Interaction of Radiation with Matter

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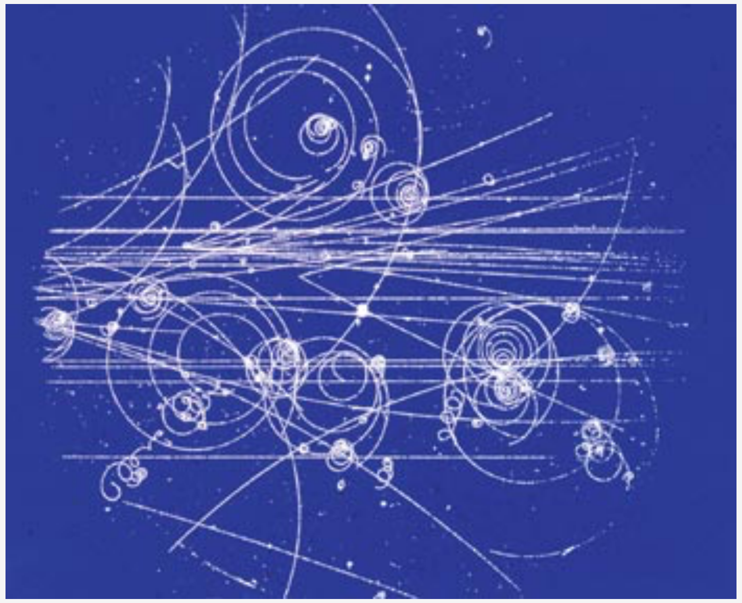
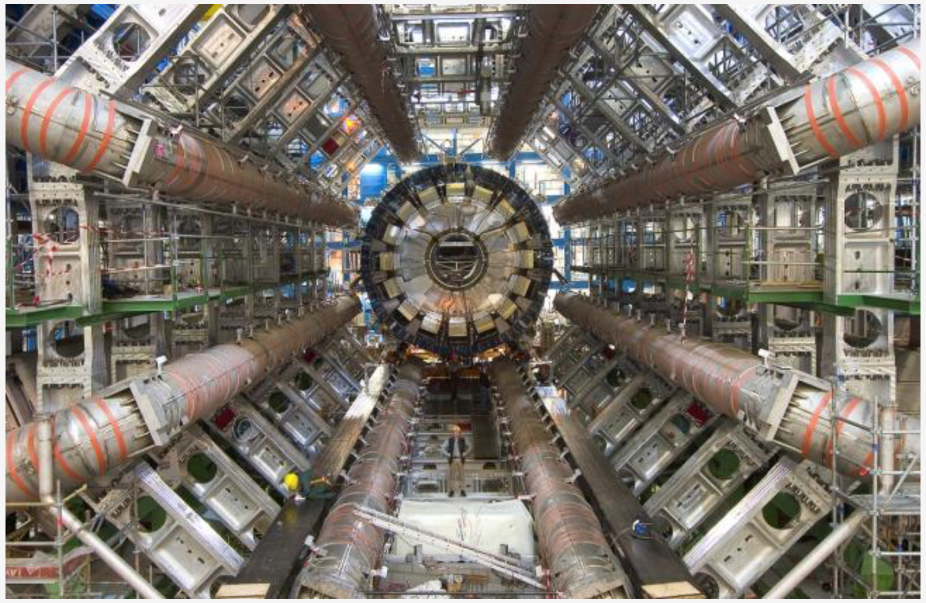
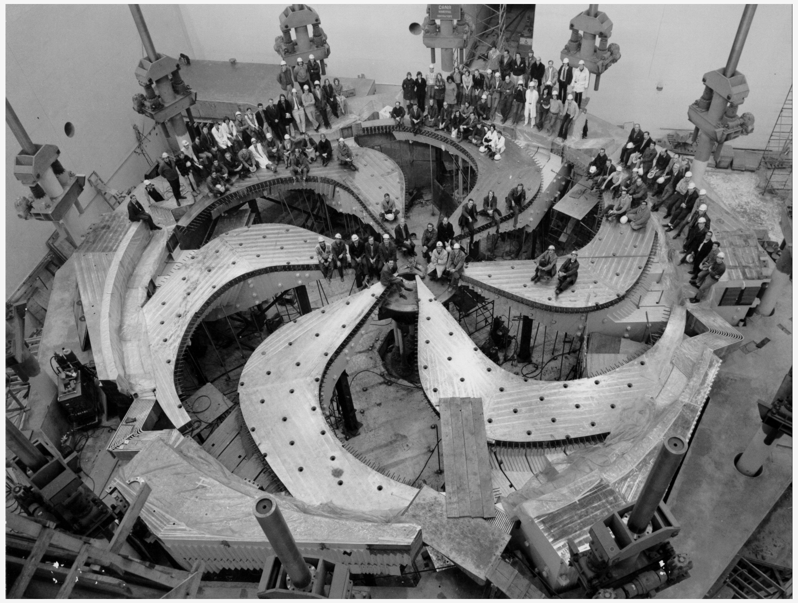
TRIUMF

Introduction

1) Radiation : General Concepts

2) Relevant Units

3) Radiation-Matter Interaction

- Heavy charged particles
- Electrons
- Photons
- Neutrons
- Neutrinos

4) Summary

General Concepts

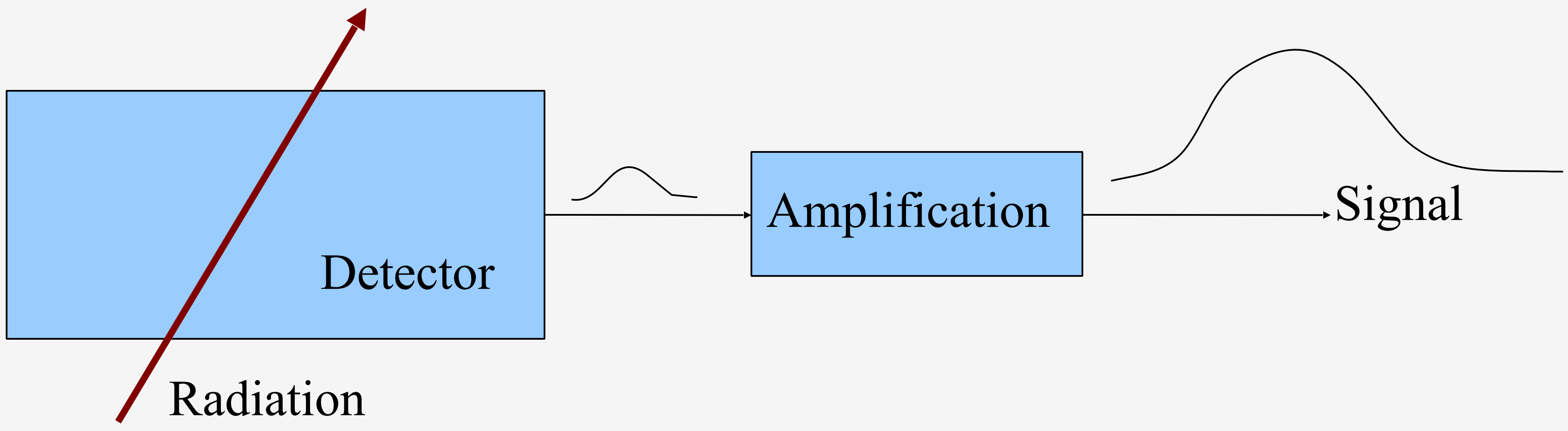
Radioactivity and Radiation :

With the term "Radiation" in physics it is intended a process in which energetic particles or waves travel in vacuum or in a medium. Radiation can be natural (cosmic rays, radioactive isotopes) or artificial (man-made isotopes, radiation produced by accelerators).

Radiation has a wide range of applications: medicine, materials science, biology, space engineering and many others. For a physicist, detecting radiation means looking at the smallest components of matter for trying to understand them.

Detectors:

Radiation is mostly invisible and we need a technique to detect it. Radiation is evident from its effects on matter. For detecting it, we need radiation detectors. The general scheme of a detector is the following:



Different radiation detectors are able to perform different measurements of physical quantities: energy, momentum, time, position, radiation identification.

Radiation effects are the working principle for particle detectors —> it is key to understand these processes for building a particle detector!

Units

<u>Common units used in radiation Physics and Radiation Protection</u>

Activity: **becquerel** (Bq) = # of disintegrations /s

Absorbed dose: **gray** (Gy) = 1 joule / kg = 6.24×10^{12} MeV / kg

Exposure: **roentgen** (R) = C/kg of air (photon fluence in terms of created charge)

Equivalent Dose: **sievert** (Sv): Gy*w (w = radiation weighting factor)

Radiation		w
X and gamma		1
e, muons		1
Neutrons		
	<10keV	5
	10-100keV	10
	>100keV-2MeV	20
	2-20MeV	10
	>20MeV	5
Protons >2MeV		5
Nuclear Fragments		20

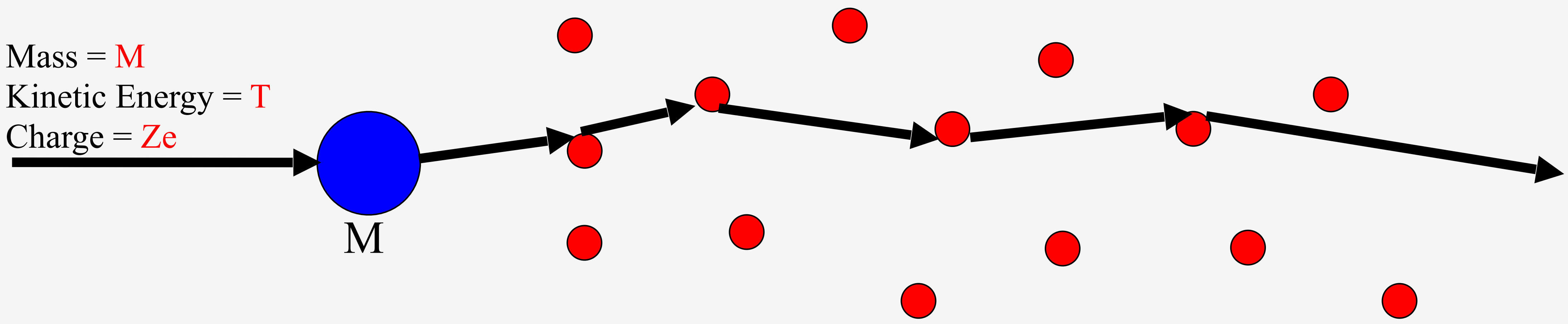
Radiation-Matter Interaction

Heavy Charged Particles :	Most common case with wide range of applications. Electromagnetic Interaction with atomic electrons.
Electrons :	Very light particles (not heavy wrt atomic electrons).
Photons :	Massless. Connected with electrons.
Neutrons:	No EM interaction, heavy.
Neutrinos:	Interact only via weak interactions.

Heavy Charged Particles (I)

Heavy Charged Particles : Protons, nuclei, charged hadrons, ..

