From QCD to Hadrons and Nuclei

Advanced Subatomic Physics Course (IV)

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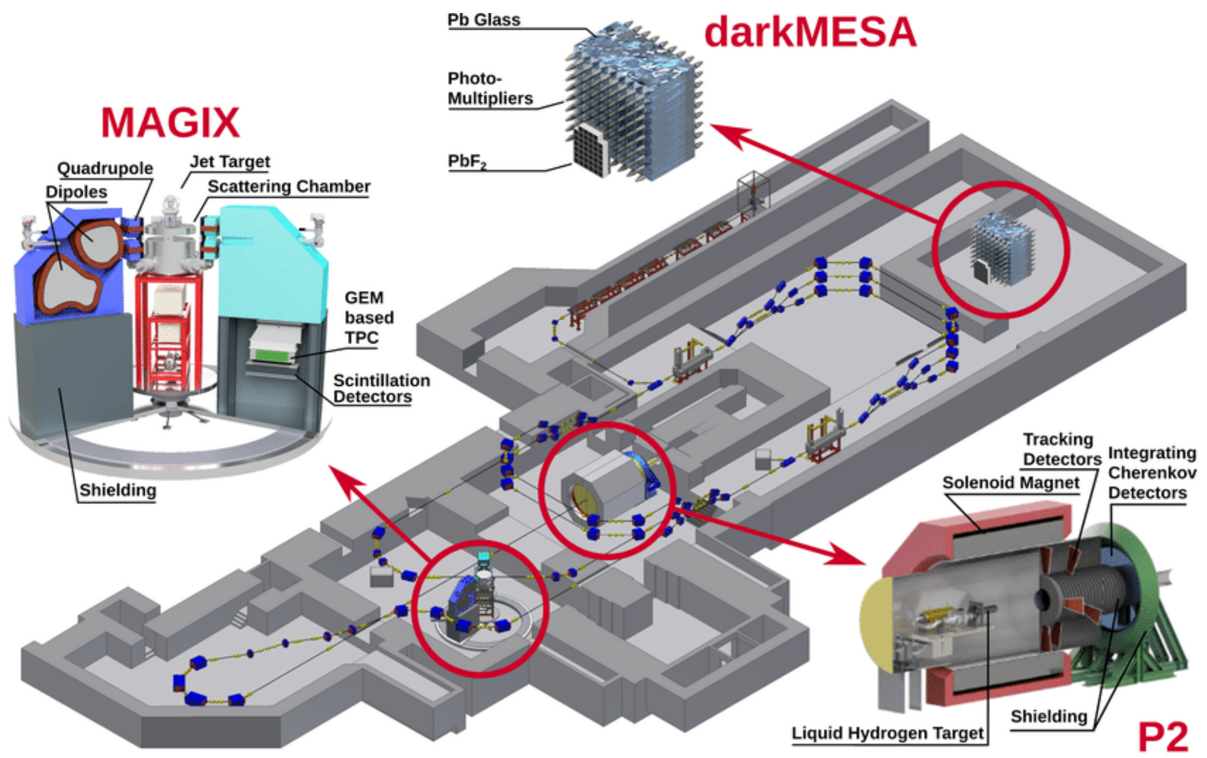
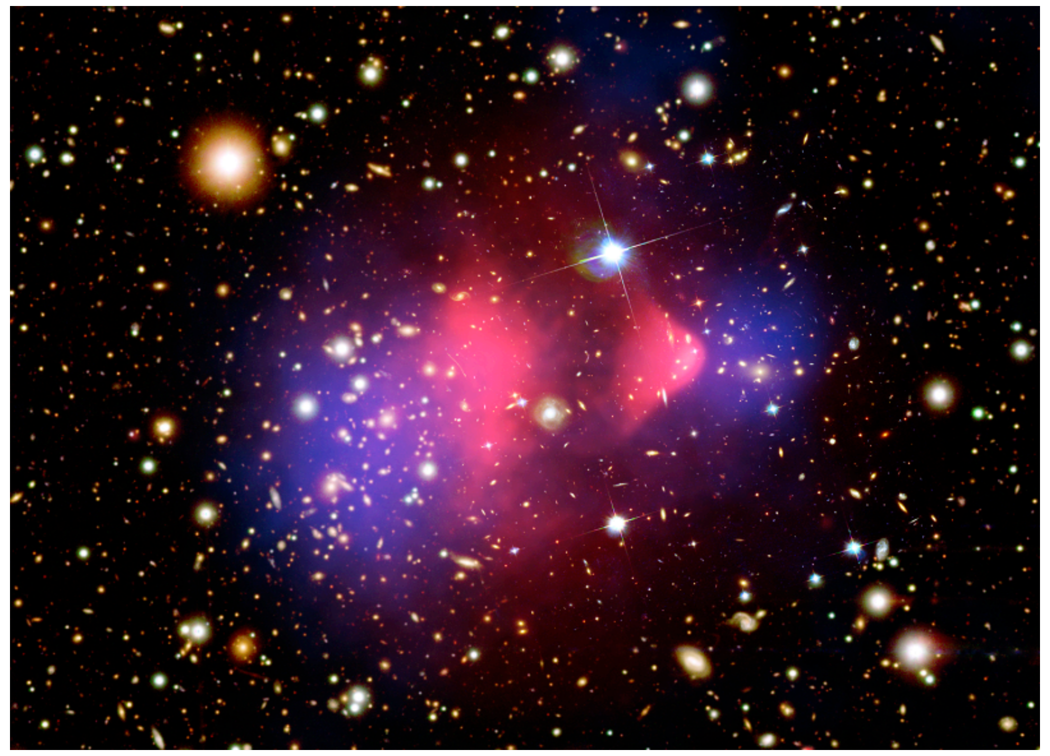
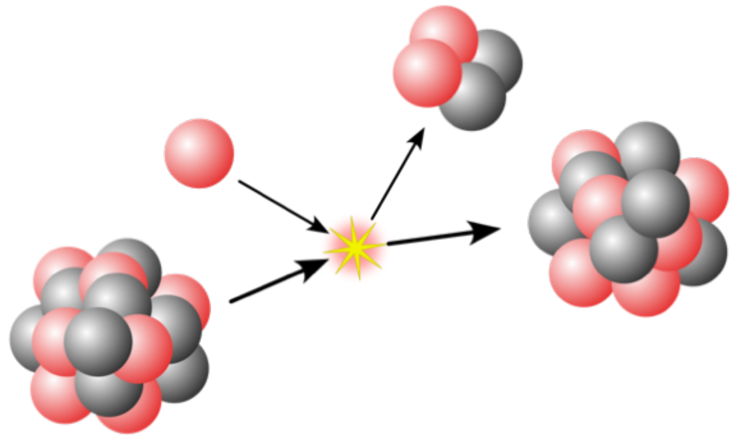
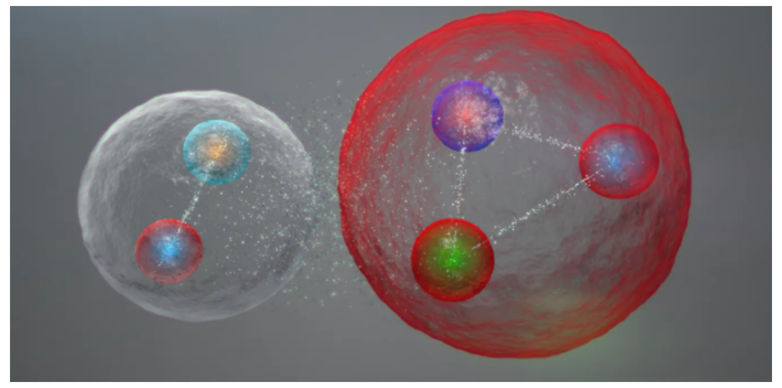








Sommersemester 2022

Syllabus

1. Introduction to strong interactions in the perturbative and non-perturbative regimes.
2. Hadrons and Nuclei
3. Electron and neutrino scattering experiments on hadrons and nuclei: form factors, elastic and inelastic scattering, resonances, deep inelastic physics.
4. Experimental methods and facilities with focus on MAMI and MESA at JGU Mainz.
5. Dark Matter
6. Search for dark matter with "intensity frontier" experiments, in particular, electron scattering experiments.
7. Search for dark matter with “direct detection” experiments with focus on argon.
8. Nuclear astrophysics and nuclear reactions of astrophysical relevance (in the Big Bang and stars).
9. Experiments for measuring astrophysical reactions with accelerators.
10. Discussion of a relevant published scientific paper on one of the topics discussed during the course.

Electron Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \left(\frac{e^2 Z^2}{4E\sin^2\theta/2}\right)^2$$

Point-like charged object:
Rutherford cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R$$

Extended charged object,
spin-less

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R |F(q)|^2$$

Extended charged object,
spin 1/2: Mott cross section

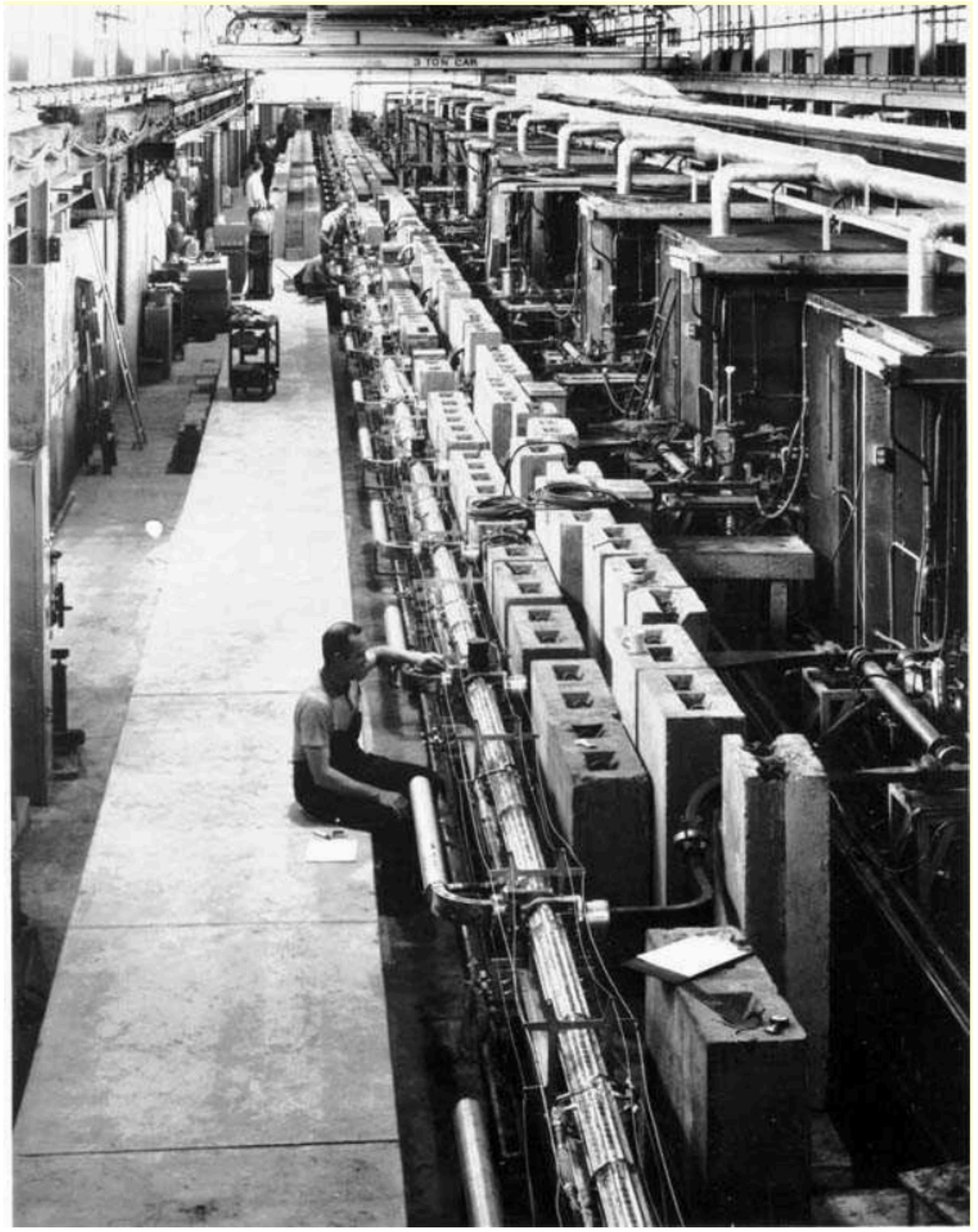
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M |F(q)|^2$$

$$F(q) = \int d^3r\ \rho(\mathbf{r}) \exp\left(i\mathbf{q}\cdot\mathbf{r}\right)$$

The form factor is the Fourier Transform of the charge distribution

Electron Scattering

Robert Hofstadter (1915-1990)
1961 Physics Nobel Prize

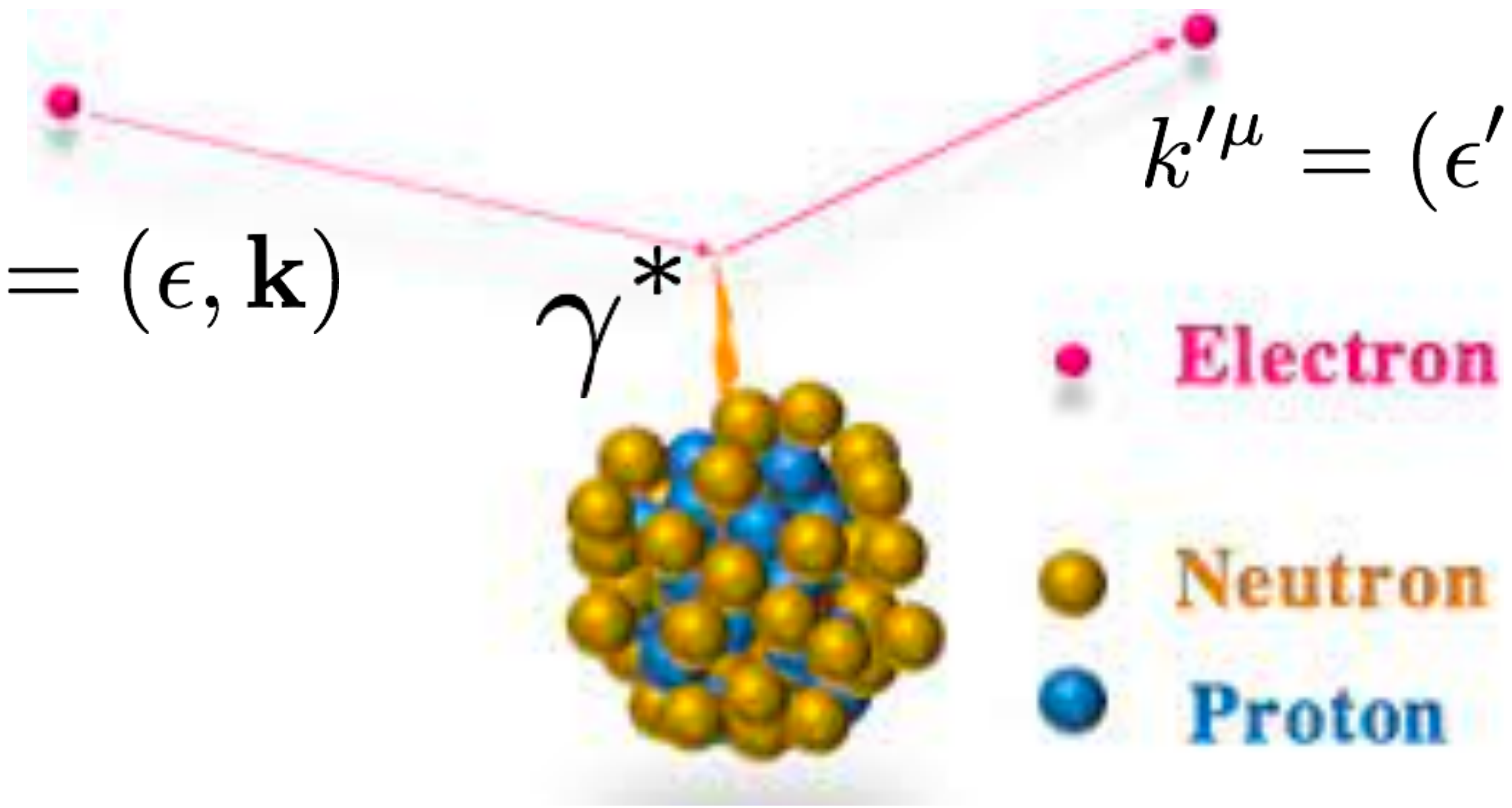


Stanford Linear Accelerator

Electron Scattering: Kinematics

$k^\mu = (\epsilon, \mathbf{k})$

$k'^\mu = (\epsilon', \mathbf{k}')$



$\omega = \epsilon - \epsilon'$ Energy transfer

$\mathbf{q} = \mathbf{k} - \mathbf{k}'$ Momentum transfer

$$\epsilon = \sqrt{\mathbf{k}^2 + m^2} \simeq |\mathbf{k}|$$
$$\epsilon' = \sqrt{\mathbf{k}'^2 + m^2} \simeq |\mathbf{k}'|$$

$$\Rightarrow \quad \omega^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2|\mathbf{k}||\mathbf{k}'|$$
$$|\mathbf{q}|^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2\,\mathbf{k}\cdot\mathbf{k}'$$
$$|\mathbf{q}|^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2\,|\mathbf{k}||\mathbf{k}'|\cos\theta$$

