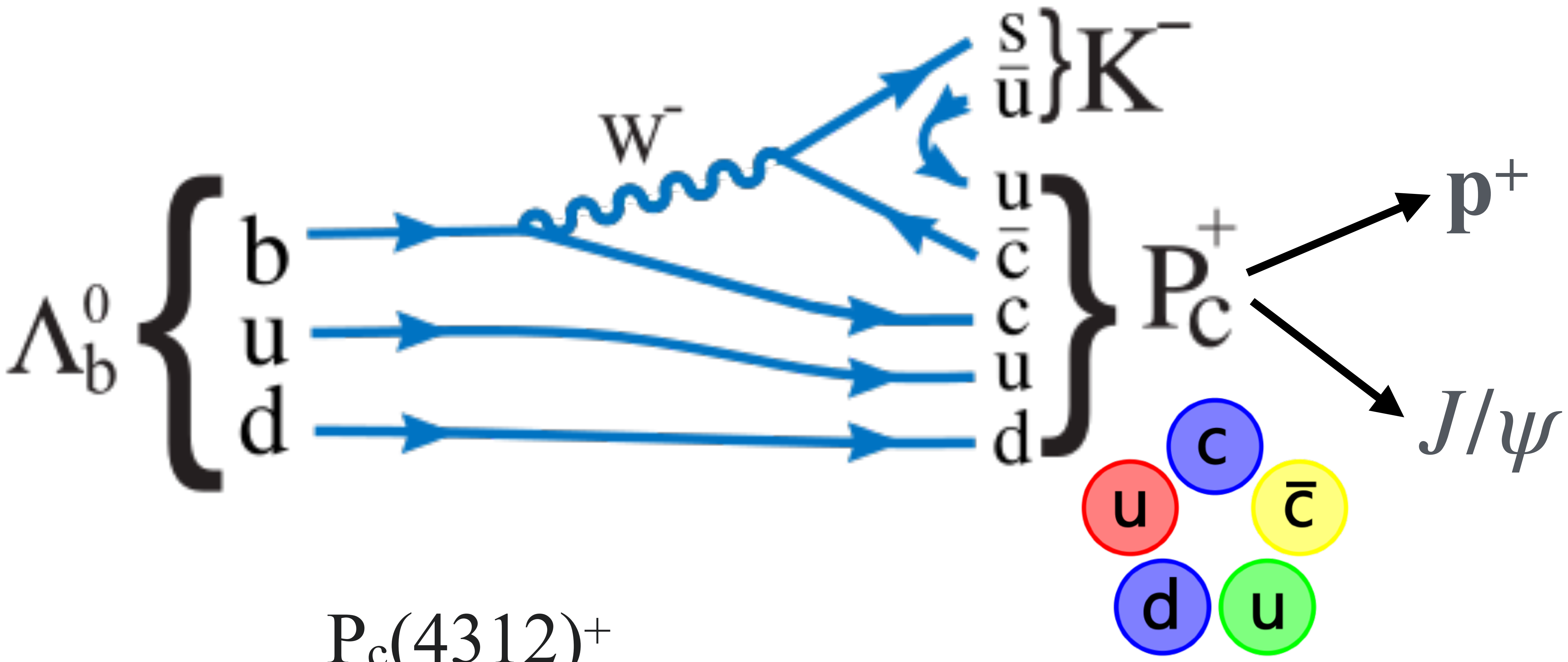
u,d,s,c,b

R = ?

Example of an exotic state: the Pentaquark

It is a state made out of 4 quarks and one anti-quark.
First claimed in 2003 but successive experiments were giving contradicting results.
Finally confirmed at LHC by LHCb in 2015 in bottom hyperon decays:





$P_c(4312)^+$

In 2019, the new pentaquark $P_c(4312)^+$ was discovered again by LHCb.

QCD at low energy and chiral symmetry

Separation of scales (EFT principle):

$$\begin{pmatrix} m_u = 0.005\,\mathrm{GeV} \\ m_d = 0.009\,\mathrm{GeV} \\ m_s = 0.175\,\mathrm{GeV} \end{pmatrix} \ll 1\,\mathrm{GeV} \leq \begin{pmatrix} m_c = (1.15 - 1.35)\,\mathrm{GeV} \\ m_b = (4.0 - 4.4)\,\mathrm{GeV} \\ m_t = 174\,\mathrm{GeV} \end{pmatrix}$$