The interaction of a heavy charge particle with a material consists mainly of its interaction with the atomic electrons. "Heavy" means that that particle mass M is heavier than the atomic electrons mass m_e : $M > m_e$



We can make the following considerations about the interaction:

1) The energy loss per collision is small.
Let's consider an head-on collision for protons. The fractional energy loss will be:

$$\frac{\Delta T}{T} = \frac{4 m_e M}{(M + m_e)^2} \approx \frac{4 m_e}{M} \approx 0.2\%$$

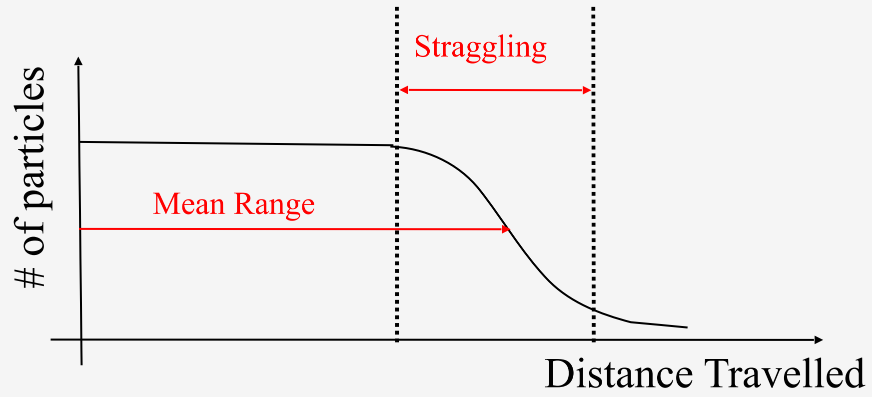
2) The heavy particle is deflected very little, therefore it follows an almost straight path

3) The interaction is mostly electric and the Coulomb force has a long range. This means that the particle interacts with many electrons at the same time. The result is a nearly gradual and continuous energy loss over time.

Heavy Charged Particles (II)

Range : The range is the distance over which the particle loses all its kinetic energy.
Because the statistical nature of the collisions, there is a variation in the range, called range straggling.

Suppose to prepare N particles with the same initial kinetic energy and measure the range for all of them.



For heavy particles, the straggling is small, typically few % of the total range.
In general, the higher the kinetic energy T, the longer the range.

After a single interaction with an atomic electron, the electron itself can be either knocked out of the atom (δ-rays) and create electron-ion pairs, or just raise the electron into higher atomic orbitals (less likely). The δ-rays themselves can ionize other atoms, etc..

The Bethe-Bloch Formula

Problem :
We are interested in calculating the energy loss of a heavy charged particle per length of material traversed: dE/dx . The energy loss per unit length is also called "stopping power" of a material.
It is a key quantity to know when dealing with radiation and radiation detectors.
Since it quantifies an energy loss, it is usually taken with the "-" sign: -dE/dx.
The stopping power depends from the incoming particle (charge, mass, kinetic energy) and from the material (density, ionizing potential, ..).

History :
The first derivation of dE/dx is due to N.Bohr with a purely classical calculation. This result works for heavy nuclei, but e.g. for protons it is not adequate. In 1930 H.Bethe proposed the first quantum derivation of the formula using first order perturbation theory. In 1932 H.Bethe derived the relativistic version of his quantum formula. The contribution of F.Bloch (in 1933) was not to the formula itself, but to a specific expression of the ionization potential of materials as a function of Z. The stopping power formula is commonly called "Bethe-Bloch" formula, but the original derivations of it are due to Bethe and Bohr.
Later on, smaller corrections to the Bethe formula were derived by different authors, with Bethe himself among them.

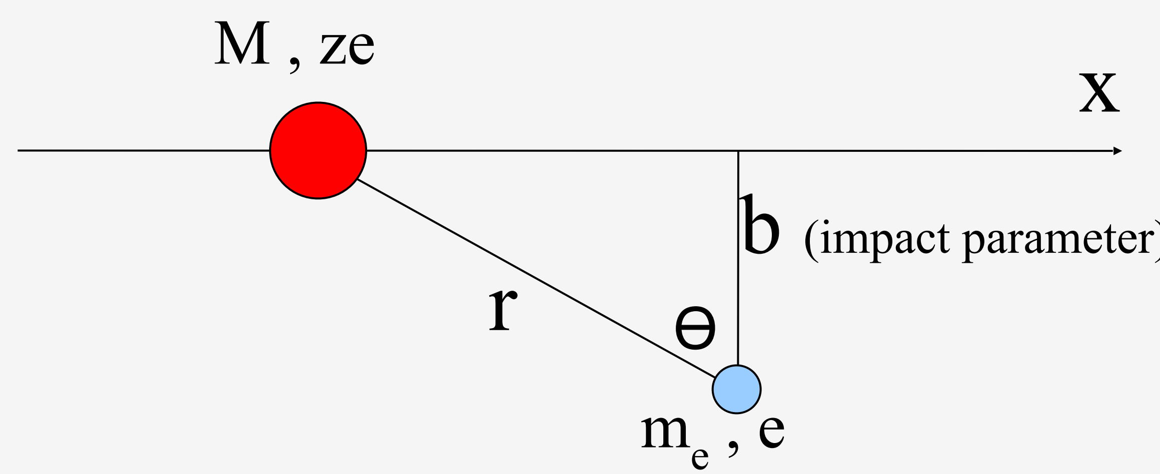
Hans A. Bethe (1906-2005)

Classical derivation for -dE/dx (I)

Hypotheses :

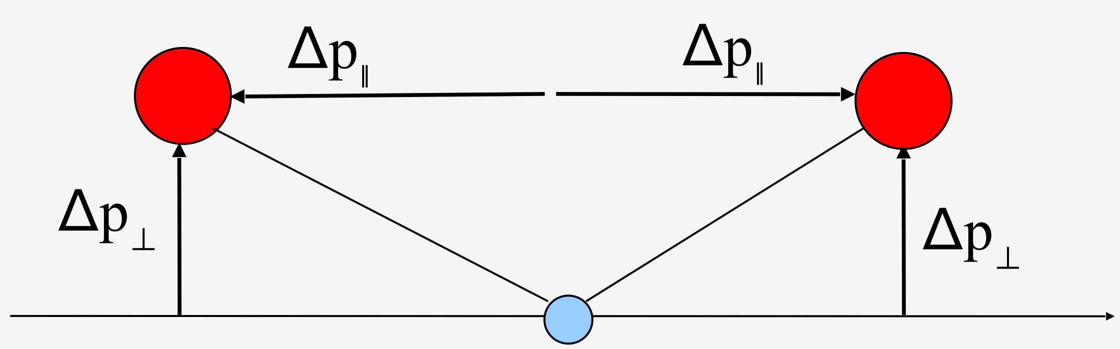
1) Energy loss via inelastic collisions (only Coulomb force F_C)
2) $M > m_e$, electron at rest and $t_{coll} < t_{orbi}$
3) Incoming particle: Mass M , $v = \beta c$, Charge = ze

Target: Charge = e , Density = ρ



Momentum variation Δp: it can be decomposed in a parallel $\Delta p_\parallel$ and transversal $\Delta p_\perp$ component with respect to the initial direction. The parallel component $\Delta p_\parallel$ averages to zero during the collision. The remaining net variation is due only to the transversal component.

$$F_C(r) = -F_C(-r)$$



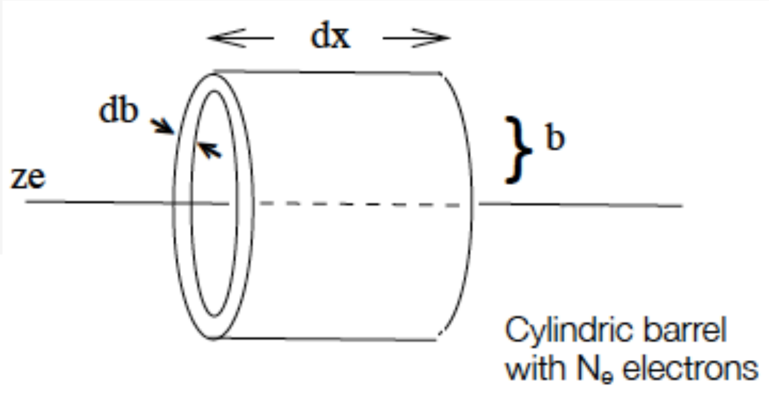
$$\Delta p_\perp = \int_{-\infty}^{+\infty} F_\perp dt = \int_{-\infty}^{+\infty} F_\perp \frac{dt}{dx} dx = \int_{-\infty}^{+\infty} F_\perp \frac{dx}{v} = \int_{-\infty}^{+\infty} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{(x^2+b^2)} \overbrace{\boxed{\frac{b}{\sqrt{x^2+b^2}}}}^{\cos\theta} \frac{dx}{v} =$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2 b}{v} \int_{-\infty}^{+\infty} \frac{1}{(x^2+b^2)^{3/2}} dx = \frac{ze^2}{2\pi b v \epsilon_0}$$

Classical derivation for -dE/dx (II)

Alternative derivation (via Gauss Theorem)

$$\left.\begin{array}{l} \int \Phi dS = ze/\epsilon_0 \\ \int E_\perp 2\pi b dx = Ze/\epsilon_0 \Rightarrow \int E_\perp dx = \dfrac{ze}{2\pi b \epsilon_0} \\ \text{Remembering: } F_\perp = eE_\perp \quad p_\perp = \int F_\perp dt = \int F_\perp \dfrac{dx}{v} \end{array}\right\} \Rightarrow \Delta p_\perp = \frac{ze^2}{2\pi b v \epsilon_0}$$



Estimate the number of atomic electrons (= Collisions N_c) $N_e = \rho_e V = \rho_e (2\pi b) db dx$

Total Energy Variation (in N_c collisions)

$$-dE(b) = \frac{\Delta p_\perp^2}{2m_e} N_c = \frac{\Delta_\perp^2}{2m_e} 2\pi \rho_e b db dx = \frac{z^2 e^4}{4\pi b^2 v^2 m_e} 2\pi \rho_e b db dx = \frac{z^2 e^4 \rho_e}{4\pi \epsilon_0^2 v^2 m_e} \frac{db}{b} dx$$

Integration on the impact parameter b:

$$-\frac{dE}{dx} = -\int_{b_{min}}^{b_{max}} db \frac{dE}{dx}(b) = \frac{z^2 e^4 \rho_e}{4\pi \epsilon_0^2 v^2 m_e} \ln \frac{b_{max}}{b_{min}}$$

Classical derivation for -dE/dx (III)

Minimum impact parameter: $b_{min} = \lambda_e = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_e v}$ (DeBroglie length of the electron)
(max energy transfer: head-on collision)

Maximum impact parameter: $b_{max} = \frac{\gamma v}{\nu_e}$
(min. energy transfer, "peripheral collision")