$\Rightarrow \mathbf{q}^2 \geq \omega^2$ Space-like photon

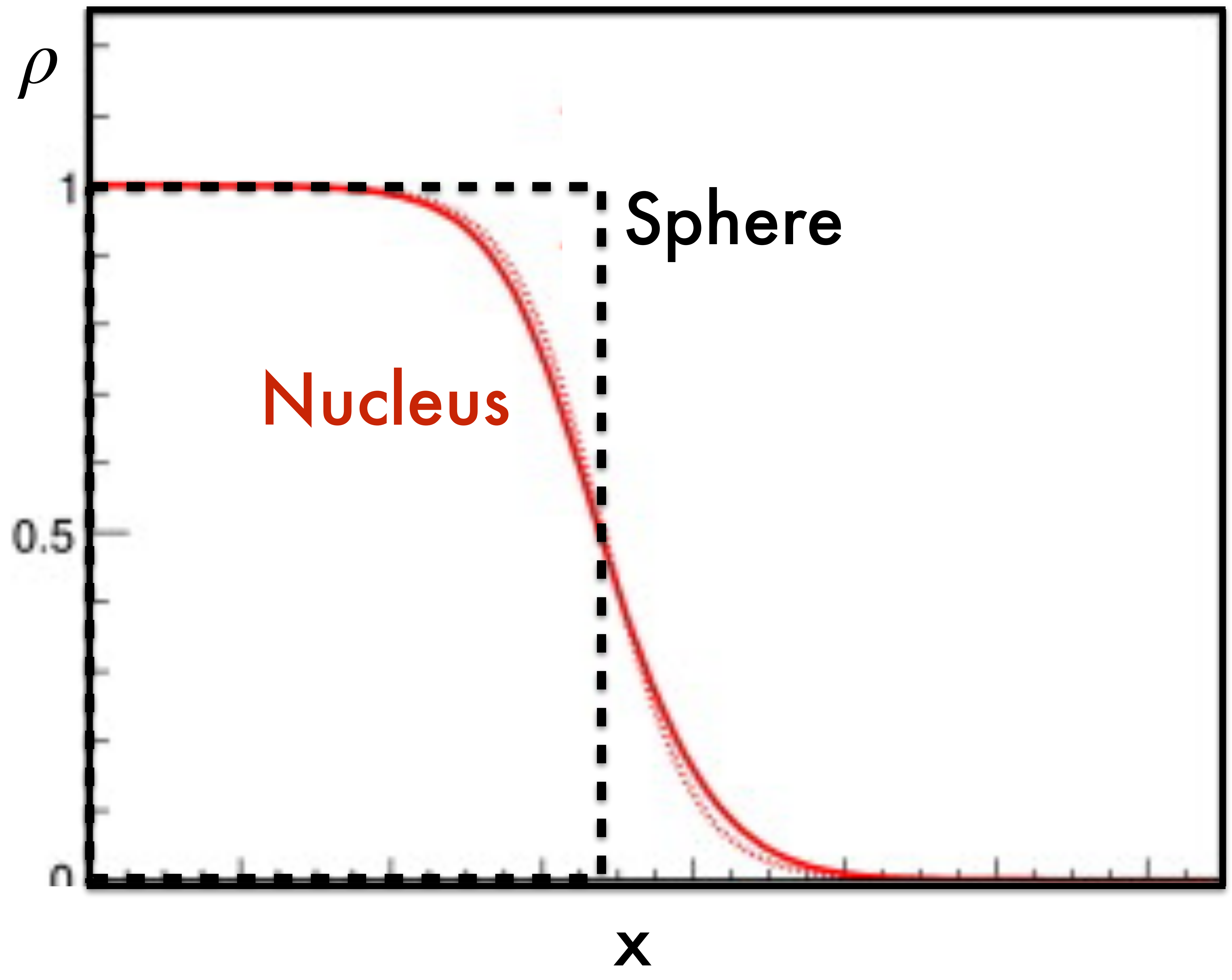
Fourier Transform with spherical symmetry

$$F(q^2) = \int dx x^2 \rho(x) \int d\phi \sin\theta d\theta e^{iqx\cos\theta}$$

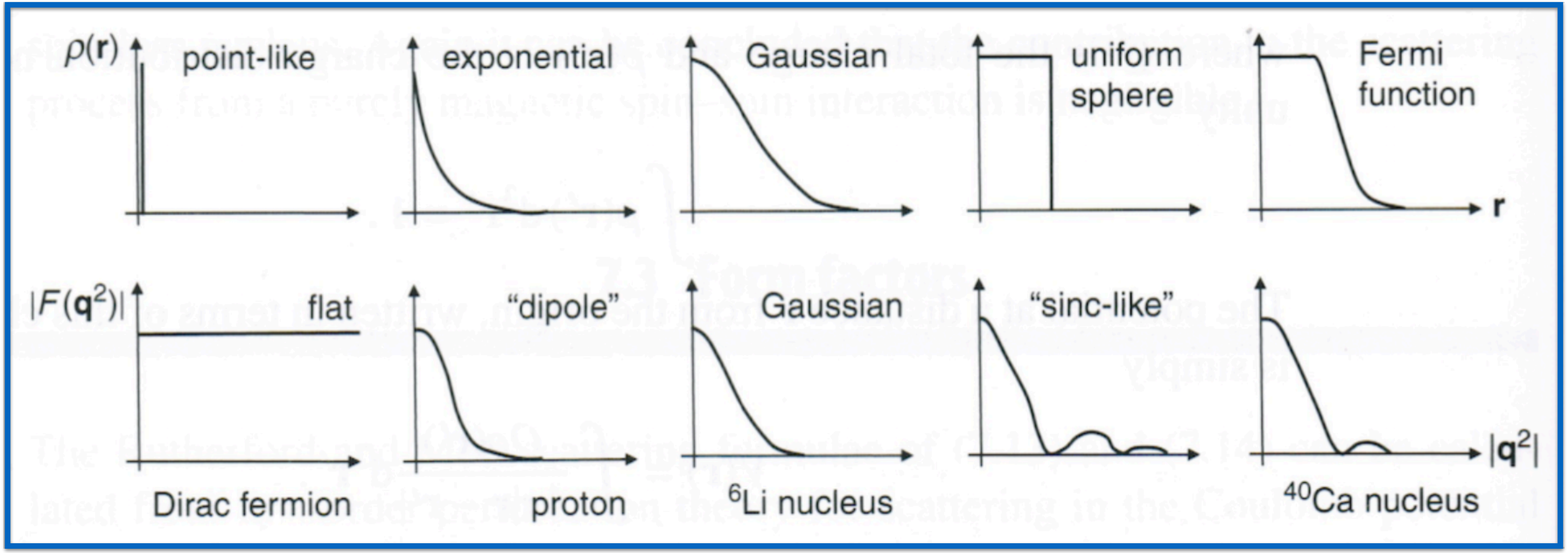
$$= 2\pi \int dx x^2 \rho(x) \int \sin\theta d\theta e^{iqx\cos\theta}$$

$$= \frac{4\pi}{q} \int dx x \rho(x) \sin qx$$

—> Only the radial (x) part contains the physics.



Examples



Common procedure: ansatz for charge distribution and then fit to the data its FF.

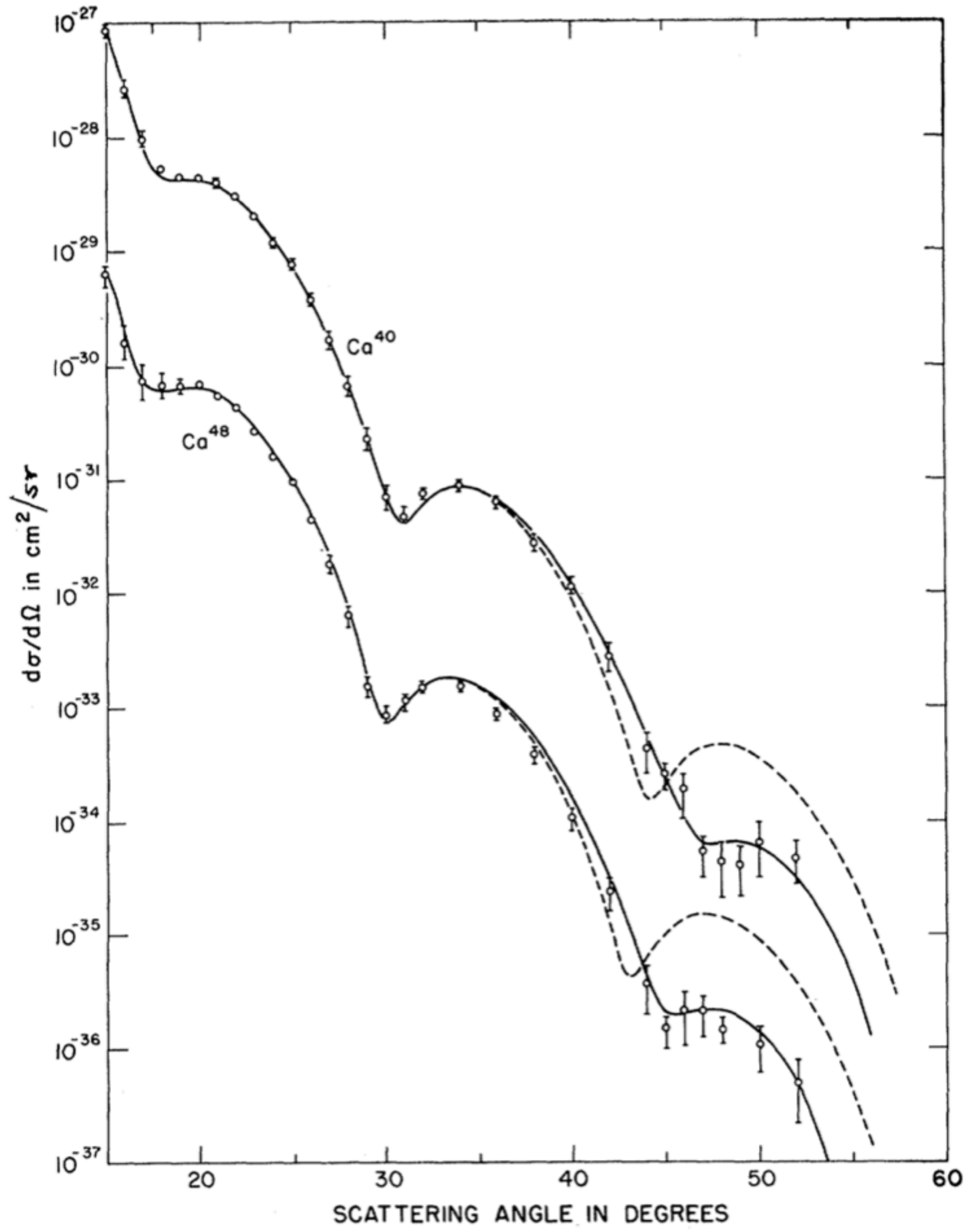
Examples

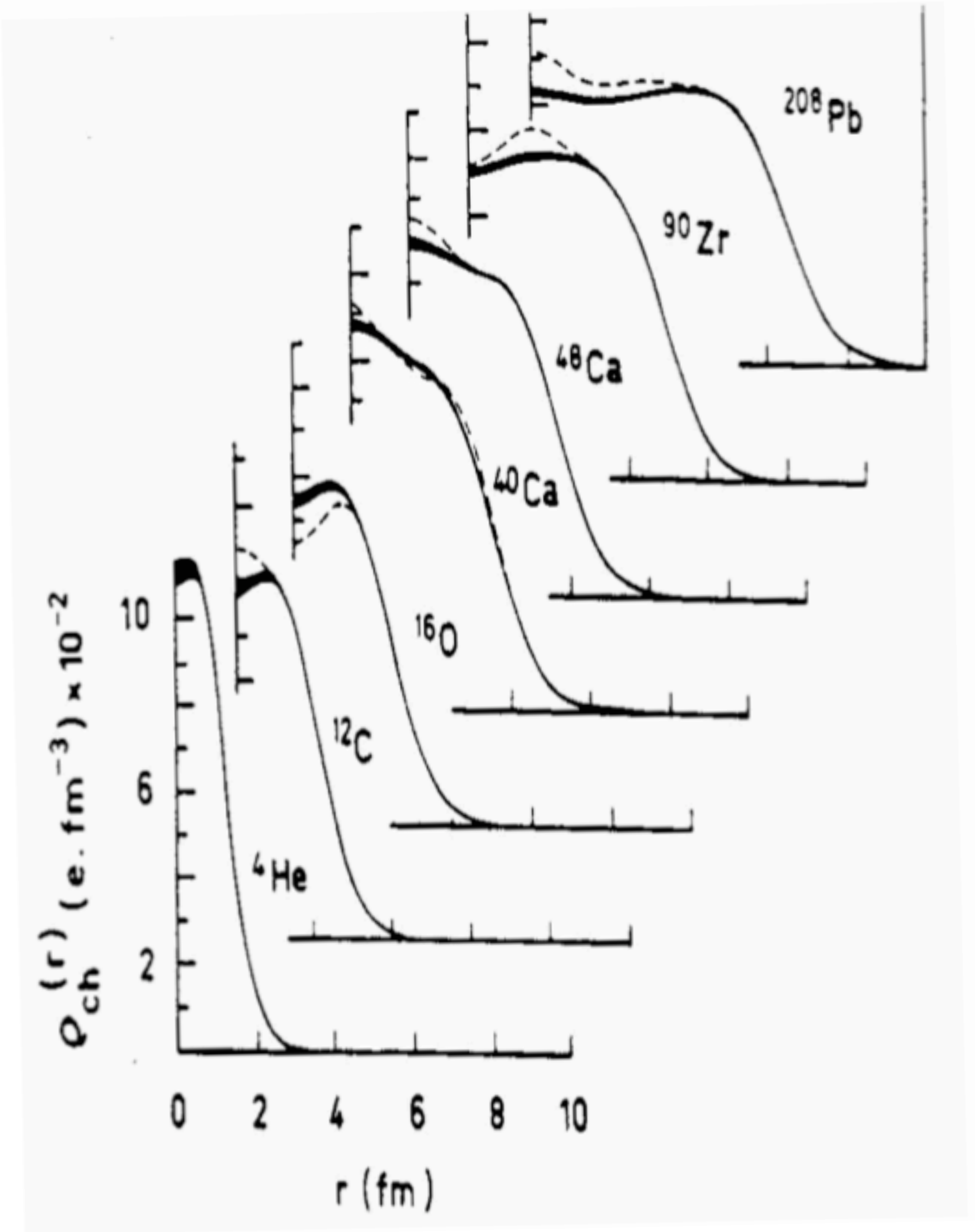
exponential, $\rho(x) = \rho_0 e^{-x/a}$;
Gaussian, $\rho(x) = \rho_0 \exp[-(x/b)^2]$;
uniform, $\rho(x) = \rho_0,\ x < kR,$
$= 0,\ x > kR$;

FF

exponential, $F = (1+q^2a^2)^{-2}$;
Gaussian, $F = \exp(-q^2b^2/4)$;
uniform, $F = 3(\sin qkR - qkR\cos qkR)/(qkR)^3$

smoothed uniform, $\rho = \rho_0[1+e^{K(x-c)}]^{-1}$;
wine-bottle, $\rho = \rho_0(1+(x/d)^4)[1+e^{K(x-c)}]^{-1}$





The Charge Radius

Considering the low-q limit (you don't "see" the details..)

$$F(q^2) = \frac{4\pi}{q}\int dx\, x\rho(x)\sin qx = \frac{4\pi}{q}\int dx\, x\rho(x)\left(qx - \frac{(qr)^2}{3!}\right) + \ldots$$

$$= 4\pi \int dx\, x^2\rho(x) - \frac{q^2}{6}4\pi\int dx\, x^4\rho(x) + \ldots$$

$$= Z(1 - \frac{q^2}{6}\langle r^2\rangle + \ldots)$$

$$\boxed{\begin{aligned}\langle r^2\rangle &= \int d^3r\ r^2\rho(\mathbf{r}) = \\ &= 4\pi\int dr\ r^4\rho(r)\end{aligned}}$$

1st correction proportional to RMS radius

The FF is just the charge at 1st order

The Charge Radius

The RMS radius can be extracted from the data measuring the form factor at low-q and derive it wrt q^2.