Starting point for an EFT of QCD: massless quarks:

$$\mathcal{L}_{QCD} = \sum_{l=u,d,s} \bar{q}_l i \gamma^\mu D_\mu q_l - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Note: this operator is flavour-blind (acts only on colour and Dirac indices only)

QCD at low energy and chiral symmetry

Symmetry transformations of the chiral lagrangian

$$\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto U_L \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} = \exp\left(-i\sum_{a=1}^{8}\Theta_a^L \frac{\lambda_a}{2}\right) e^{-i\Theta^L} \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$$

$$\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto U_R \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} = \exp\left(-i\sum_{a=1}^{8}\Theta_a^R \frac{\lambda_a}{2}\right) e^{-i\Theta^R} \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

Symmetry transformations of the chiral lagrangian from Nöther's theorem.

$$\begin{aligned} L^{\mu,a} &= \bar{q}_L \gamma^\mu \frac{\lambda^a}{2} q_L, \quad \partial_\mu L^{\mu,a} = 0, \\ R^{\mu,a} &= \bar{q}_R \gamma^\mu \frac{\lambda^a}{2} q_R, \quad \partial_\mu R^{\mu,a} = 0 \end{aligned}$$

$2 \times (8+1)$ conserved currents

QCD at low energy and chiral symmetry

Instead of the chiral currents, it is useful to consider the linear combinations

“Vector” current $$V^{\mu,a} = R^{\mu,a} + L^{\mu,a} = \bar{q}\gamma^\mu \frac{\lambda^a}{2} q,$$

“Axial” current $$A^{\mu,a} = R^{\mu,a} - L^{\mu,a} = \bar{q}\gamma^\mu \gamma_5 \frac{\lambda^a}{2} q.$$

NOTE: The axial current is only conserved classically. After quantisation, this is no longer true. The non-conservation of a classical current in the quantum version of the theory is called **anomaly**.

$$\partial_\mu A^\mu = \frac{3g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathcal{G}_a^{\mu\nu} \mathcal{G}_a^{\rho\sigma}$$

Chiral Symmetry Breaking due to Quark Masses

If the quark masses are considered, the corresponding mass term is not chirally invariant:

$$\mathcal{L}_M = -\bar{q}Mq = -(\bar{q}_R M q_L + \bar{q}_L M q_R)$$

and the currents (including the anomaly) are not conserved:

$$\begin{aligned}
\partial_\mu V^{\mu,a} &= i\bar{q}[M, \frac{\lambda_a}{2}]q, \\
\partial_\mu A^{\mu,a} &= i\left(\bar{q}_L\{\frac{\lambda_a}{2}, M\}q_R - \bar{q}_R\{\frac{\lambda_a}{2}, M\}q_L\right) = i\bar{q}\{\frac{\lambda_a}{2}, M\}\gamma_5 q, \\
\partial_\mu V^\mu &= 0, \\
\partial_\mu A^\mu &= 2i\bar{q}M\gamma_5 q + \frac{3g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathcal{G}_a^{\mu\nu}\mathcal{G}_a^{\rho\sigma}
\end{aligned}$$

Summary on Chiral Symmetry

1) In the massless limit, the QCD lagrangian has SU(3)xSU(3) chiral symmetry.
2) 18 currents are conserved classically, corresponding to SU(3)xSU(3) and U(1)xU(1)
3) Quantum-mechanically, the axial current is not conserved (anomaly).
4) The single flavour currents $\bar{u}\gamma u$, $\bar{d}\gamma d$, and $\bar{s}\gamma s$ are always conserved, even in the case of massive quarks. This reflects the flavour independence of the strong interactions and the diagonality of the mass matrix.
5) If we consider $m_u = m_d = m_s$ (non-zero), all the vector currents are conserved.
   This is the SU(3) symmetry (the "eightfold way") originally advocated by Gell-Mann (flavour symmetry, no color)

Symmetry Breaking

Quark masses **explicitly** break chiral symmetry. There is another type of breaking which happens when the ground state is not invariant under the same group of the hamiltonian. In this case, we say that a (continuous) symmetry is **spontaneously** broken.