- Approximate interaction time: $T_i \sim b/v$
- Electron revolution time: $T_r \sim \gamma/v_e$
- Adiabatic approximation: $T_i \sim T_r$

Electron density: $\rho_e = N_A \rho \frac{Z}{A}$

Effective ionization potential: $I \sim h\nu_e$

Relativistic velocity: $v = \beta c$

(Classical) Bethe Formula:

$$\boxed{-\frac{dE}{dx} = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi z^2 Z N_A \rho}{m_e c^2 \beta^2 A} \ln\left(\frac{m_e c^2 \beta^2 \gamma^2}{I}\right)}$$

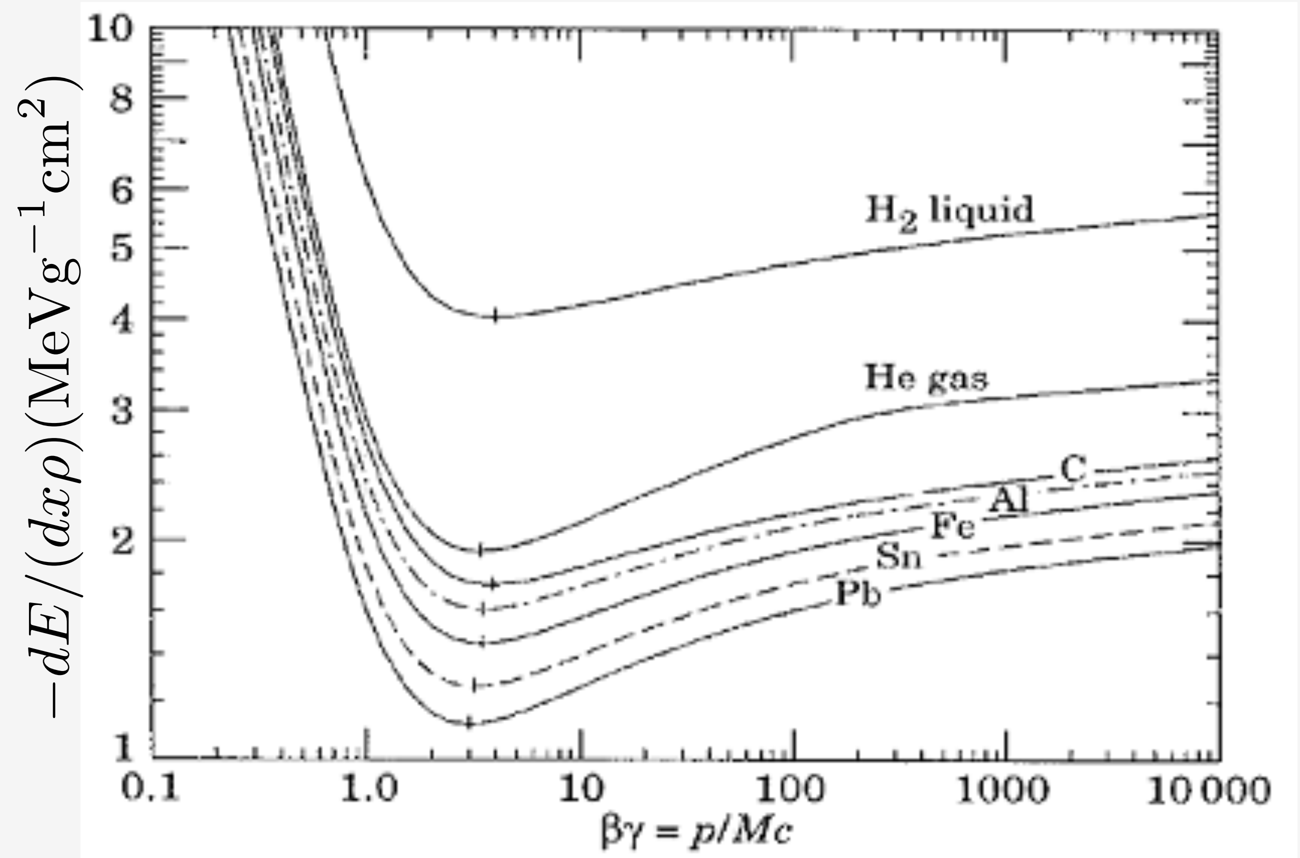
The Bethe-Bloch Formula

Saturation effect (Medium Polarization)

$$-\frac{dE}{dx} = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi z^2 Z N_A \rho}{m_e c^2 \beta^2 A}\left[\ln\left(\frac{m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

"Fall term" "Relativistic Rise term" Corrections

me = electron mass
ze = charge of the projectile
Z = atomic number of the material
A= atomic mass of the material
I = ionization energy of the material
β = v/c
$\gamma = 1/\sqrt{(1-\beta^2)}$
N_A = Avogadro's Number
ρ = density of the material

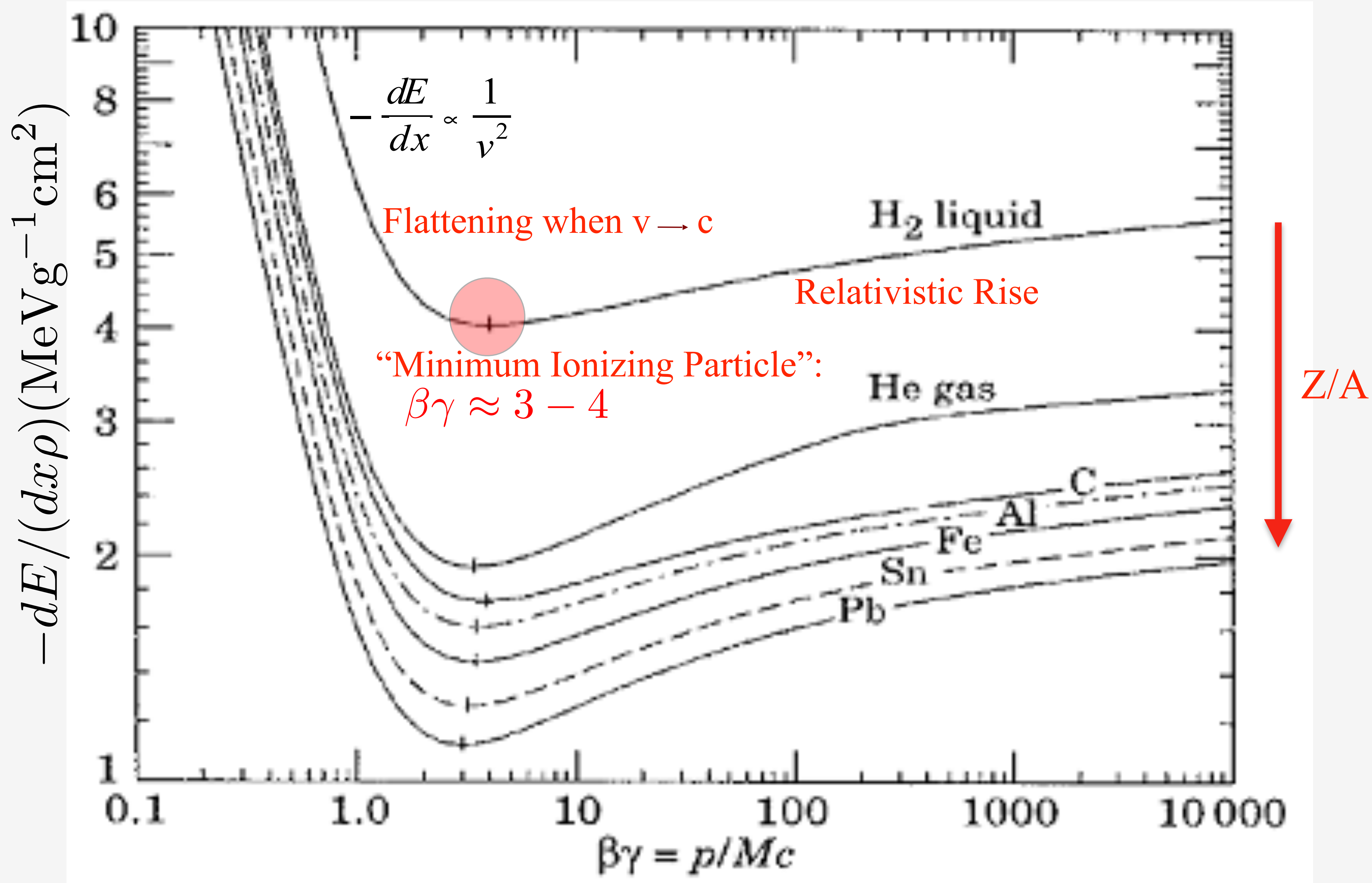


Approximation:
If v<<c we can ignore the small logarithmic term and assuming Z/A~1/2 (valid for most materials):

$$-\frac{dE}{dx} \propto \frac{z^2\rho}{v^2} \longrightarrow$$

- High stopping power for highly charged particles and high-density materials
- High stopping power for slow particles