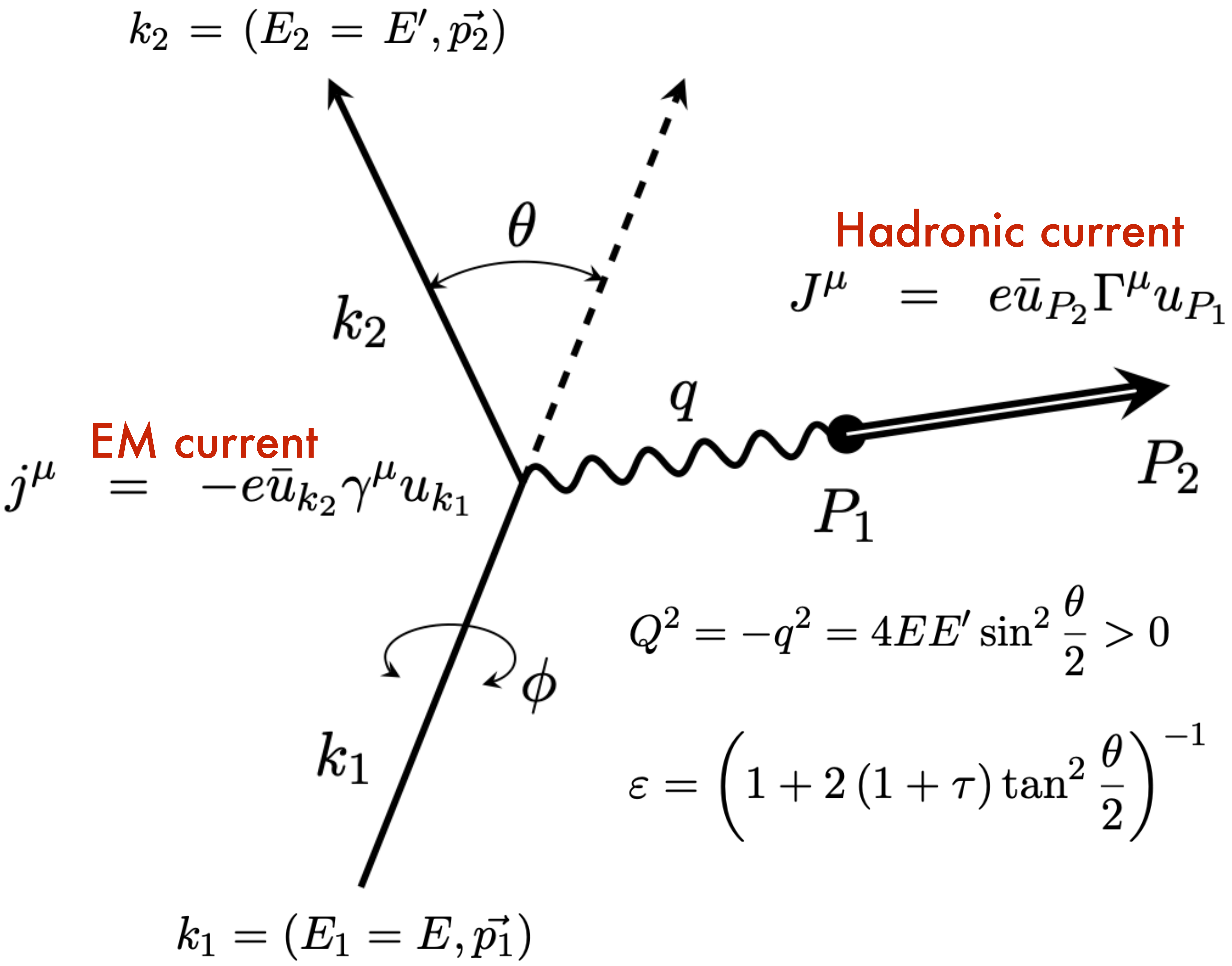
$$\langle r^2 \rangle = -6 \frac{dF(q^2)}{dq^2}\bigg|_{q=0}$$

For nuclei: $R \simeq r_0 A^{1/3}$ with $r_0 = 1 - 1.25\text{fm}$

This prediction (liquid drop model) is not followed by exotic nuclei (e.g. “halo nuclei” far from the stability valley).

Nucleon Form Factors

Kinematics



$$Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2} > 0$$

$$\varepsilon = \left(1 + 2\,(1+\tau)\tan^2 \frac{\theta}{2}\right)^{-1}$$

Dynamics

Born invariant transition amplitude (1st order) $\mathcal{M} = j_\mu \frac{g^{\mu\nu}}{q^2} J^\nu = j_\mu \frac{1}{q^2} J^\mu$

photon propagator

Hadronic "form factor":

$$\Gamma^\mu = P_1^\mu \Gamma_1 + P_2^\mu \Gamma_2 + \gamma^\mu \Gamma_3$$

There are three 4-vectors involved, so it depends from 3 structures proportional to them.
In the case of on-shell particles, energy and momentum are linked, so we can choose that they depend only from Q^2.
From current conservation:

$q_\mu \bar{u}_{P_2} \Gamma^\mu u_{P_1} = 0$ and this implies $\Gamma_1 = \Gamma_2$

Since only two structures are independent, we can parameterise the hadronic FF as

$\Gamma^\mu = \gamma^\mu F_1\left(Q^2\right) + i\sigma^{\mu\nu} q_\nu \frac{\kappa}{2m_p} F_2\left(Q^2\right)$ where F_1 is the Dirac FF and F_2 the Pauli FF.

Dynamics

Using the derived invariant structures for calculating the cross section

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \left[\left(F_1^2 + \tau\left(\kappa F_2\right)^2\right) + 2\tau\left(F_1 + \kappa F_2\right)^2 \tan^2\frac{\theta}{2}\right] \qquad \tau = Q^2/(4m_p^2)$$

Where the Mott cross section (Rutherford scattering with spin 1/2) is

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} = \frac{4Z^2\alpha^2 E'^2}{Q^4}\frac{E'}{E}\left(1 - \beta^2\sin^2\left(\frac{\theta}{2}\right)\right)$$

Introducing the linear combinations (also called Sachs Form Factors)

$$G_E = F1 - \tau\kappa F2 \quad \text{Electric Form Factor}$$

$$G_M = F1 + \kappa F2 \quad \text{Magnetic Form Factor}$$

ep Cross section

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}}\left[\frac{G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)}{1+\tau} + 2\tau G_M^2\left(Q^2\right)\tan^2\frac{\theta}{2}\right] = \boxed{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}}\frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon\left(1+\tau\right)}}$$

Form Factors

Advantages of defining the Sachs Form Factors:

1) Eliminate the mixing term F_1F_2 from the cross section. CS depends only on the squares of the Sachs FFs.
2) Physical interpretation: at $Q^2=0$, G_E is the electric charge and G_M the magnetic moment.

$$\boxed{G_E(0) = 1 \qquad\qquad G_M(0) = \mu_p}$$

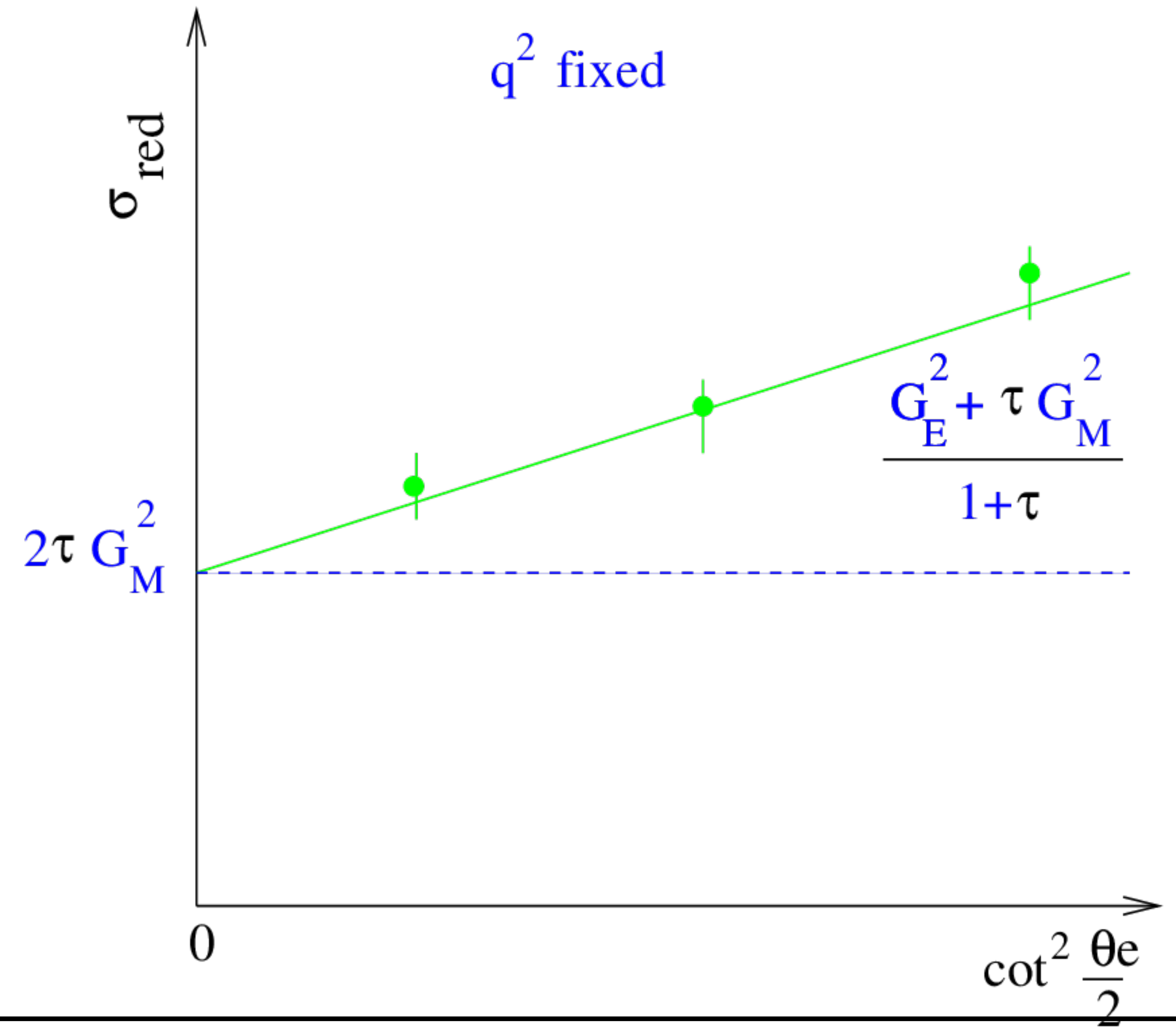
FFs are a fundamental strong-interaction property of nucleons. How to measure them?

Traditional method: Rosenbluth Separation.

Other methods: Polarisation, Model Fitting, ...

Rosenbluth Separation

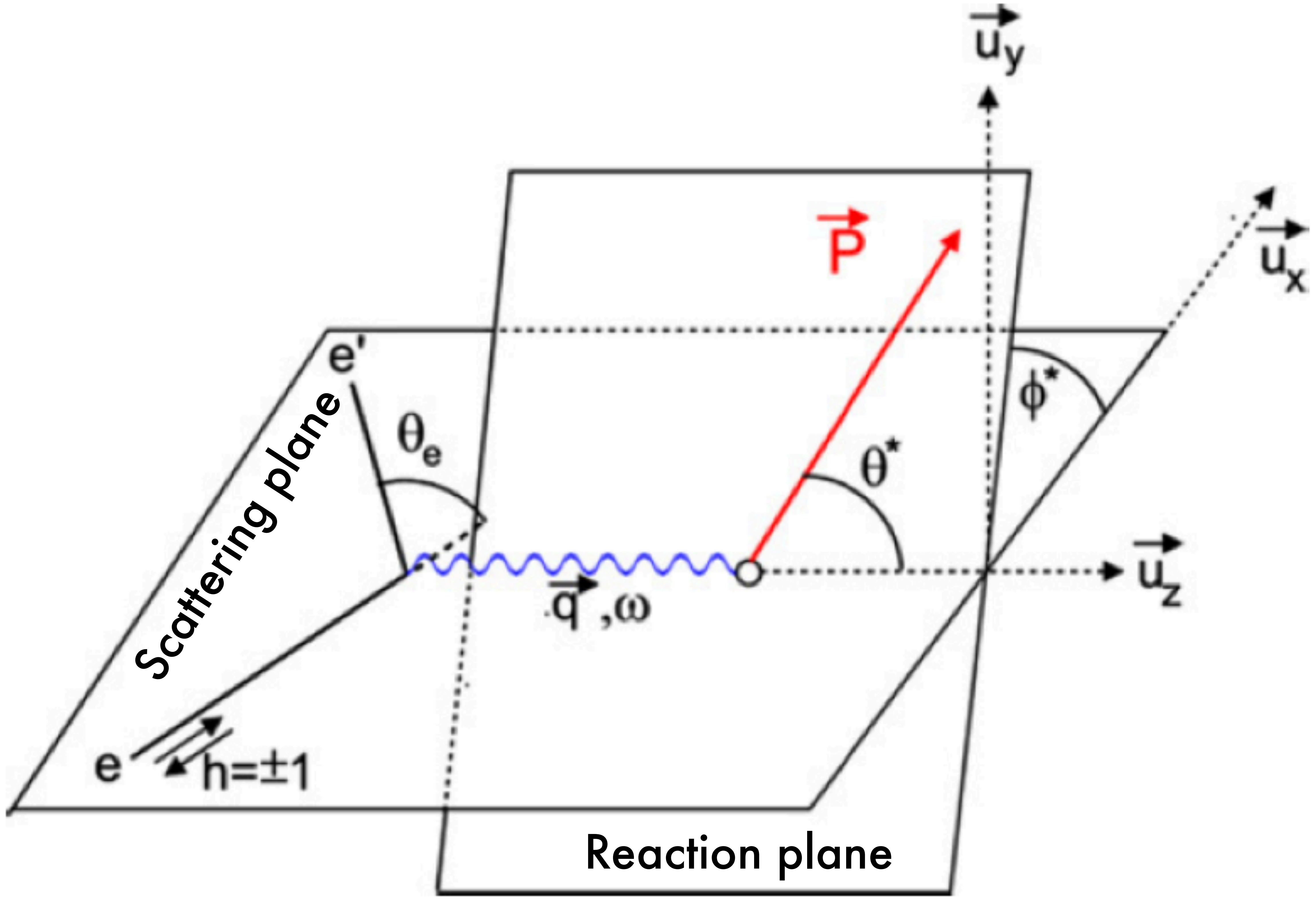
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon\,(1+\tau)} \Rightarrow \boxed{\varepsilon\,(1+\tau)\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} = \left(\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right)}$$



Linear in ϵ:

Measure at constant Q^2 and varying ϵ. G_E is the slope and G_M the intercept of a linear fit to the data.