We will consider only a simpler abelian case, considering KG the lagrangian

$$\mathscr{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - \underbrace{\boxed{m^2 \phi^* \phi - \lambda (\phi^* \phi)^2}}_{-V}$$

which is invariant under the (global) U(1) transformation: $\phi \rightarrow \phi' = e^{i\Lambda} \phi$

Symmetry Breaking

The minimum of the potential is given by: $$\frac{\partial V}{\partial|\phi|} = 2m^2|\phi| + 4\lambda|\phi|^3 = 0$$

Two cases:

$m^2 > 0$

$$\boxed{\phi = \phi^* = 0}$$

$m^2 < 0$

$$\boxed{\phi^*\phi = |\phi|^2 = -\frac{m^2}{2\lambda} = a^2}$$

In this case, the VEV is not zero:

$$|\langle 0|\phi|0\rangle|^2 = a^2$$

Symmetry Breaking

In order to understand what is happening in the $m^2 < 0$ case, we change field variables: from a complex field to its polar representation:

$$\phi(x) = \rho(x)\mathrm{e}^{i\theta(x)}$$

with the VEVs: $\langle 0|\,\rho\,|0\rangle = a; \quad \langle 0|\,\theta\,|0\rangle = 0$

For getting back a field with zero VEV, we change again coordinates and "shift" the field value as:

$$\phi(x) = (\rho'(x) + a)\mathrm{e}^{i\theta(x)}$$

This can be seen as an expansion around the ground state: $\rho \approx a + \rho'$.
The VEVs are now $\langle 0|\,\rho'\,|0\rangle = \langle 0|\,\theta\,|0\rangle = 0$

Symmetry Breaking

Let's now plug in the lagrangian the new shifted field remembering that in this case $-m^2/(2\lambda) = a^2$:

$$
\begin{aligned}
(\partial_\mu \phi)(\partial^\mu \phi^*) &= (\partial_\mu \rho' \mathrm{e}^{i\theta} + (\rho' + a) i\mathrm{e}^{i\theta} \partial_\mu \theta)(\partial^\mu \rho' \mathrm{e}^{-i\theta} + (\rho' + a) i\mathrm{e}^{-i\theta} \partial^\mu \theta) \\
&= (\partial_\mu \rho')(\partial^\mu \rho') + (\rho' + a)(\partial_\mu \theta)(\partial^\mu \theta) + (\rho' + a) i (\partial_\mu \theta \partial^\mu \rho' - \partial_\mu \rho' \partial^\mu \theta) \\
&= (\partial_\mu \rho')(\partial^\mu \rho') + (\rho' + a)(\partial_\mu \theta)(\partial^\mu \theta)
\end{aligned}
$$

$$
\begin{aligned}
V &= m^2 \phi^* \phi + \lambda (\phi^* \phi)^2 = m^2 (\rho' + a)^2 + \lambda (\rho' + a)^4 \\
&= -2\lambda a^2 (\rho'^2 + 2\rho' a + a^2) + \lambda (\rho'^4 + 4a\rho'^3 + 6a^2 \rho'^2 + 4a^3 \rho' + a^4) \\
&= \lambda (\rho'^4 + 4a\rho'^3 + \boxed{4a^2 \rho'^2} - a^4).
\end{aligned}
$$

Symmetry Breaking

After symmetry breaking, we have two fields, where one is massless and one is massive:

$$\boxed{\begin{aligned} m_\theta^2 &= 0 \\ m_\rho^2 &= 4\lambda a^2 \end{aligned}}$$

This is an example of the **Goldstone Theorem**: for every broken symmetry generator, one massless particle (a Goldstone boson) emerges in the particle spectrum. The case of broken <u>gauge symmetries</u> is covered by the **Higgs mechanism**.
<u>For QCD</u>: light mesons can be seen as the Goldstone bosons of the broken chiral symmetry.

The Chiral Lagrangian

Witten-Vafa Theorem:

In the chiral limit, the ground state is invariant under $SU(3)_V x U(1)_V$.

Coleman's theorem:

If the vacuum is invariant under $SU(3)_V x U(1)_V$ so is the Hamiltonian (not the contrary)

The ground state of QCD is non invariant under axial transformations: from Goldstone's Theorem we expect 8 massless bosons.

The Chiral Lagrangian

Symmetry breaking pattern:

$$SU(3)_R \times SU(3)_L \times U(1)_V \longrightarrow SU(3)_V \times U(1)_V$$

Symmetry group required in the chiral limit. Must contain 8 pseudo-scalar degrees of freedom transforming as an octet under SU(3)V

Invariance of the ground state

SU(3) matrix:

$$U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right)$$

Fields

$$\phi(x) = \sum_{a=1}^{8} \lambda_a \phi_a(x) \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

The Chiral Lagrangian

Most general, chirally invariant, effective Lagrangian with the minimal number of derivatives is:

$$\mathcal{L}_{\text{eff}} = \frac{F_0^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)$$

Expanding the exponentials:

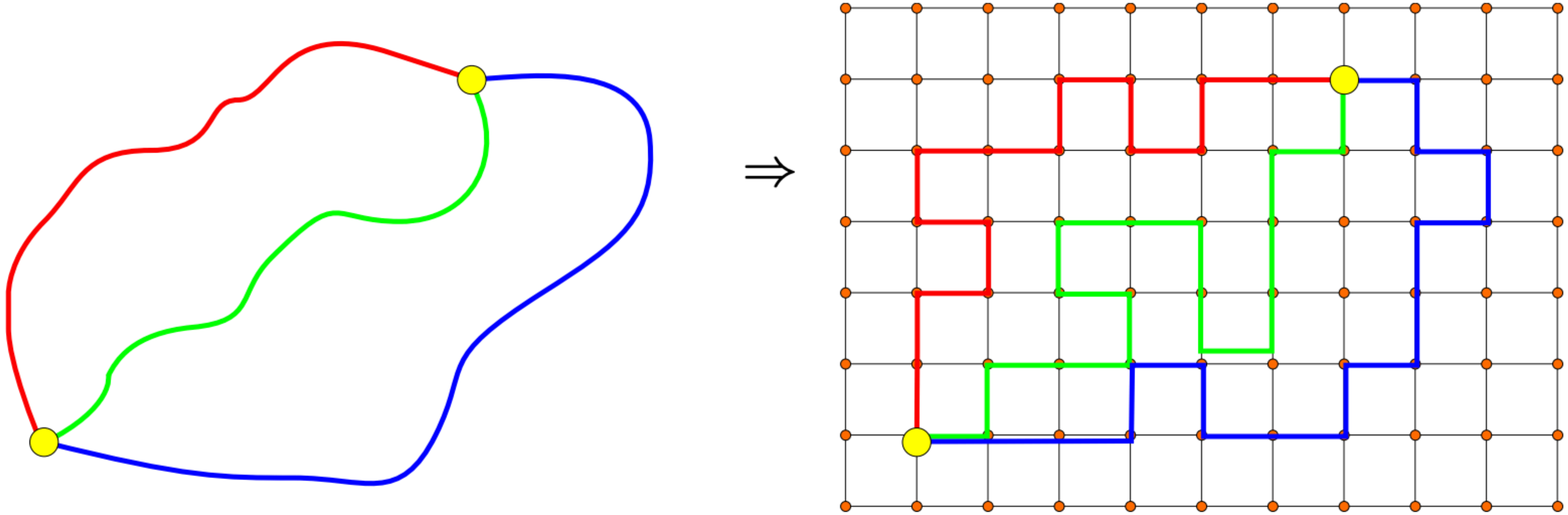
$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{F_0^2}{4} \mathrm{Tr}\left[\frac{i\partial_\mu \phi}{F_0}\left(-\frac{i\partial^\mu \phi}{F_0}\right)\right] + \cdots = \frac{1}{4}\mathrm{Tr}(\lambda_a \partial_\mu \phi_a \lambda_b \partial^\mu \phi_b) + \cdots \\ &= \frac{1}{4}\partial_\mu \phi_a \partial^\mu \phi_b \mathrm{Tr}(\lambda_a \lambda_b) + \cdots = \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a + \mathcal{L}_{\text{int}},\end{aligned}$$

Standard kinetic term

Goldstone bosons interactions (starting with 4-fields terms)

Lattice QCD

In the strong coupling (confinement) regime, perturbative techniques are not allowed. A direct (or "brute-force") approach consists in discretising space-time and solve numerically (with a MC technique) QCD with the path-integral formalism.



Lattice QCD

Path integral representation for transitions: $\psi(x_2,t_2) = \frac{1}{Z}\int e^{iS}\psi(x_1,t_1)\ \mathcal{D}x$

with action: $S = \int_{t_1}^{t_2} L(x,\dot{x},t)\ \mathrm{d}t$

Go to imaginary time (Euclidean path integral) for using MC techniques:

$$t \to i\tau$$

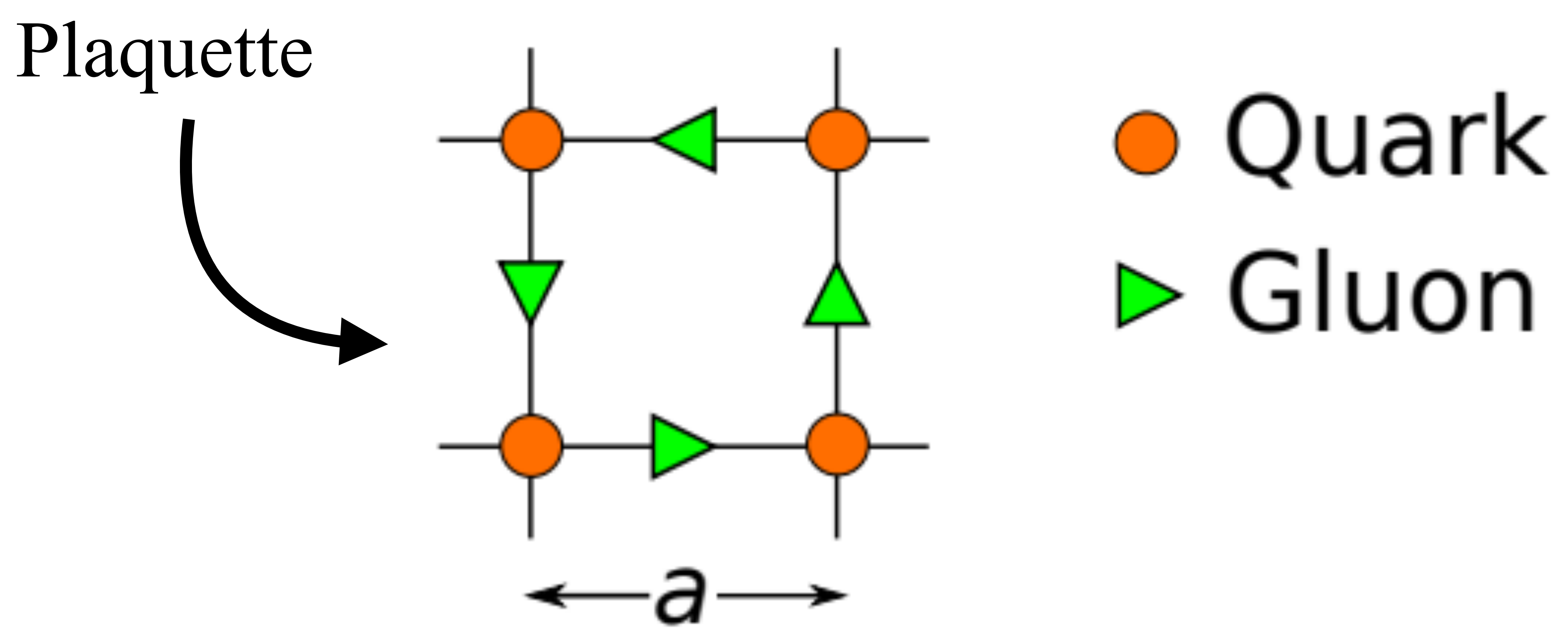
$$-(\mathrm{d}t^2) + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \to \mathrm{d}\tau^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

Define gauge fields through link variables: $U_\mu = \exp\left(iaG_\mu\left(n+\frac{\hat{\mu}}{2}\right)\right)$

Discretize the derivatives: $S = \int \bar{u}(iD_\mu\gamma_\mu + m)u\,d^4x \quad \to \quad D_\mu = \frac{1}{2a}\left[U_\mu(x)q(x+a\hat{\mu}) - U_\mu(x-a\hat{\mu})^\dagger q(x-a\hat{\mu})\right]$

Note: fermion doubling problem!