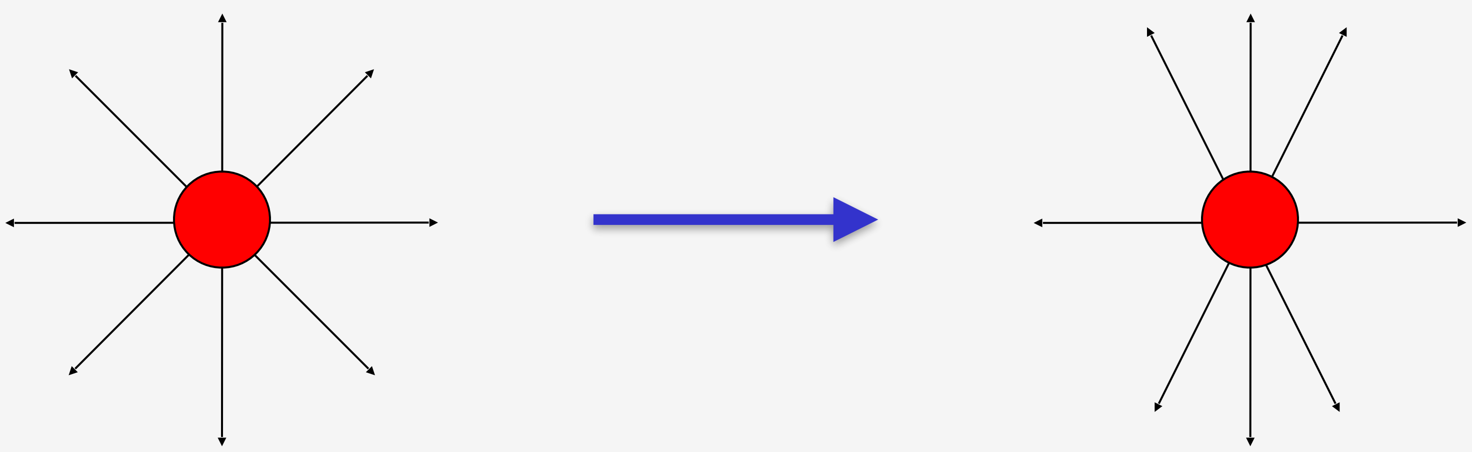
Understanding Bethe-Bloch Formula

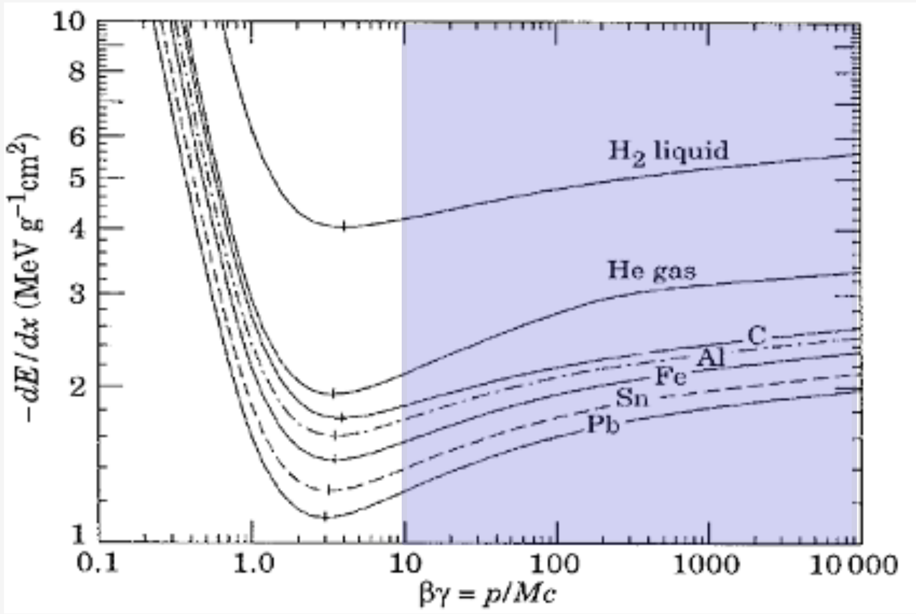


Example: a MIP looses about 13 MeV/cm in Copper (density = 8.94 g/cm3)

The relativistic rise

The rise of the stopping power at high velocities is due to a relativistic effect. If a charged particle is boosted to a reference frame moving close to the speed of light, the electric field E will be modified. E has a spherical symmetry in the rest frame, while in the moving frame the lengths will contract in the direction of motion.





The component of the E field transversal to the direction of motion increases and therefore the same will happen to the stopping power.
The moving charged particle exerts a larger force on the atomic electrons and this results in a larger energy loss.

A couple of Examples

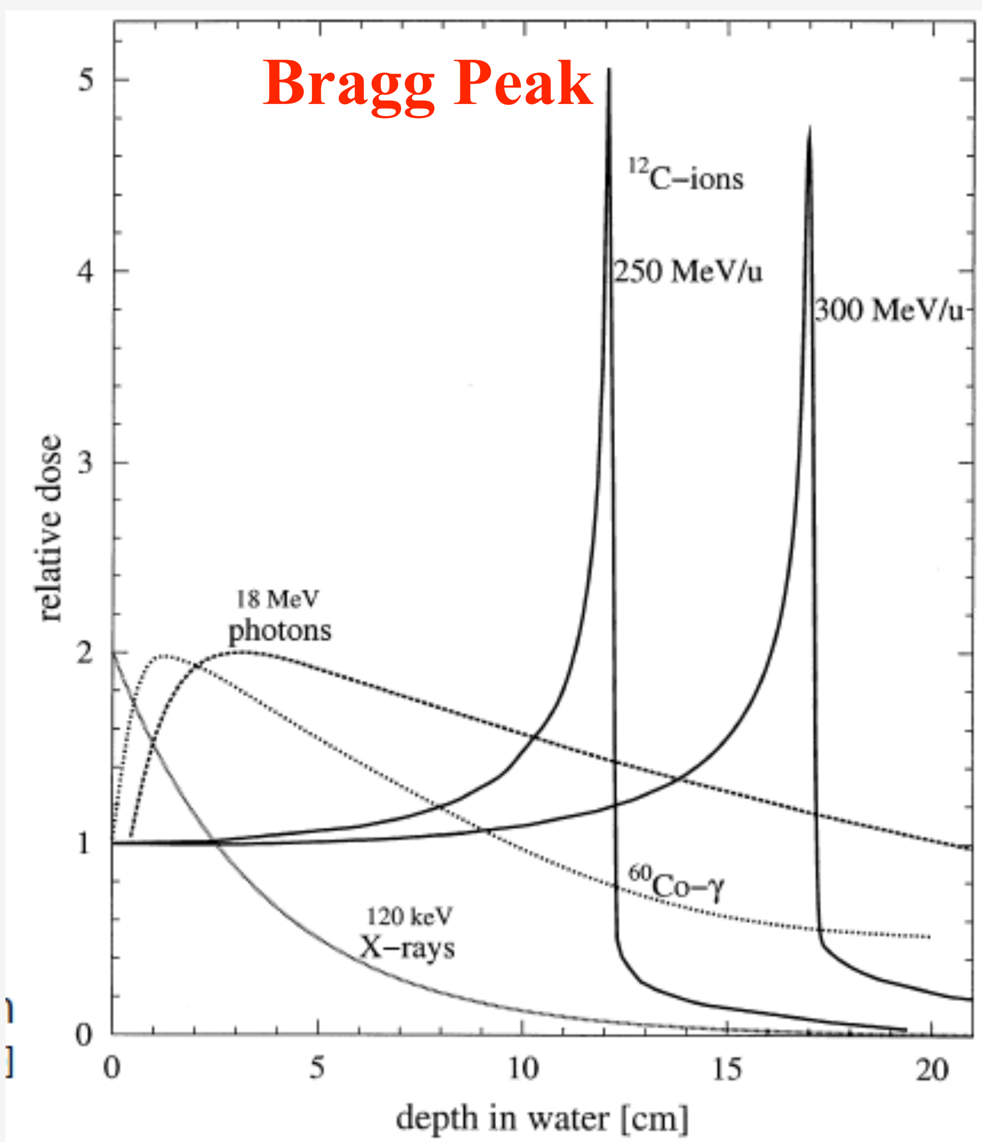
Example 1
Minimum Ionizing particles: ~1-2 MeV/g cm^{-2}
If the density is ~ 1g/cm^3
dE/dx ~ 1-2 MeV/cm

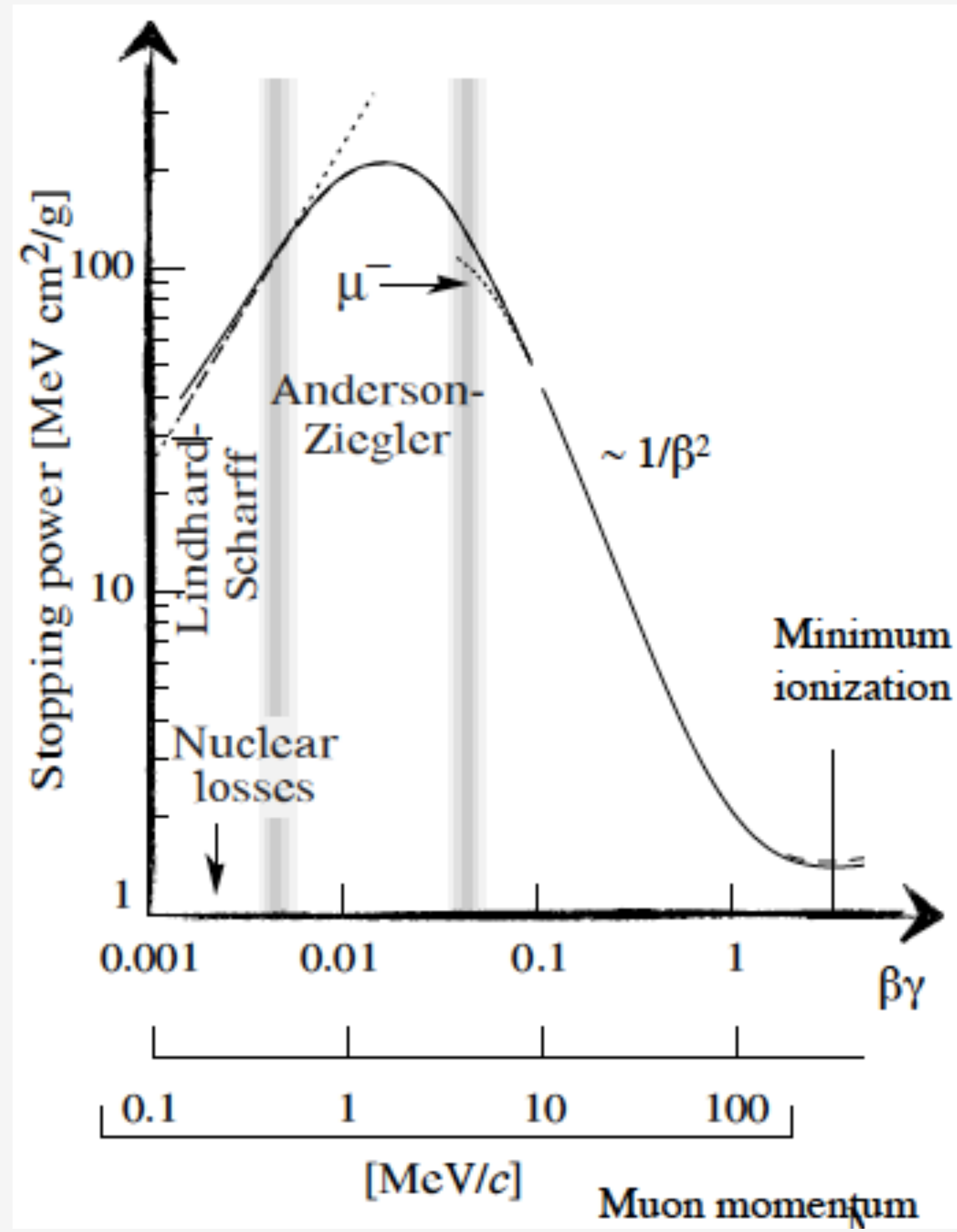
Example 2:
Iron (density=7.87 g/cm^3) with thickness 1m
dE ~ 1.4 MeV $g^{-1}cm^2$ * 1m * 7.87g/cm^3 ~ 1.1GeV

--> A 1GeV muon (typical cosmic ray) can easily traverse 1m of Iron !

Energy Loss at Small Momenta

At small momenta, the energy deposition increases.
This means that most of the energy is deposited at the end of the particle track in the material.