Polarisation Method



$$\boxed{\frac{G_E}{G_M} = -\frac{P_t}{P_l}\frac{E_e + E_e'}{2M}\tan\frac{\theta_e}{2}}$$

$$\begin{aligned}
P_n &= 0 \\
\pm h P_l &= \pm h \left(\frac{E_e + E_e'}{M}\right) \frac{\sqrt{\tau(1+\tau)}\, G_{Mp}^2(Q^2) \tan^2\frac{\theta_e}{2}}{G_{Ep}^2(Q^2) + \frac{\tau}{\epsilon} G_{Mp}^2(Q^2)} \\
\pm h P_t &= \mp h \frac{2\sqrt{\tau(1+\tau)}\, G_{Ep} G_{Mp} \tan\frac{\theta_e}{2}}{G_{Ep}^2(Q^2) + \frac{\tau}{\epsilon} G_{Mp}^2(Q^2)}
\end{aligned}$$

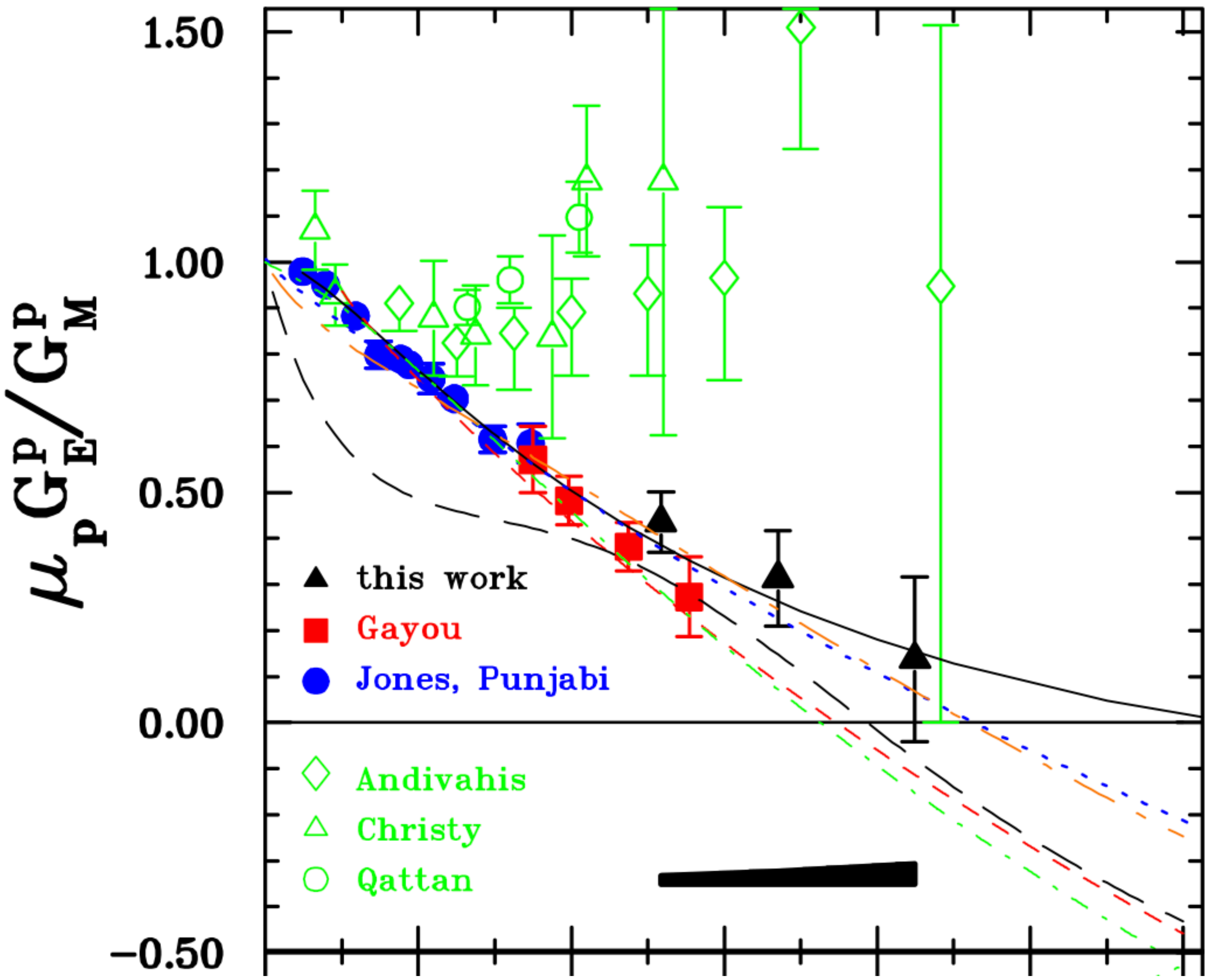
Use a polarised electron beam and measure the recoil polarisation components of the proton.

The transverse and longitudinal polarisation components are relative to the momentum transfer in the scattering plane.

First "crisis": polarisation data

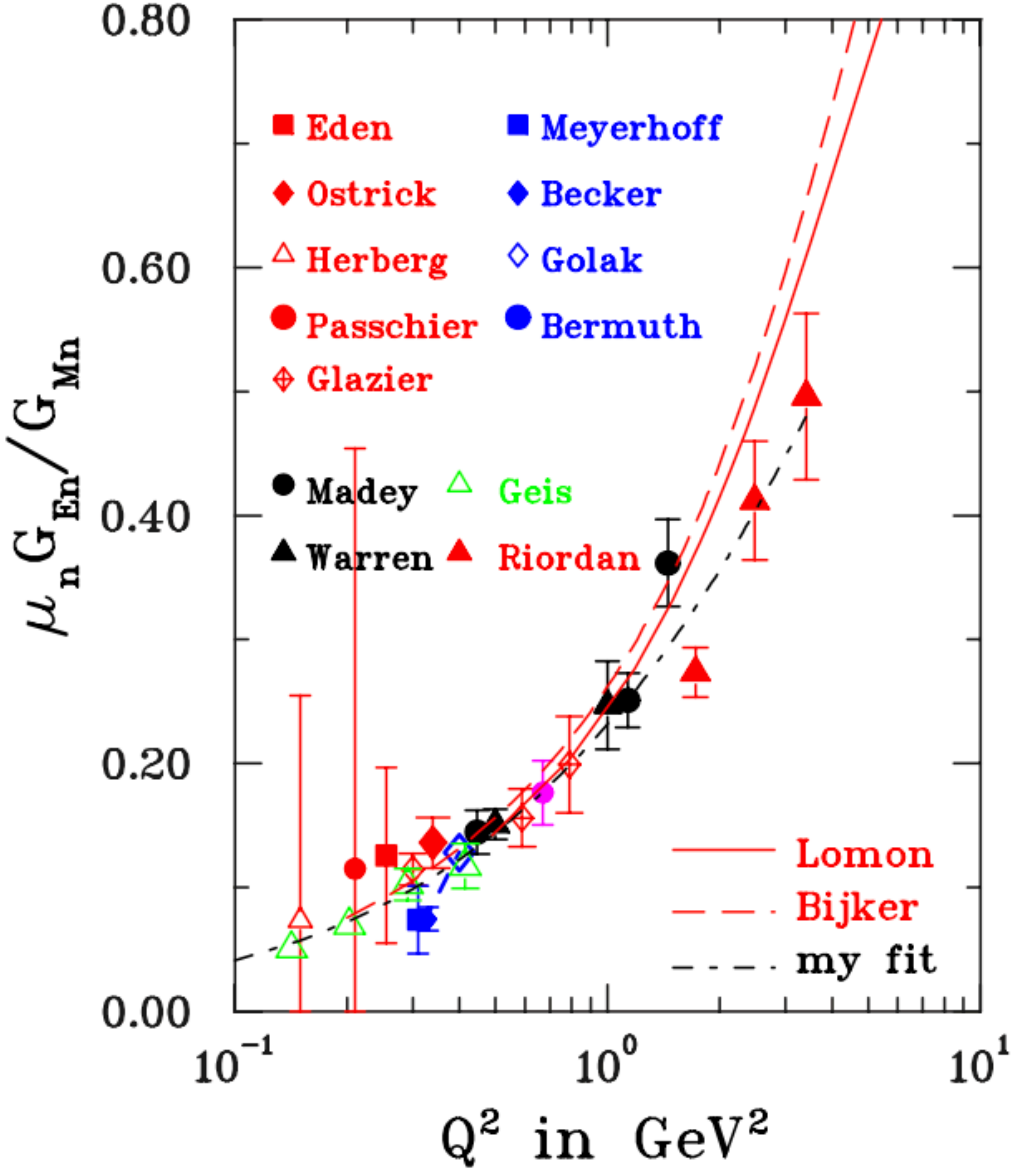
Unexpected! Rosenbluth data almost flat, while polarisation data shows linear decrease:

- Rosenbluth separation very sensitive to radiative corrections. G_E difficult to obtain.
- In particular two-photon exchange very important and ϵ- dependent.
- Polarisation data less sensitive to two-photon exchange.

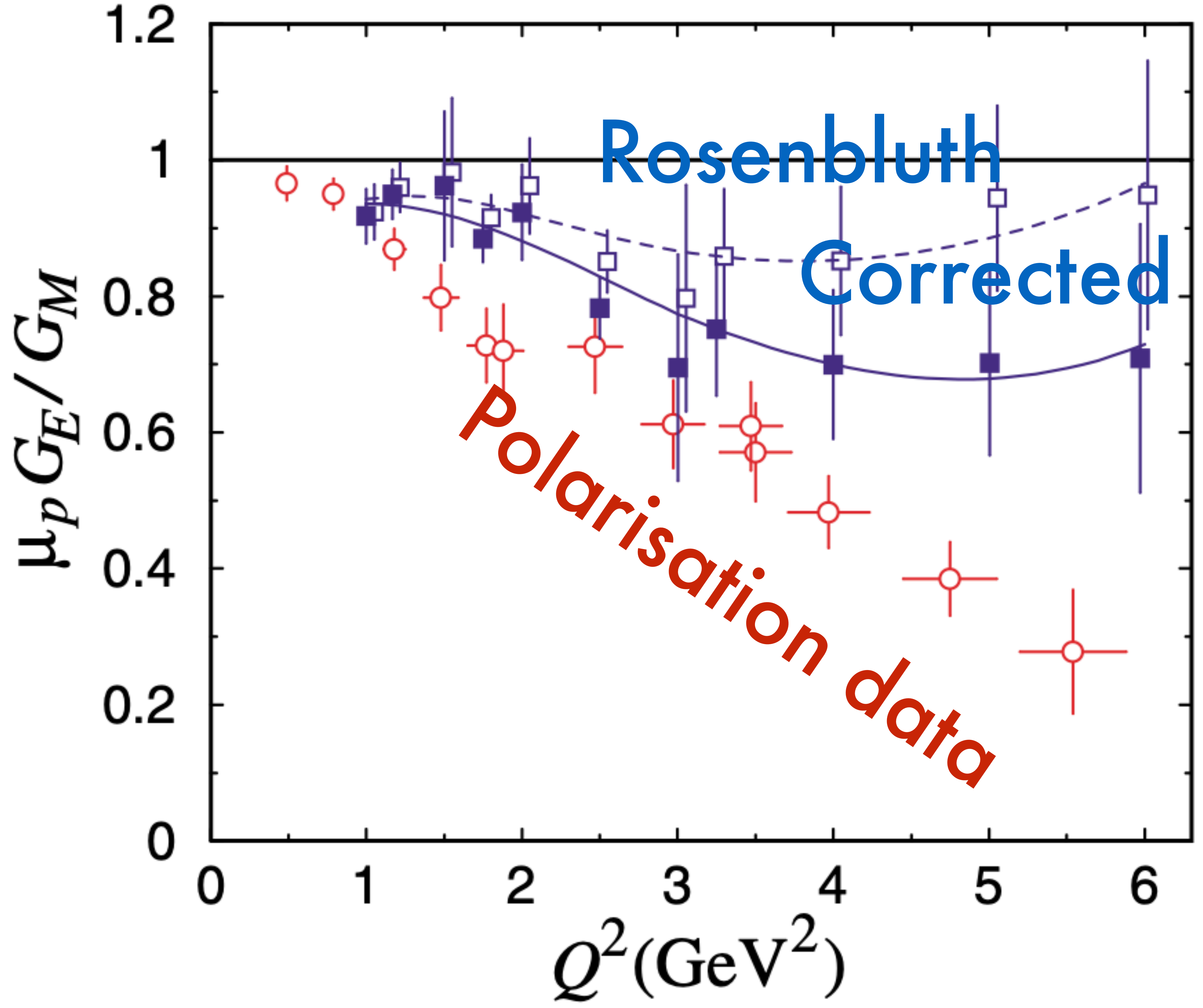


Experimental Results

For the neutron, results are more consistent given the absence of charge.



Two-photon exchange



VOLUME 91, NUMBER 14 PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2003

Two-Photon Exchange and Elastic Electron-Proton Scattering

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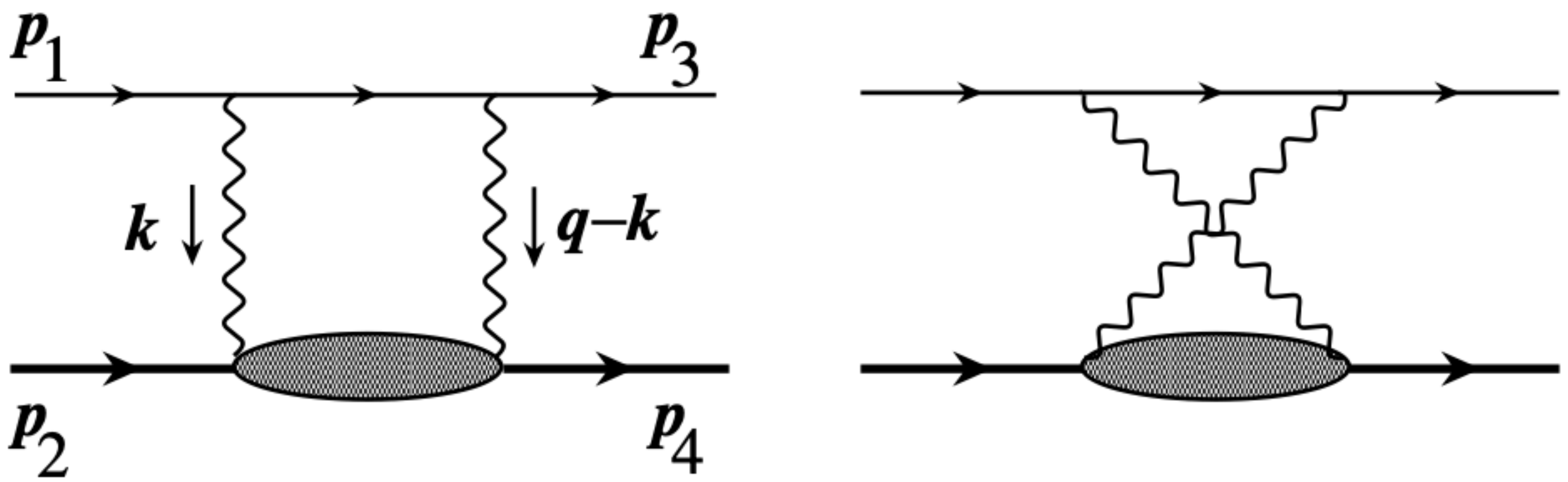
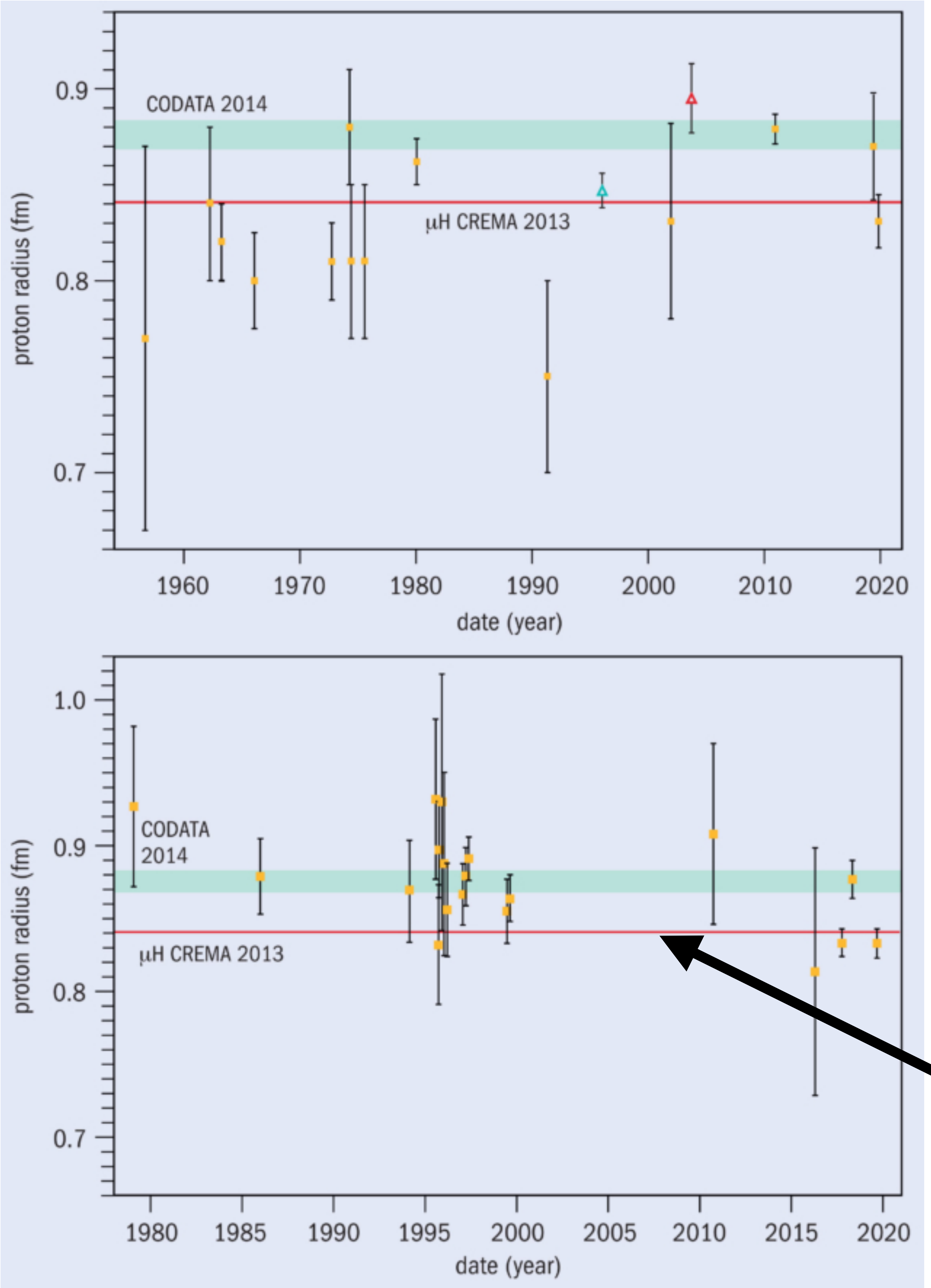
(Received 25 June 2003; published 3 October 2003)

Two-photon exchange contributions to elastic electron-proton scattering cross sections are evaluated in a simple hadronic model including the finite size of the proton. The corrections are found to be small in magnitude, but with a strong angular dependence at fixed Q^2. This is significant for the Rosenbluth technique for determining the ratio of the electric and magnetic form factors of the proton at high Q^2, and partly reconciles the apparent discrepancy with the results of the polarization transfer technique.