Lattice QCD

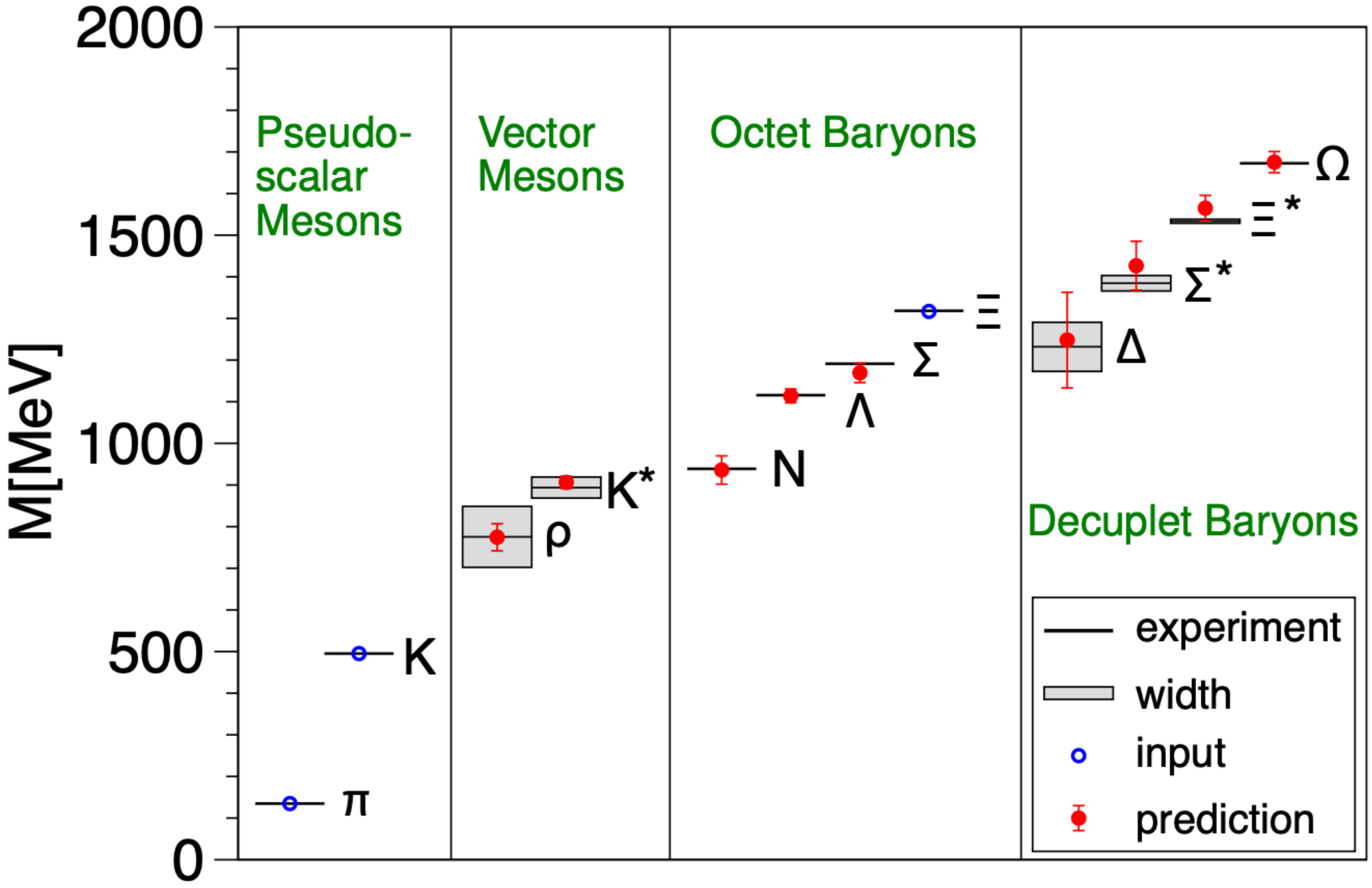


Gluon action: sum over plaquettes:

$$S = -\frac{1}{2g^2}\mathrm{Tr}\int G_{\mu\nu}G^{\mu\nu}d^4x \quad \rightarrow \quad S_L = -\frac{1}{2g^2}\sum a^4\mathrm{Tr}\left(1-U_{\mu\nu}(n)\right)$$

Discretization allows solution of the path integral. High dimensionality requires MC methods.
Calculations can be performed at different lattice spacings and the continuum result is obtained with extrapolation.
Discretization allows for different versions of the discretised lagrangian and appropriate choices for optimising numerical accuracy and speed have to be made.
Discretization of the fermionic part is also not unique and different lattice fermions can be used, with different computational properties.

Lattice QCD



S. Durr, et al., Science 322 (2008)

Summary

* Quantum chromodynamics (QCD) is based on the non-abelian group SU(3).
* Local gauge invariance is a fundamental property of QFTs describing elementary particles.
* The basic degrees of freedom of QCD are quarks and gluons.
* Quarks have 6 flavours (up, down, strange, charme, bottom, top) and are fermions with spin 1/2 .
* Gluons are 8 (like the generators of SU(3)) and carry color, therefore they interact also with themselves.
* In QED, the photon does not carry electric charge, so it does not interact with himself.

* The non-abelian structure of QCD implies the charge of its gauge bosons and further consequences are:
* Asymptotic freedom: the force becomes small for high momenta/energies and a perturbative treatment is possible.
* Confinement: the force becomes large at small momenta/energies. The theory becomes non-perturbative.

* Confinement implies that we cannot observe free coloured particles: quarks are confined in uncoloured objects like hadrons (made by three quarks) and mesons (quark-antiquark states). Other more exotic combinations can exist.