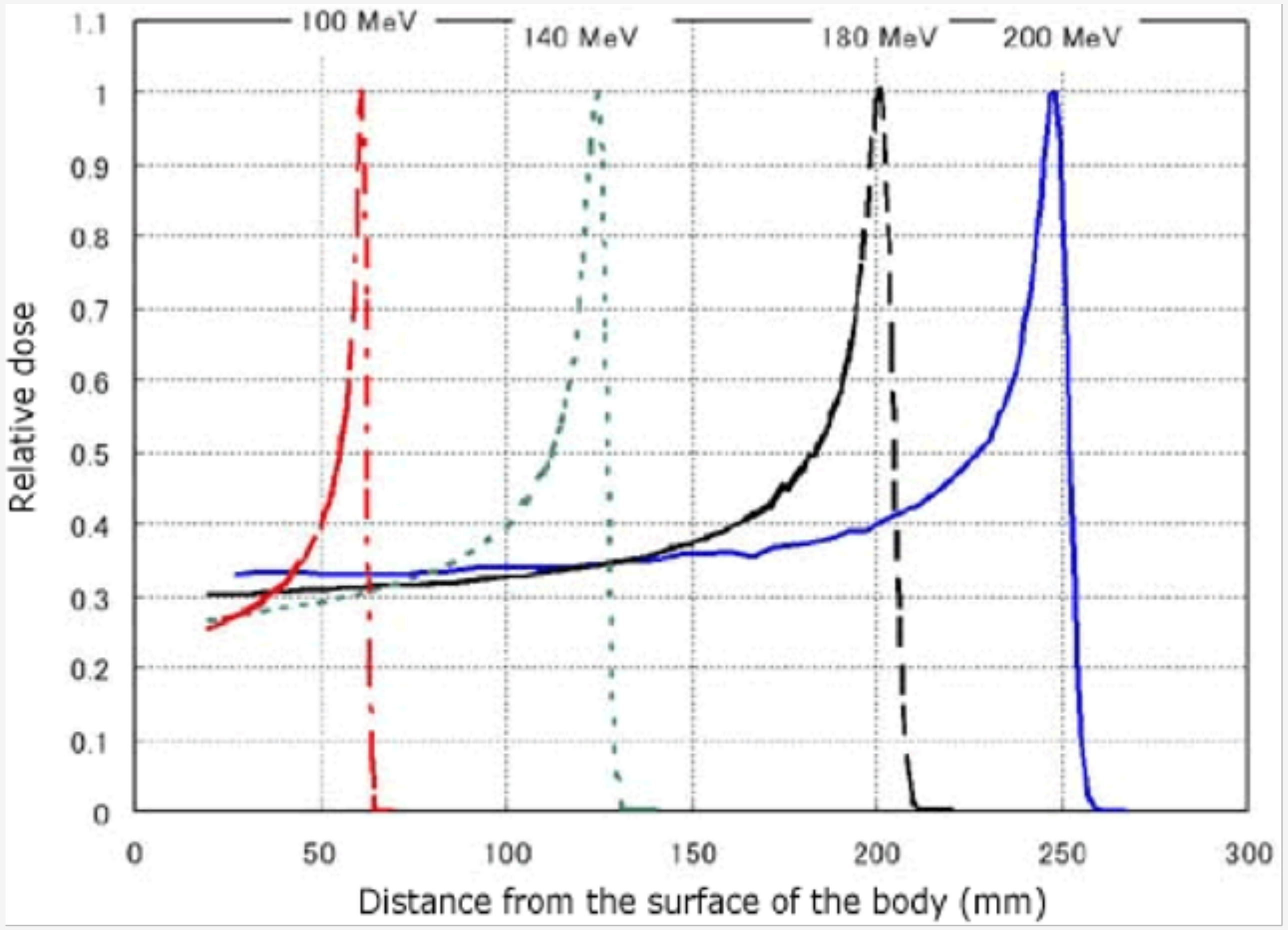




The Bragg Peak

A charged particle entering a material starts to loose energy according to the Bethe-Bloch formula. Since $-dE/dx \sim 1/v^2$, as the particle slows down it looses more and more energy, eventually coming to a stop. From the latter consideration, it is clear that the majority of the energy is lost in the last part of the particle path just before stopping, while the energy loss at the beginning of its path in the material is smaller. The maximum in dE/dx as a function of the path length in the material is called the **Bragg Peak**.

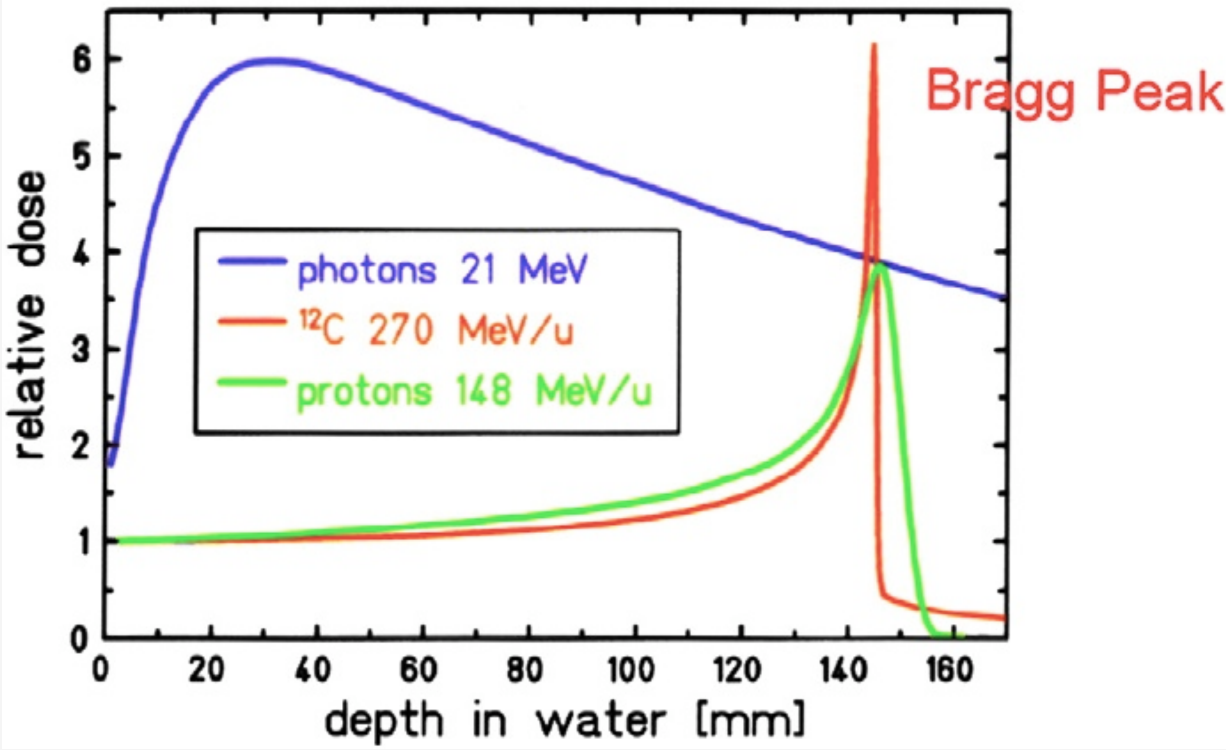
In the figure, the Bragg peaks of protons in human tissue are plotted. As the energy of the proton beam is varied, the Bragg peak moves deeper into the tissue.
This property of dE/dx of heavy charged particles suggests an important application in medical physics: beams of protons or heavy nuclei can be used for cancer therapy. The technique is known as **hadrontherapy**.



Application: Hadrontherapy

Hadrontherapy consists in irradiating a tumor mass with a beam of protons or heavy nuclei. The existence of the Bragg peak gives substantial advantages with respect to the standard radiation therapy based on X/gamma rays (photons). Photons deposit energy in the tissue in a totally different way, as it can be seen in the figure.

It is clear that in the case of radiation therapy the major part of the dose is delivered on the surface of the tissue that can be eventually be damaged. In this way, the tumor mass will receive energy deposit inefficiently.
A treatment with protons or nuclei deposits most of the energy exactly on the tumor mass and thus damaging less the surrounding tissues.
Varying the beam energy, the position (depth) of the Bragg peak can be varied: this allows a full-3D scan of the mass by the beam.

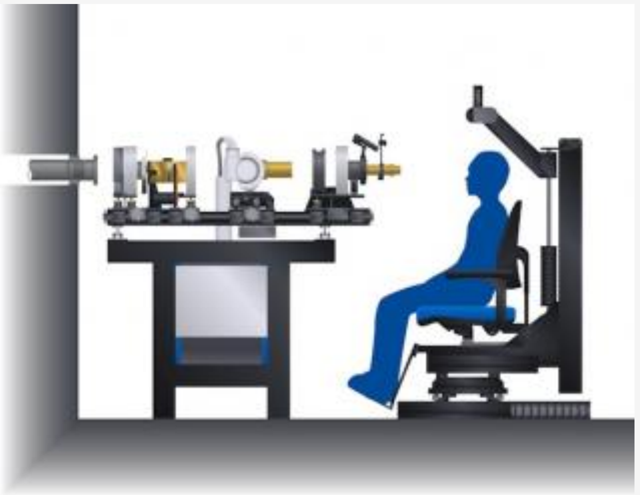


Hadrontherapy works because the radiation damages the DNA of the irradiated cells making them incapable of reproducing themselves. Since cancer cells divide very rapidly, they are more susceptible to radiation than normal cells.
Medical physicists refer to dE/dx as "Linear Energy Transfer" (LET). The LET of protons or nuclei is more densely localized, ending in more DNA damage than conventional radiation therapy.
Only advantages? Unfortunately no:

- Up to now, only non-moving body parts can be irradiated with hadrontherapy treatments (no chest/abdomen)
- The hospital needs an expensive infrastructure: accelerator, beamlines, treatment stations.