Second "crisis": muonic atoms

Pohl et al. ,Nature 466 213 (2010)



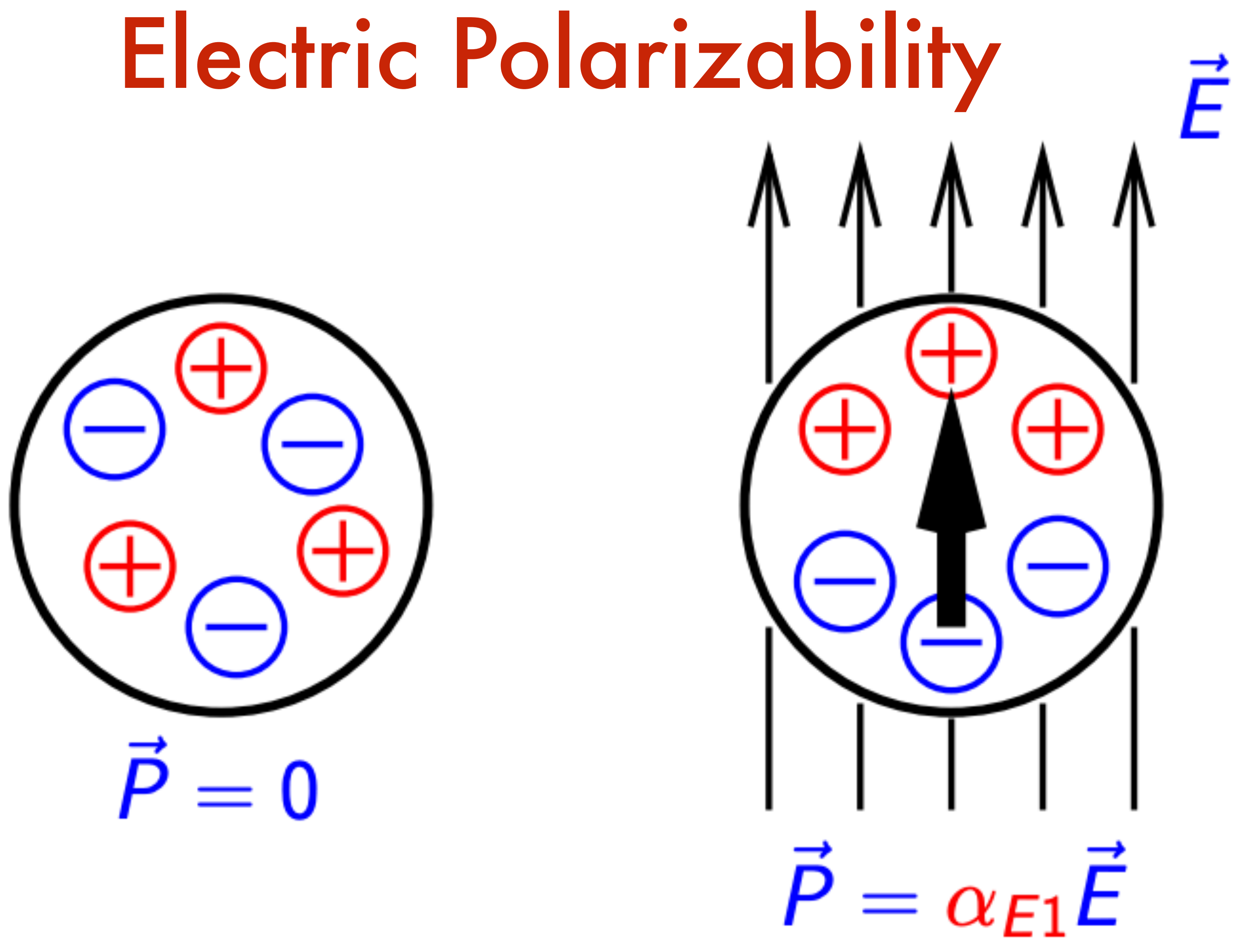
CODATA (2002): 0.8750 ± 0.0068 fm

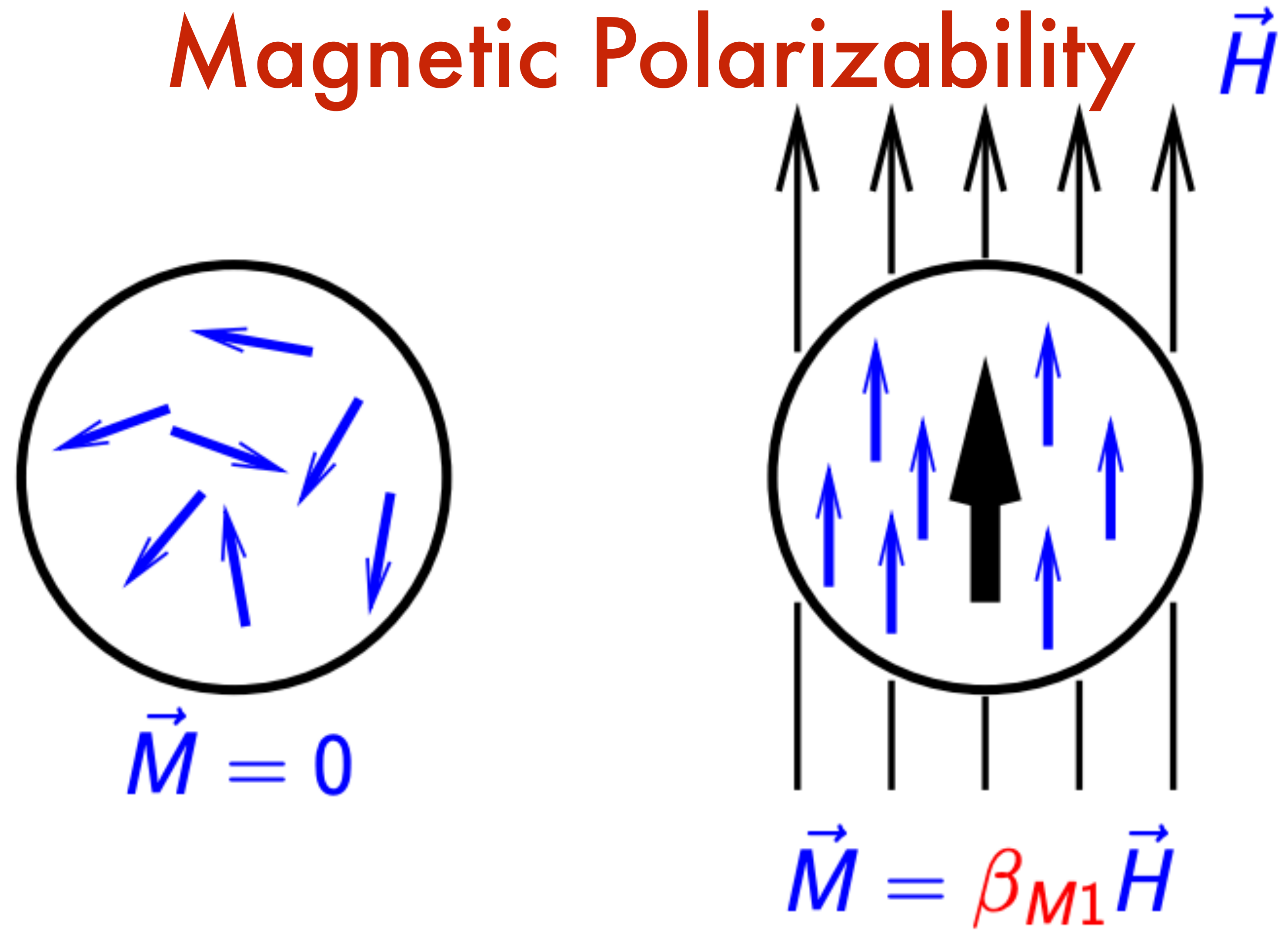
2010: CREMA muonic atoms measurement
2011: : Most precise ep scattering (MAMI)
2017: MPI Munich atomic spectra. agrees with muons
2019: York atomic spectroscopy exp. agrees with muons
2018: CODATA revises to 0.8414 ± 0.0019 fm
2019: JLab result consistent with muonic hydrogen
Other muonic atoms results coming..
Disp. Relations Fits to scattering data OK with lower R.
MUSE @ PSI coming up.
Story towards an end?
What was exactly the problem with electron scattering?

CREMA: 0.84184 ± 0.00067 fm

Polarizabilities

Polarizabilities





How to apply an electric/magnetic field to a nucleon? Use photons!
How large would be such a field? For a 100 MeV photon:

$$E = \frac{V}{d} = \frac{100MV}{10^{-15}} \approx 10^{23} V/m$$

Quite large. But nucleons are strongly-interacting systems and thus very "stiff": we need large fields to see an effect.

Polarizabilities

How to calculate them?

Expand the interaction Hamiltonian in incident photon energy:

0th order: mass and charge

1st order: magnetic moment

2nd order: scalar polarizabilities (analog to classical ones) $H^{(2)} = -4\pi\left[\frac{1}{2}\boxed{\alpha_{E1}}E^2 + \frac{1}{2}\boxed{\beta_{M1}}H^2\right]$

3rd order: spin (or vector) polarizabilities.

$$H^{(3)}_{\text{eff}} = -4\pi\left[\tfrac{1}{2}\gamma_{E1E1}\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \tfrac{1}{2}\gamma_{M1M1}\vec{\sigma}\cdot(\vec{H}\times\dot{\vec{H}}) - \gamma_{M1E2}E_{ij}\sigma_i H_j + \gamma_{E1M2}H_{ij}\sigma_i E_j\right]$$

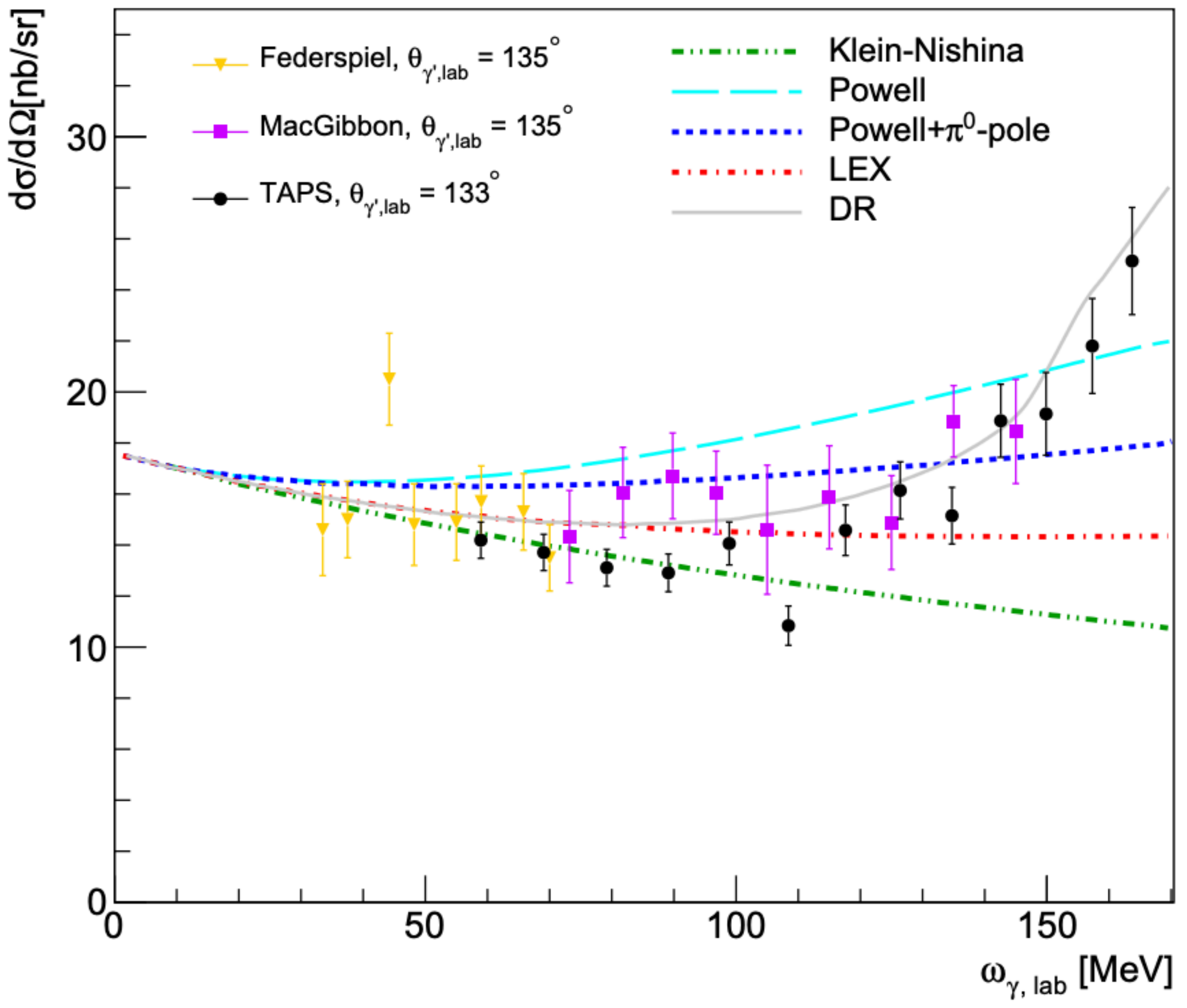
More difficult to visualise. A classical analog might be the induced precession of the nucleon spin.

Polarizabilities

$$\frac{d\sigma}{d\Omega}(\nu,\theta) = \frac{d\sigma}{d\Omega}^{\text{Born}}(\nu,\theta) - \nu\nu'\left(\frac{\nu'}{\nu}\right)\frac{e^2}{2m}\left[(\alpha_{E1}+\beta_{M1})(1+z^2)(\alpha_{E1}-\beta_{M1})(1-z^2)\right]$$

Low Energy Expansion (LEX)

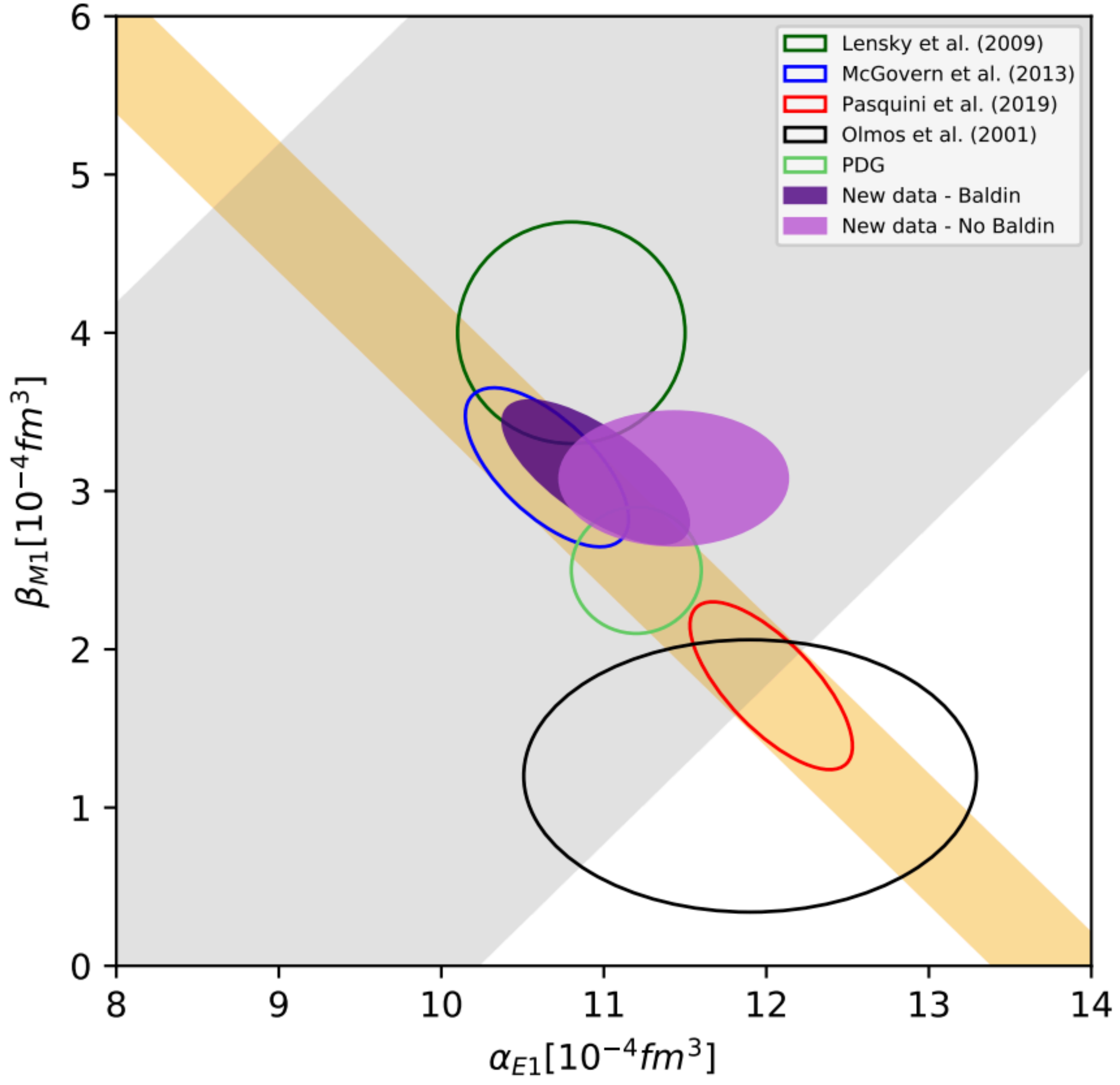
$$z = \cos\theta$$

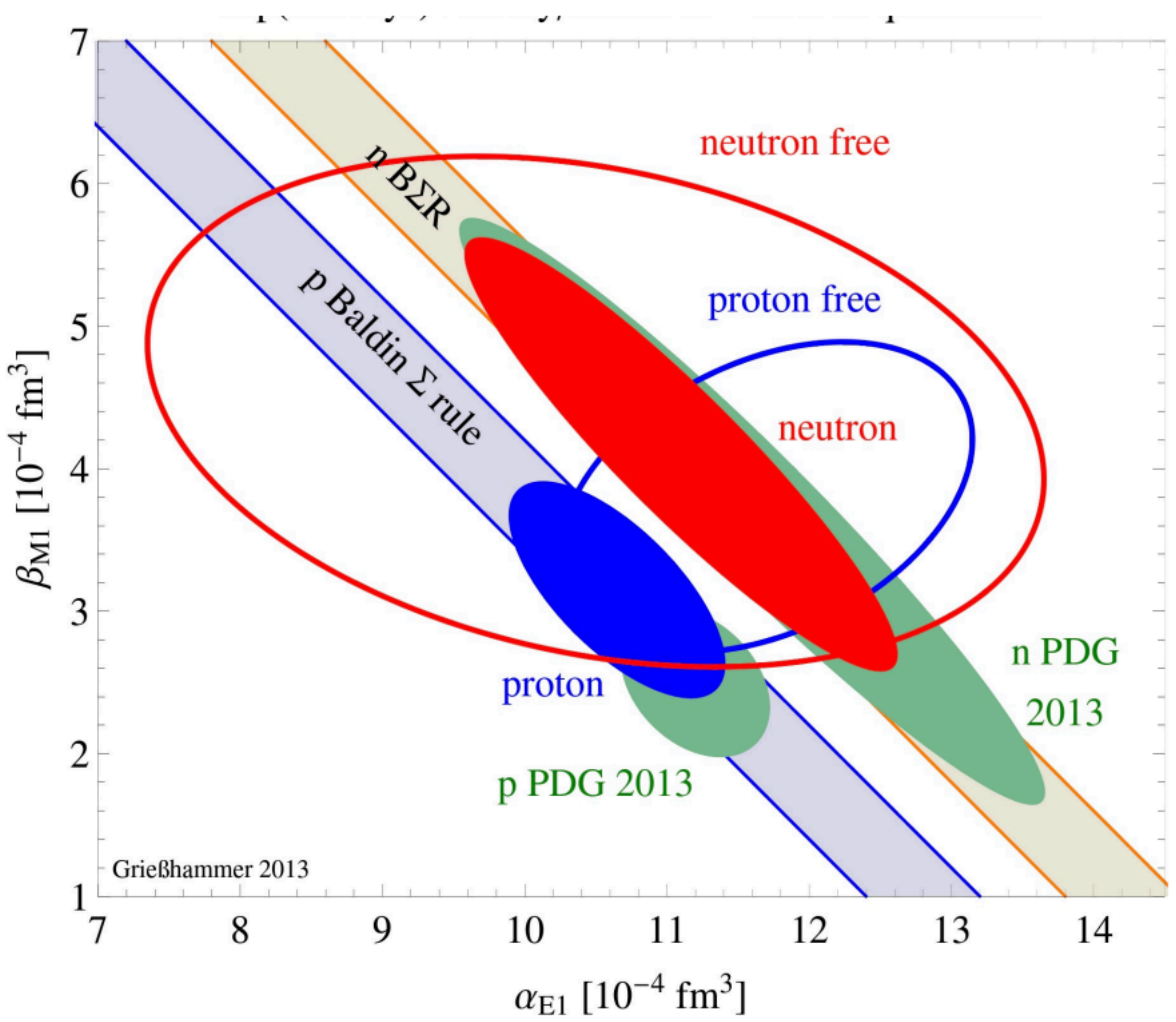


The (sum and difference of) polarizabilities can be extracted from precise measurements of <u>unpolarised</u> cross sections and asymmetries. The difference (at low E) between Born and LEX quantifies the polarisability effect.

Polarizabilities

Baldin Sum Rule: $\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\omega_0}^{\infty} \frac{d\omega}{\omega^2} \sigma_{tot}(\omega)$





EPJ A49, 13 (2013)

Polarizabilities: Polarised Cross Sections

Unpolarized target

Polarized beam and target: $$\frac{d\sigma}{d\Omega}\bigg|_{\mathrm{pol}} = \frac{d\sigma}{d\Omega}\bigg|_{\mathrm{unpol}} \Big\{ 1 + \xi_3\Sigma_3 + \zeta_y\Sigma_y + \xi_1(\zeta_x\Sigma_{1x} + \zeta_z\Sigma_{1z}) + \xi_2(\zeta_x\Sigma_{2x} + \zeta_z\Sigma_{2z}) + \xi_3\zeta_y\Sigma_{3y} \Big\}.$$

Unpolarized target $$\Sigma_3 = \frac{d\sigma^{\parallel} - d\sigma^{\perp}}{d\sigma^{\parallel} + d\sigma^{\perp}}$$ Direction of the photon polarisation wrt scattering plane.

Target polarised in y direction and un polarized beam $$\Sigma_y = \frac{d\sigma_y - d\sigma_{-y}}{d\sigma_y + d\sigma_{-y}}$$ Proton polarisation axis

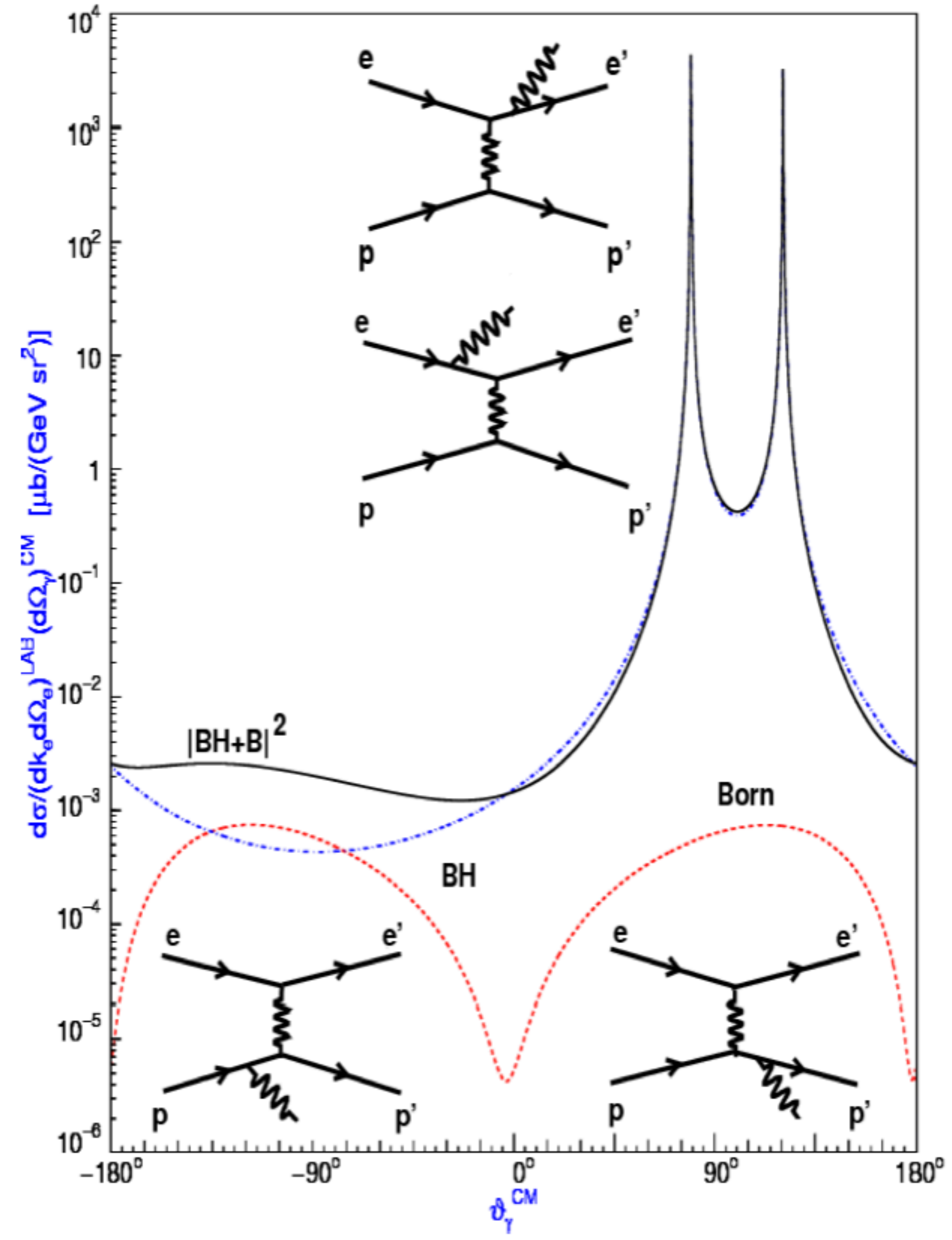
Circularly polarized photon and polarized target $$\Sigma_{2x} = \frac{d\sigma_x^R - d\sigma_x^L}{d\sigma_x^R + d\sigma_x^L} \qquad \Sigma_{2z} = \frac{d\sigma_z^R - d\sigma_z^L}{d\sigma_z^R + d\sigma_z^L}$$

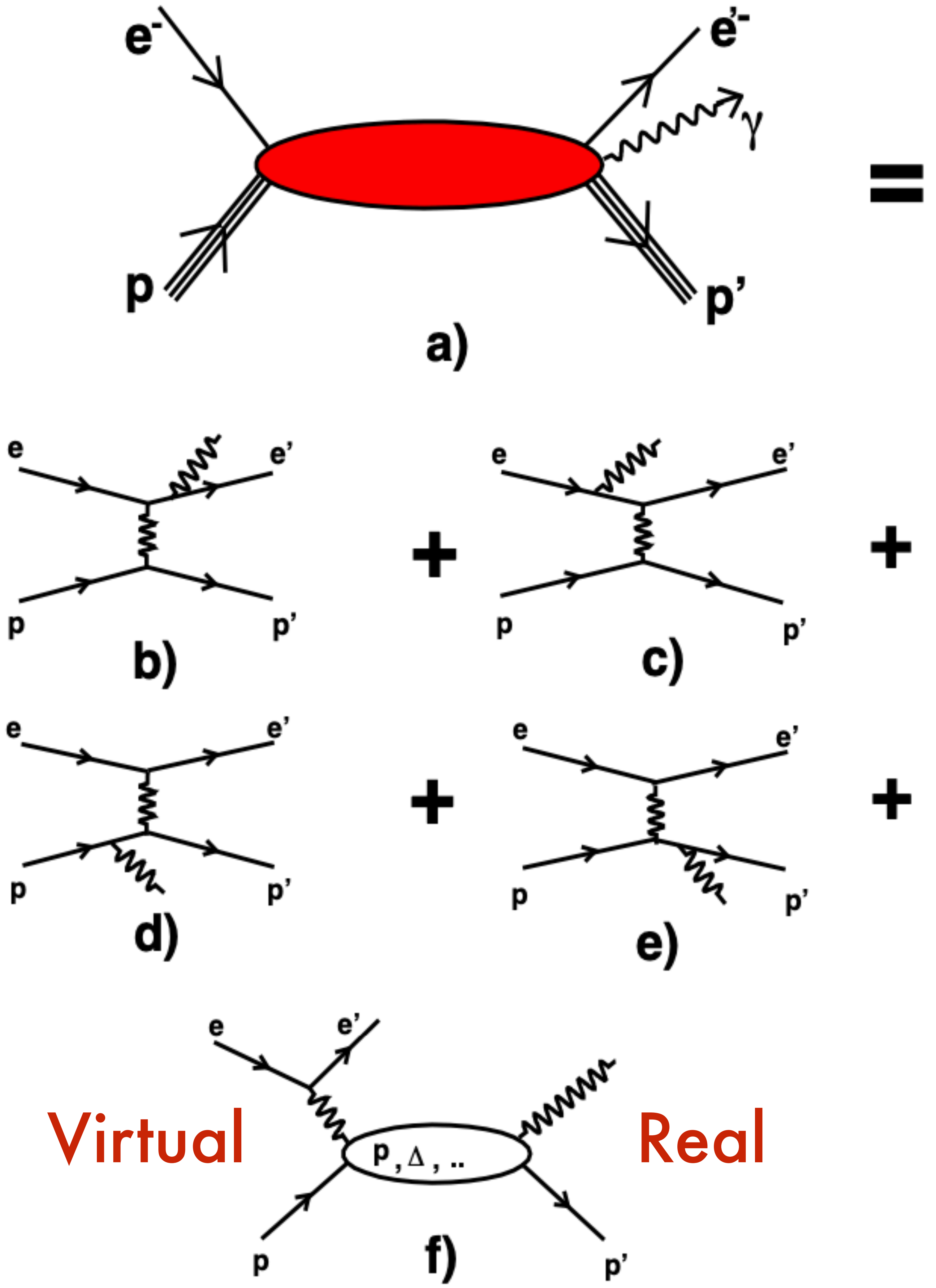
+ 3 double polarizations with linearly polarised photon and target.