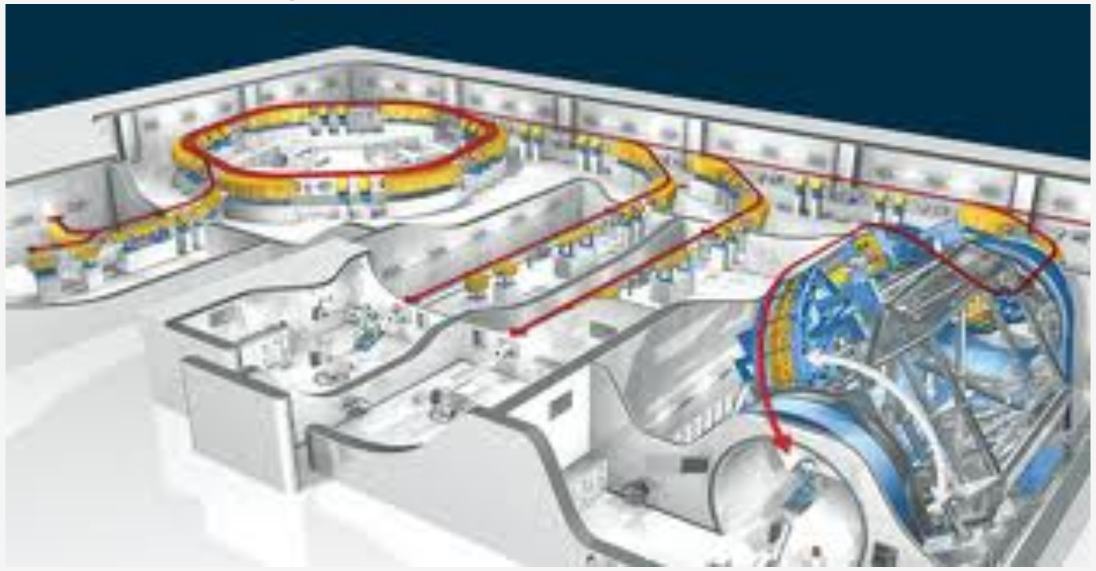
Application: Hadrontherapy

TRIUMF Eye Cancer Treatment Facility



Rotating gantry at the Paul Scherrer Institute

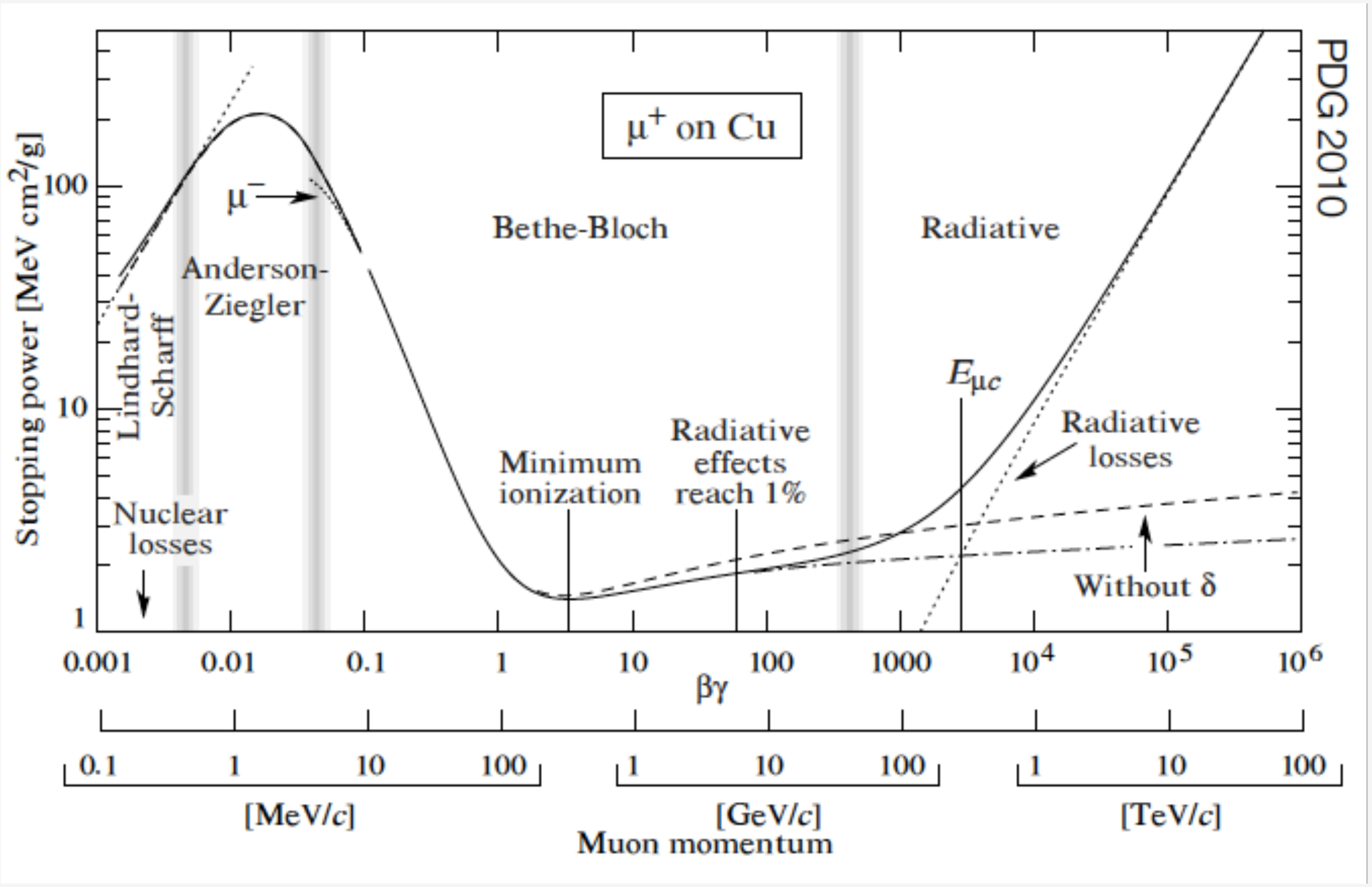
Heidelberg Hadrontherapy Center



Charged Particles: Summary

Heavy charged particles interaction:

1) Particles experience small deflections
2) The range straggling is small
3) There is an energy loss peak at the end of the range: the Bragg peak
4) For slow (v<<c) particles: $$-\frac{dE}{dx} \propto \frac{z^2\rho}{v^2}$$



Energy Loss by Radiation

- Bremsstrahlung

- Synchrotron Radiation

- Cerenkov Radiation

- Transition Radiation

Electrons and Positrons

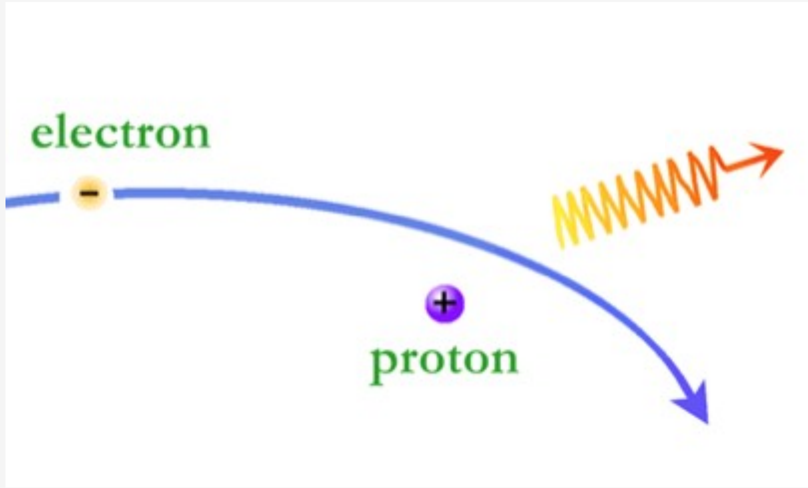
Electrons and positrons also interact with the atomic electrons of materials. Now we are dealing with collisions among particles with the same mass and therefore large deflections are expected. This turns out into large energy losses, in comparison to the previous case (heavy charged particles). Also the range straggling (statistical variation of the range) is larger.

Being the electrons relatively light, it is easy to accelerate them to relativistic velocities (v~c). For example, electrons and positrons from nuclear beta decay are relativistic.

When a relativistic charge is accelerated (for example in a collision), it can radiate photons. This process is called **Bremsstrahlung** ("braking radiation" in German).
The total energy loss can be divided into two components, one due to the Coulomb scattering and one due to the Bremsstrahlung.

$$\left(\frac{dE}{dx}\right)_{Tot} = \underbrace{\left(\frac{dE}{dx}\right)_{Coulomb}}_{\propto \ln(E)} + \underbrace{\left(\frac{dE}{dx}\right)_{Radiation}}_{\propto \frac{E}{m^2}}$$

Very relevant for electrons!



Radiative Energy Loss

The **radiative energy loss** dominates the stopping power for electrons and positrons already above few MeV energy for some dense materials. It can be expressed as:

$$\left(\frac{dE}{dx}\right)_{Radiation} = \frac{E}{X_0} \quad [1]$$

From formula [1] it is possible to calculate the average energy of an electron with initial energy E_0 after travelling a distance x in a material:

$$E = E_0 e^{-\frac{x}{X_0}} \quad [2]$$

From [2], X_0 is the thickness of the material over which the average energy decreases by a factor e. X_0 is given by the approximate formula:

$$\rho X_0 \approx \frac{716.4A}{Z(Z+1)\ln(287/\sqrt{Z})} g/cm^3$$

Where A is the atomic mass and Z the atomic number of the material.

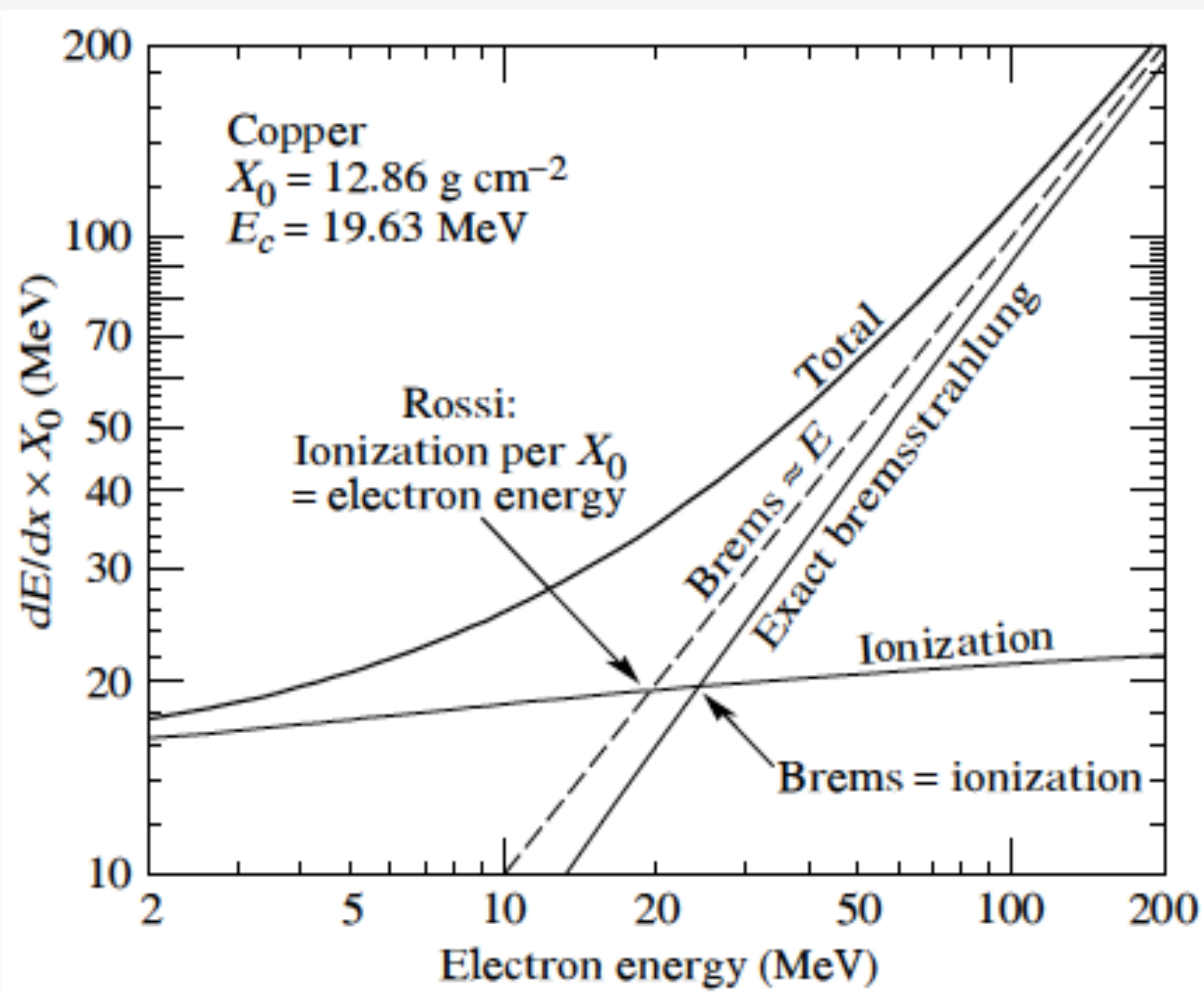
Critical Energy

The critical energy E_c is the electron energy at which the energy losses from radiation and ionization are equal. If the electron energy is bigger than the critical energy, radiation energy loss dominates. The critical energy is approximately given by the following formula:

$$E_C \approx 660/Z \quad MeV$$

The relative sizes of the Coulomb and radiation energy losses for relativistic electrons is given by:

$$\frac{dE/dx(rad)}{dE/dx(Coulomb)} \approx \frac{T + m_e c^2}{m_e c^2} \frac{Z}{1600}$$



From the last formula, it can be observed that energy loss by radiation is enhanced for high kinetic energies and high-Z materials.

Bremsstrahlung Application (I)

An important application of Bremsstrahlung is **Synchrotron Radiation.**
A synchrotron is a circular particle accelerator where particles are guided by magnetic fields into a circular path. Steering particles involves an (centripetal in this case) acceleration which turns into the emission of (synchrotron) radiation.

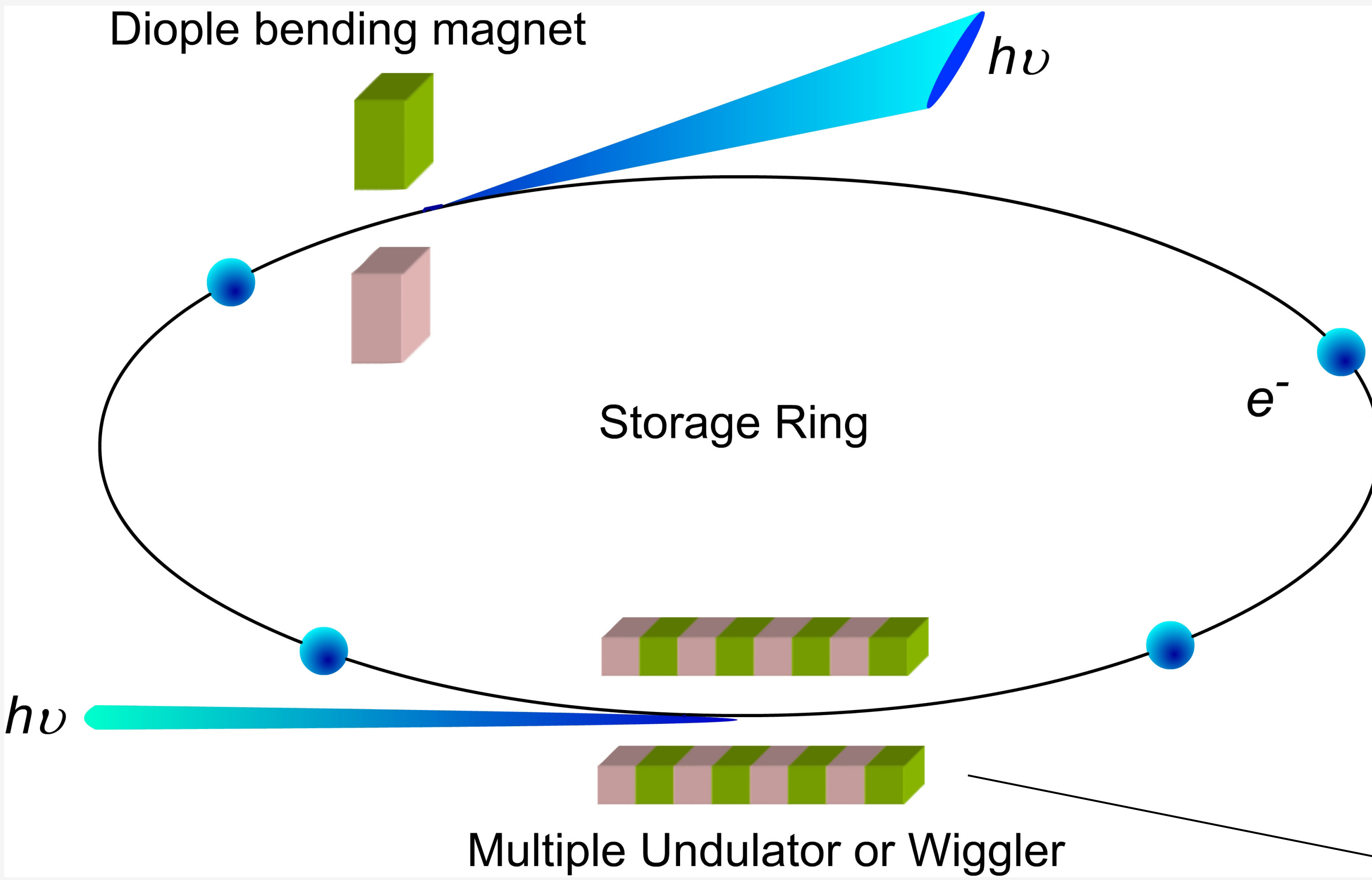
This radiation is a problem if you would like to accelerate particles because part of the energy provided is lost by radiation. Especially light particles experience a sizable energy loss by radiation. In particular, for electrons the energy loss per turn in an accelerator of radius R is approximately:

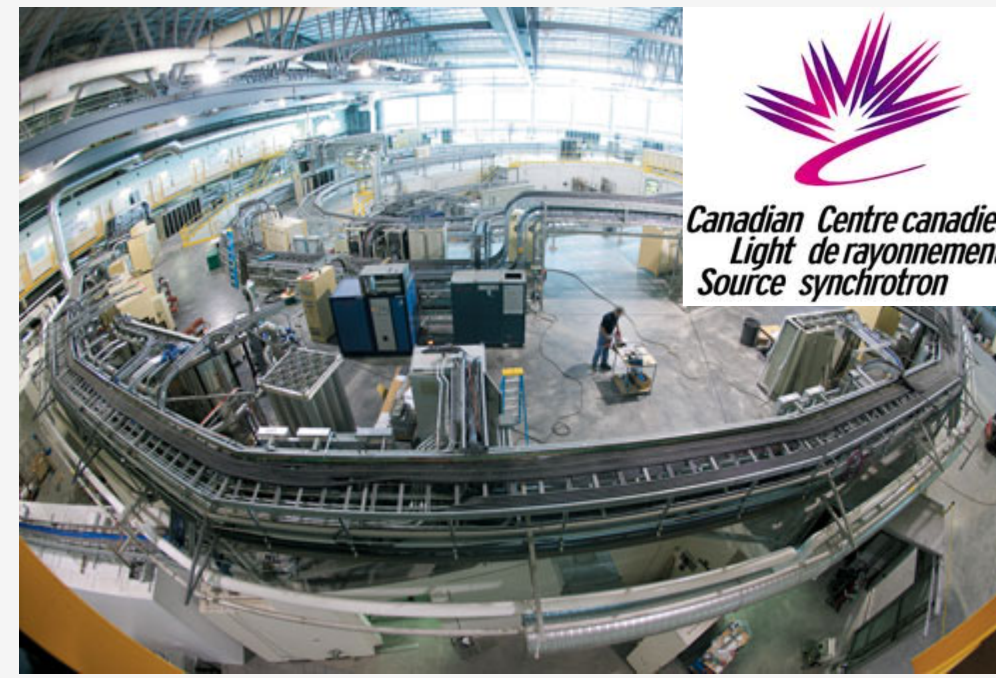
$$\delta E(MeV) \approx 0.0885 \frac{E^4(MeV)}{R(m)}$$

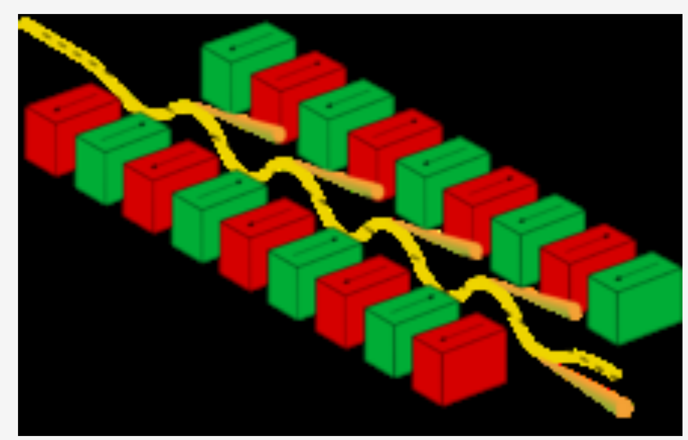
Synchrotron radiation can be also a very useful property if a high-power source of radiation is needed. This is indeed the case in many fields of physics (solid-state, biophysics, chemical physics, ...) and there are synchrotrons dedicated to this task ("Light Sources").