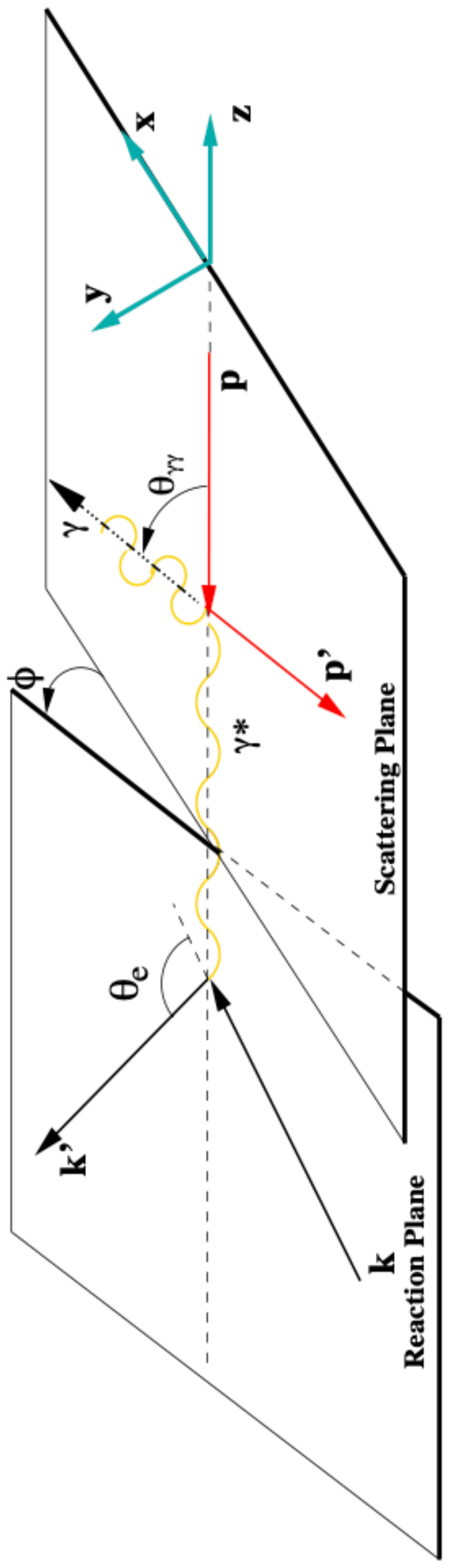
Generalised Polarizabilities

Photon	Electron (virtual photon: E,q)	Fourier Transform
Charge/Magnetization	Form factor	Charge/Magnetization space distributions
Polarizability	Generalised Polarizabilities	Polarizabilities space distributions

Generalised Polarizabilities







Generalised Polarizabilities

5-fold cross section (coincidence detection of e and p)

$$\frac{d^5\sigma^{exp}}{dk'_{lab}d\Omega_{k'_{lab}}d\Omega_{p'}} = \underbrace{\frac{(2\pi)^{-5}}{64M}\frac{k'_{lab}}{k_{lab}}\frac{s-M^2}{s}}_{\phi}\mathcal{M}$$

$$d^5\sigma^{Exp} = d^5\sigma^{BH+B} + \phi q'[\underbrace{\left(\mathcal{M}_0 - \mathcal{M}^{BH+Born}\right)}_{\text{Structure effects: GPs}} + \mathcal{O}(q')]$$

Rosenbluth-like Separation:

$$\Psi_0 = \mathcal{M}_0 - \mathcal{M}^{BH+Born} = v_{LL}\left[P_{LL}(q) - \frac{1}{\epsilon}P_{TT}\right] + v_{LT}P_{LT}(q)$$

Generalised Polarizabilities

Structure functions

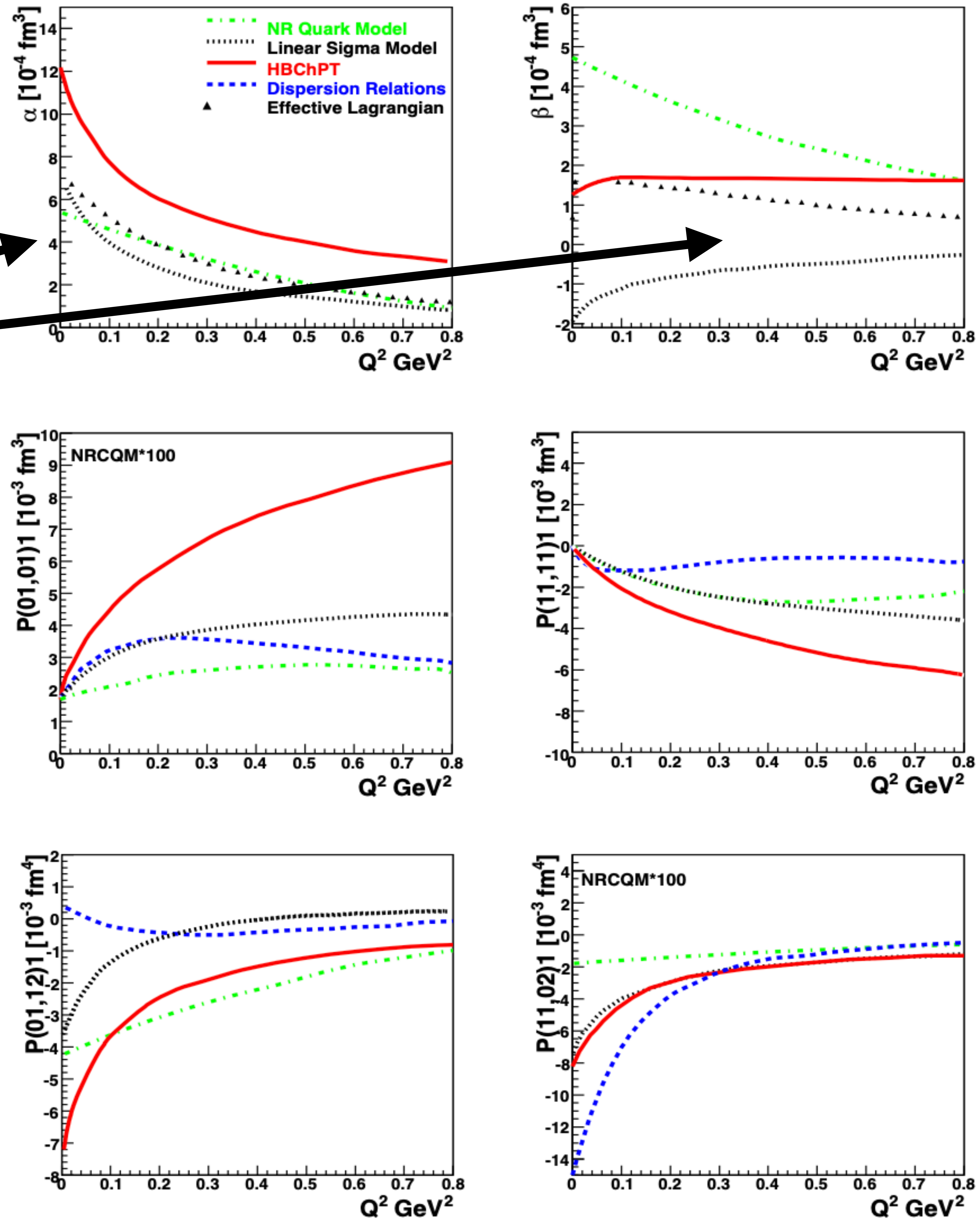
$$
\begin{aligned}
P_{LL}(q) &= -2\sqrt{6} M G_E P^{(01,01)0}(q)\,, \\
P_{TT}(q) &= -3 G_M \frac{q^2}{\tilde{q}_0}\left(P^{(11,11)1}(q) - \sqrt{2}\tilde{q}_0 P^{(01,12)1}(q)\right) \\
P_{LT}(q) &= \sqrt{\frac{3}{2}}\frac{Mq}{\tilde{Q}} G_E P^{(11,11)0}(q) + \frac{3}{2}\frac{\tilde{Q}q}{\tilde{q}_0} G_M P^{(01,01)1}
\end{aligned}
$$

Unpolarized measurement

$$
\begin{aligned}
P_{LT}^{z}(q) &= \frac{3\tilde{Q}q}{2\tilde{q}_0} G_M P^{(01,01)1}(q) - \frac{3Mq}{\tilde{Q}} G_E P^{(11,11)1} \\
P_{LT}^{\prime z}(q) &= -\frac{3}{2}\tilde{Q} G_M P^{(01,01)1}(q) + \frac{3Mq^2}{\tilde{Q}\tilde{q}_0} G_E P^{(11,11)1} \\
P_{LT}^{\prime \perp}(q) &= \frac{3q\tilde{Q}}{2\tilde{q}_0} G_M \left(P^{(01,01)1}(q) - \sqrt{\frac{3}{2}}\tilde{q}_0 P^{(11,02)1}(q)\right)
\end{aligned}
$$

Polarization needed

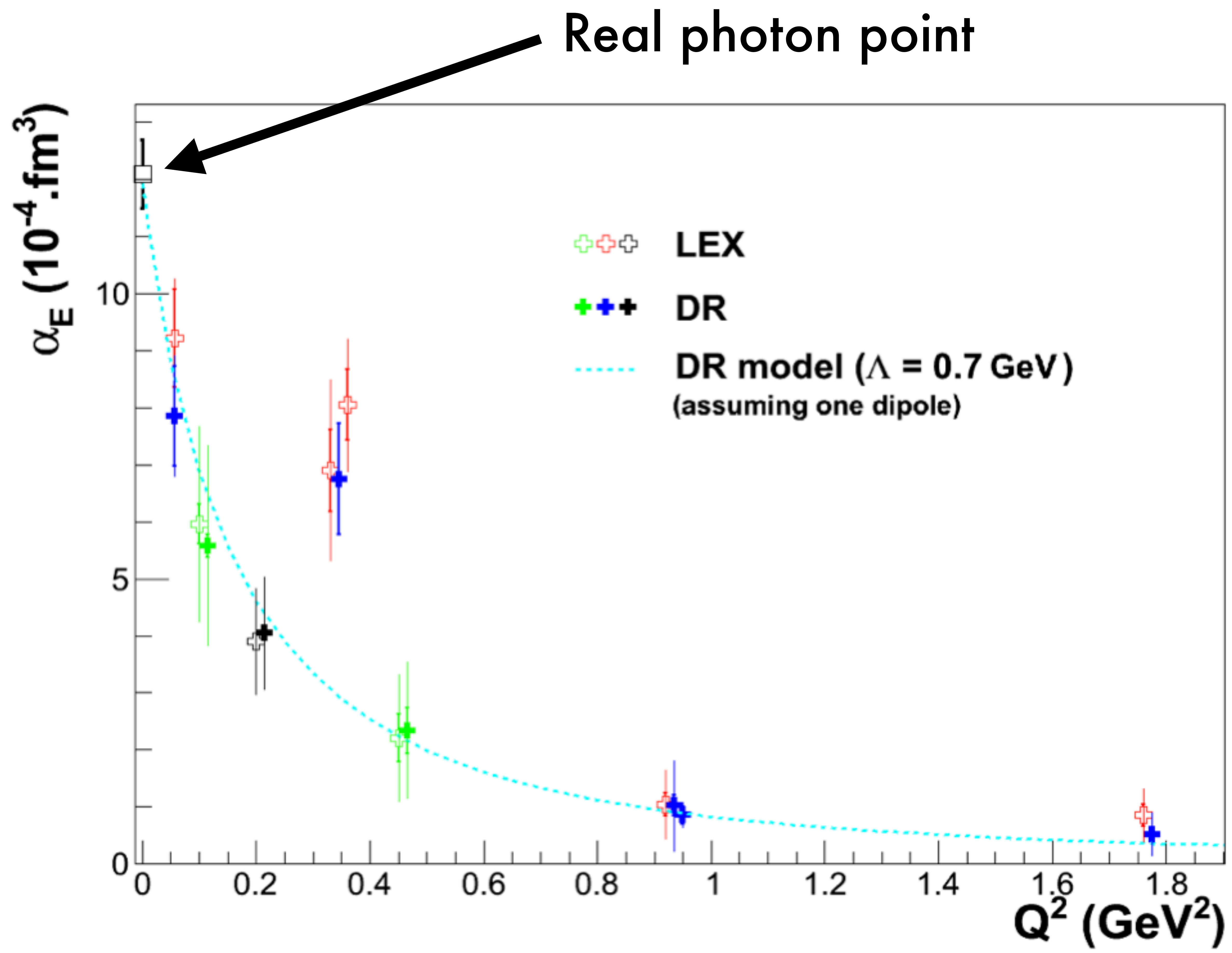
Generalised Polarizabilities

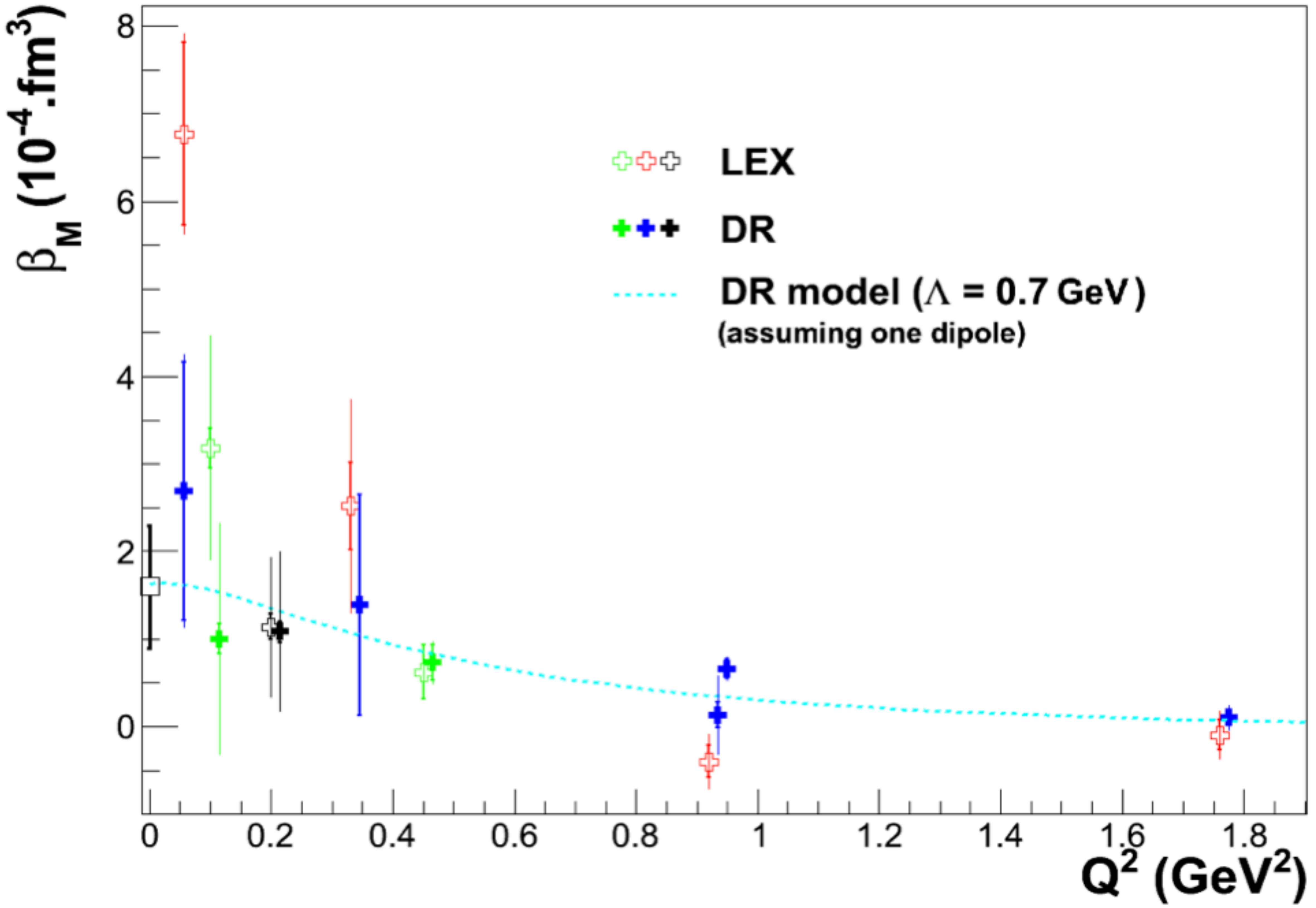


Evolution of electric and magnetic polarizabilities with q (FF=space distribution)

6 Independent structure functions:
2 generalised polarizabilities
4 “spin” generalised polarizabilities.

Generalised Polarizabilities: Experimental results



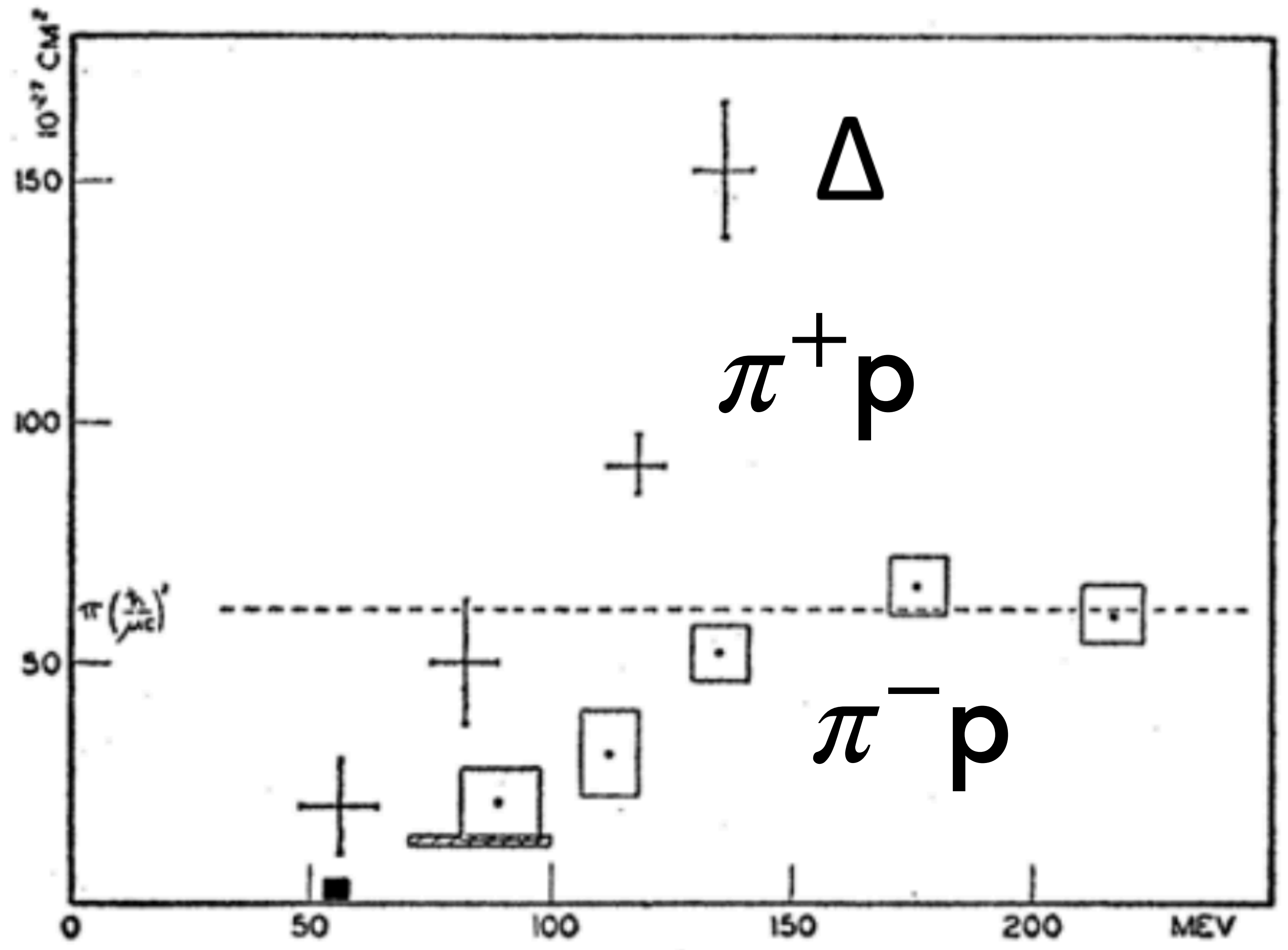


Resonances

Resonances

1952: First hint of the DELTA resonance in pion-nucleon scattering
1964: Resonances important for the quark model
2010: Lattice QCD reproduces the barion spectrum
2015+ : Resonances relevant for QGP physics

Total cross section



Total Cross Sections of Positive Pions in Hydrogen*

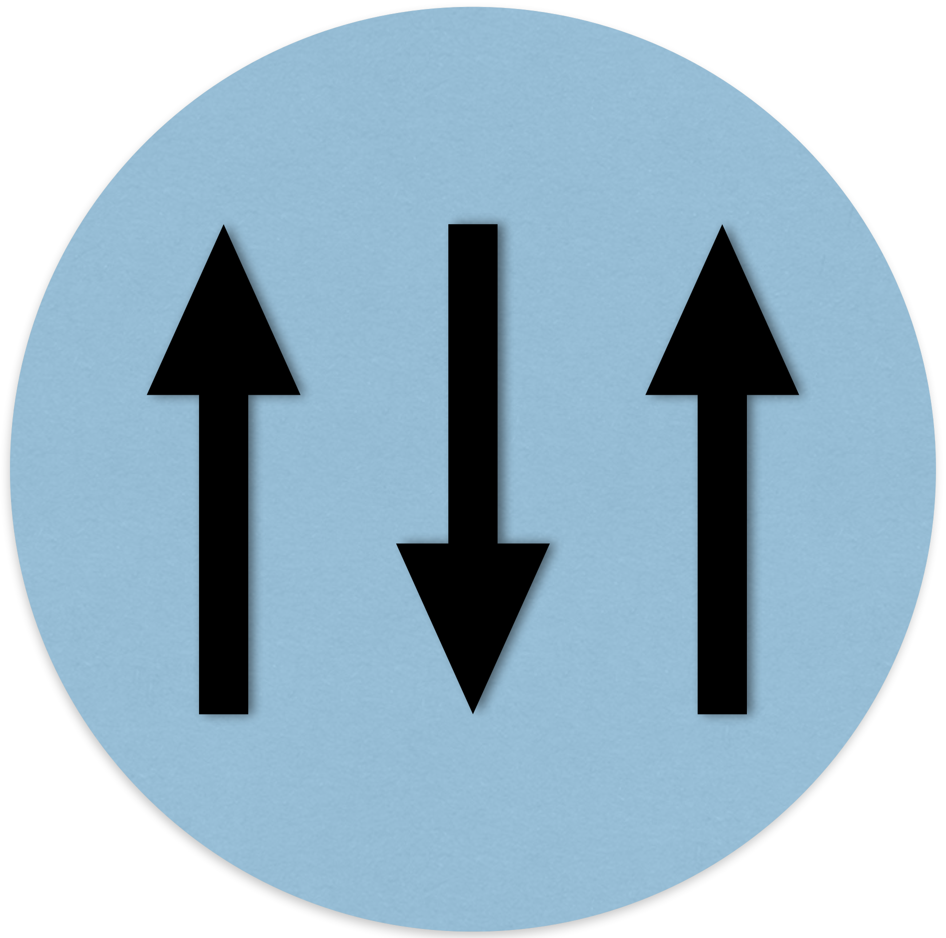
H. L. ANDERSON, E. FERMI, E. A. LONG,† AND D. E. NAGLE
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 21, 1952)

Anderson, H.L.; Fermi, E.; Long, E.A.; Nagle, "Total cross-sections of positive pions in hydrogen". *Physical Review*. **85** (5): 936 (1952).

The Delta resonance

Proton



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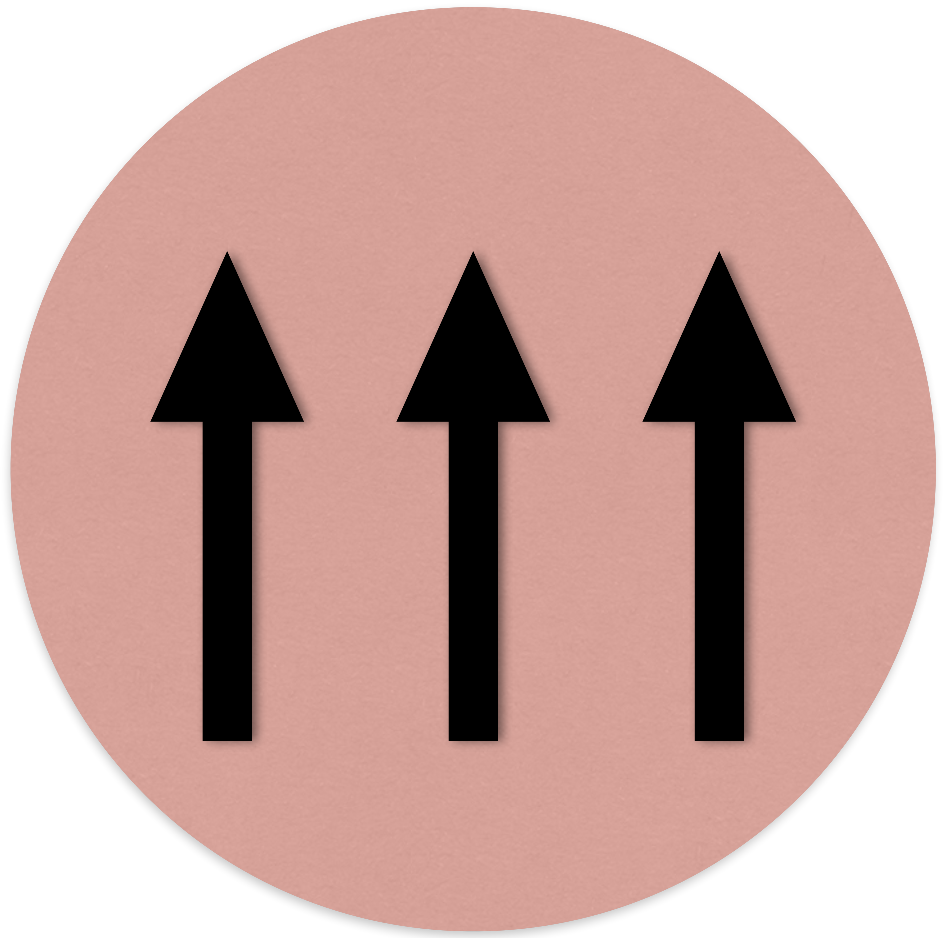
Δ^{++} Resonance



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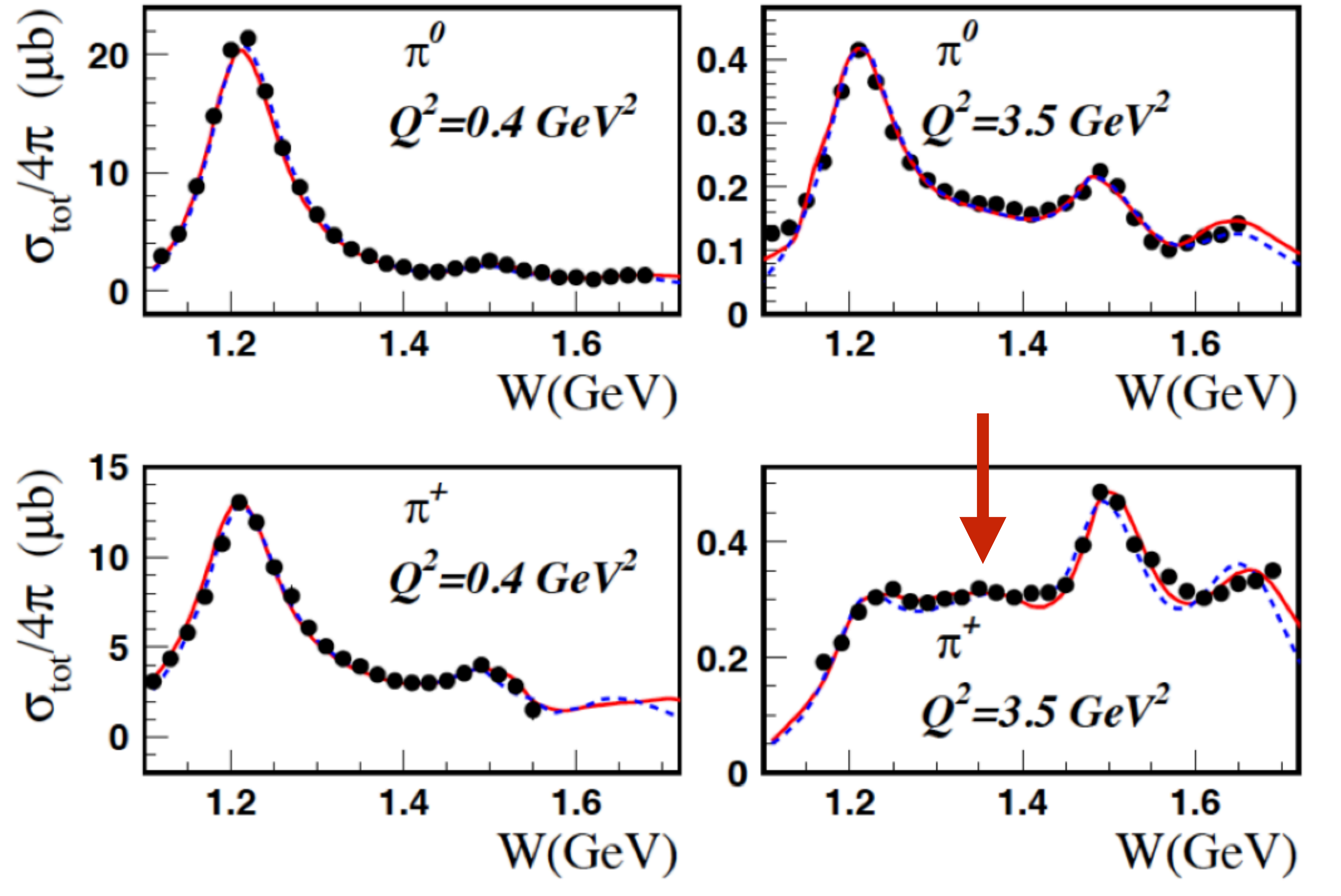
<u>Relevant for the quark model:</u>

- Charge +2 = 2/3+2/3+2/3
- Spin 3/2: alignment of 3 1/2 spins
- Pauli principle: new quantum number should exist: color.

The Roper resonance

- Discovered in 1963
- Looks like a proton but 50% heavier: mass value unexplained.
- First radial excitation of the proton, but mass hard to explain with 3 valence quarks.
- <u>Emerging picture:</u> quark core + meson cloud which reduces the core mass

JLAb Data

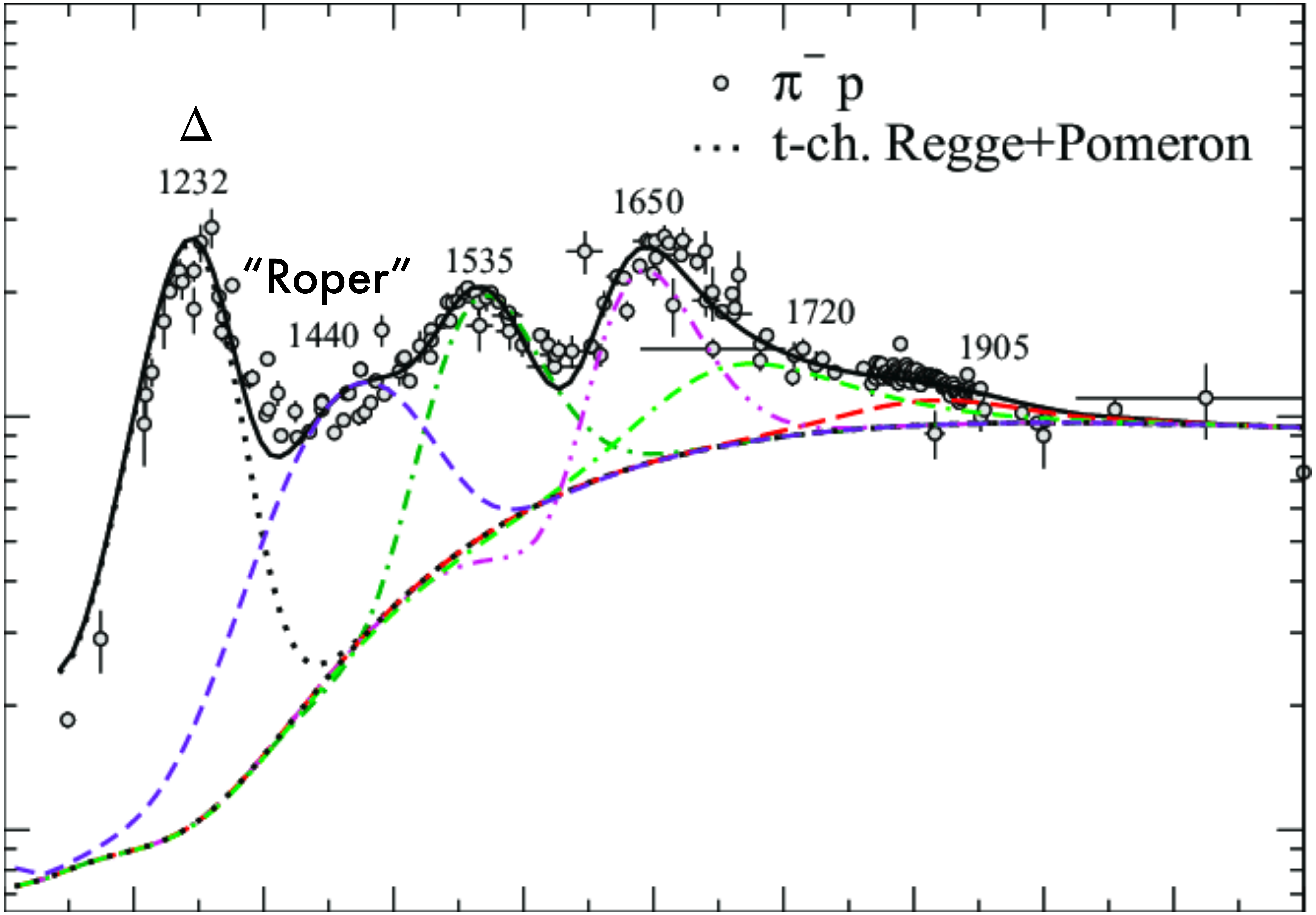


$p\pi^0$

$n\pi^+$

Original paper:
Roper, L. D., R. M. Wright, and B. T. Feld, 1965, Phys. Rev. 138, B190.

Resonances



Resonances in photo/electron production