Bremsstrahlung Application (II)

Synchrotron Radiation: Wigglers



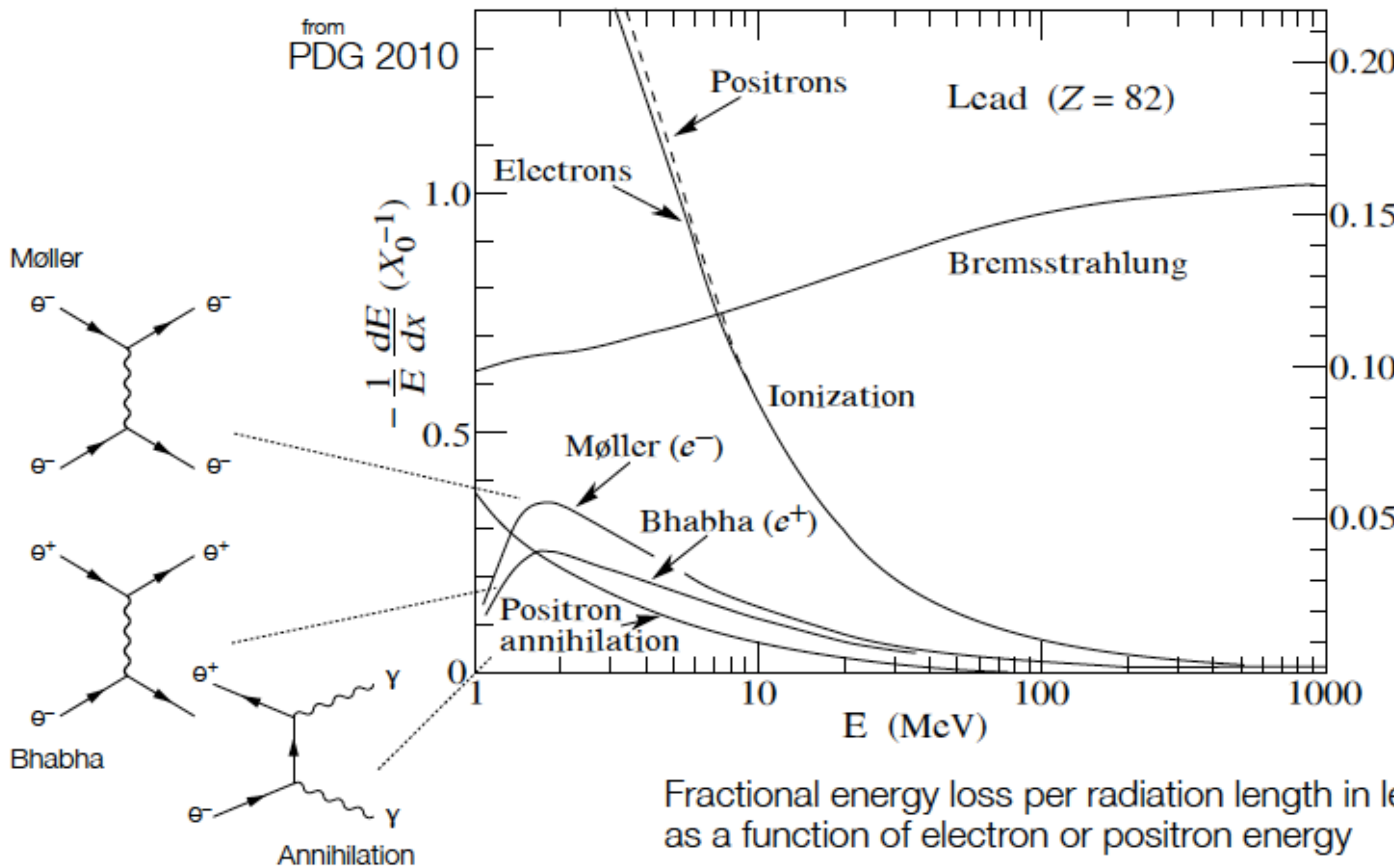




Thanks to the Lorentz boost of the accelerated electrons, the synchrotron radiation is emitted mainly forward, in a narrow cone with the same direction as the particle's velocity.

NOTE: Other application: Free Electron Laser (FEL)

Total Energy Loss for Electrons



Fractional energy loss per radiation length in lead as a function of electron or positron energy

Cerenkov Radiation

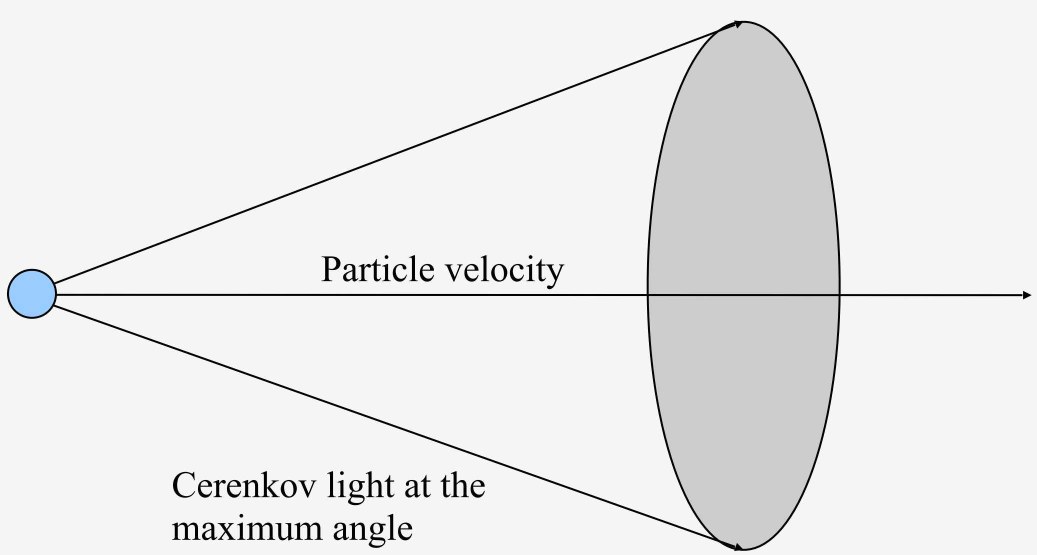
Cerenkov radiation is emitted by a particle when its speed is greater than the speed of light in the traversed material.

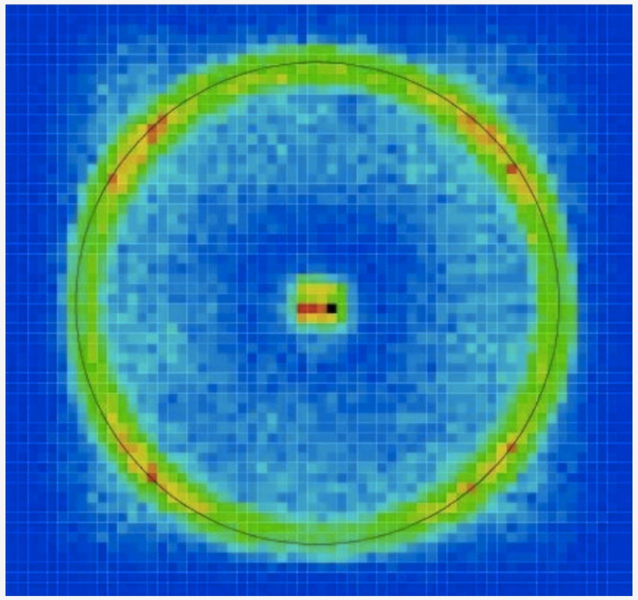
Speed of light in a material with refractive index n : $c_m = c/n$

Cerenkov radiation is emitted in a cone opening in the direction of the particle's velocity. The cone apex half angle has a relation with particle's velocity and the material's refractive index:

$$\cos\theta = \frac{1}{\beta n}$$

At the minimum velocity β=1/n the cone has θ=0. As the velocity increases, also the cone aperture increases up to the maximum of **arccos(1/n)**.

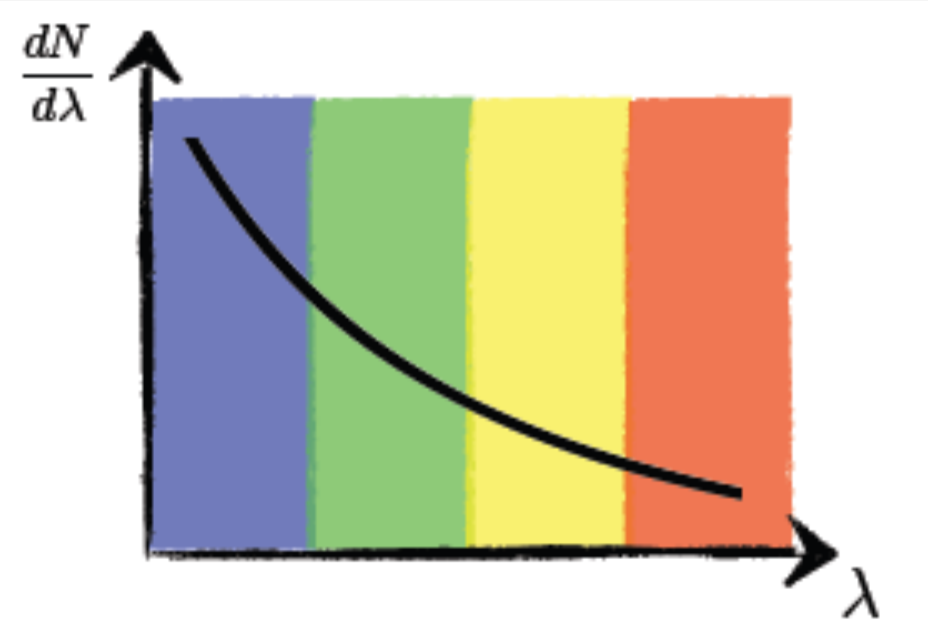




Cerenkov Radiation

Number of emitted photons per unit length per wavelength:

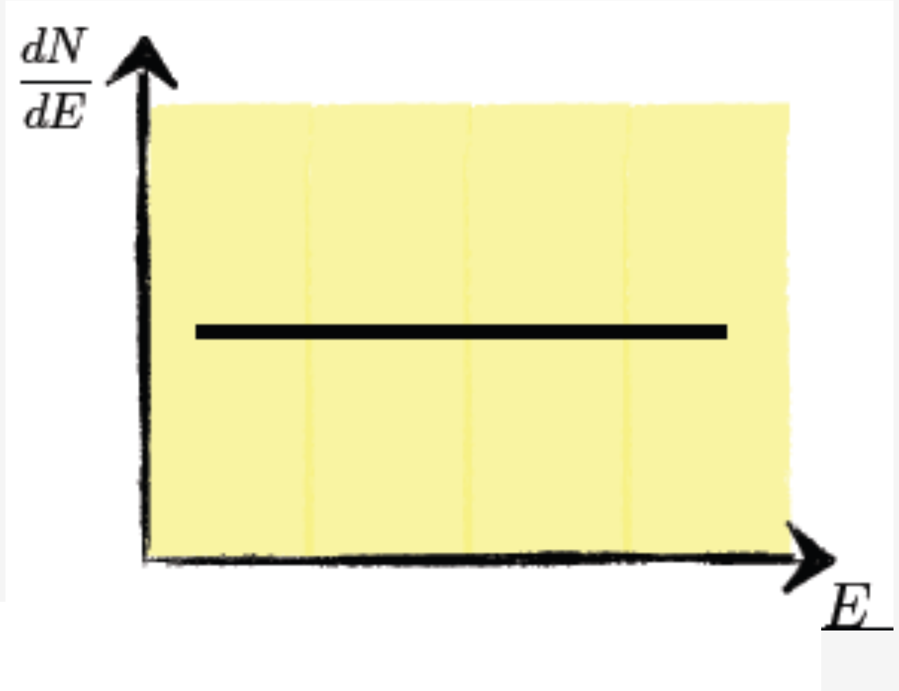
$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_C$$



Number of emitted photons per unit length per energy:

$$\frac{d^2N}{dEdx} = \frac{z^2\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{z^2\alpha}{\hbar c}\sin^2\theta_C$$

$$\approx \text{const}$$

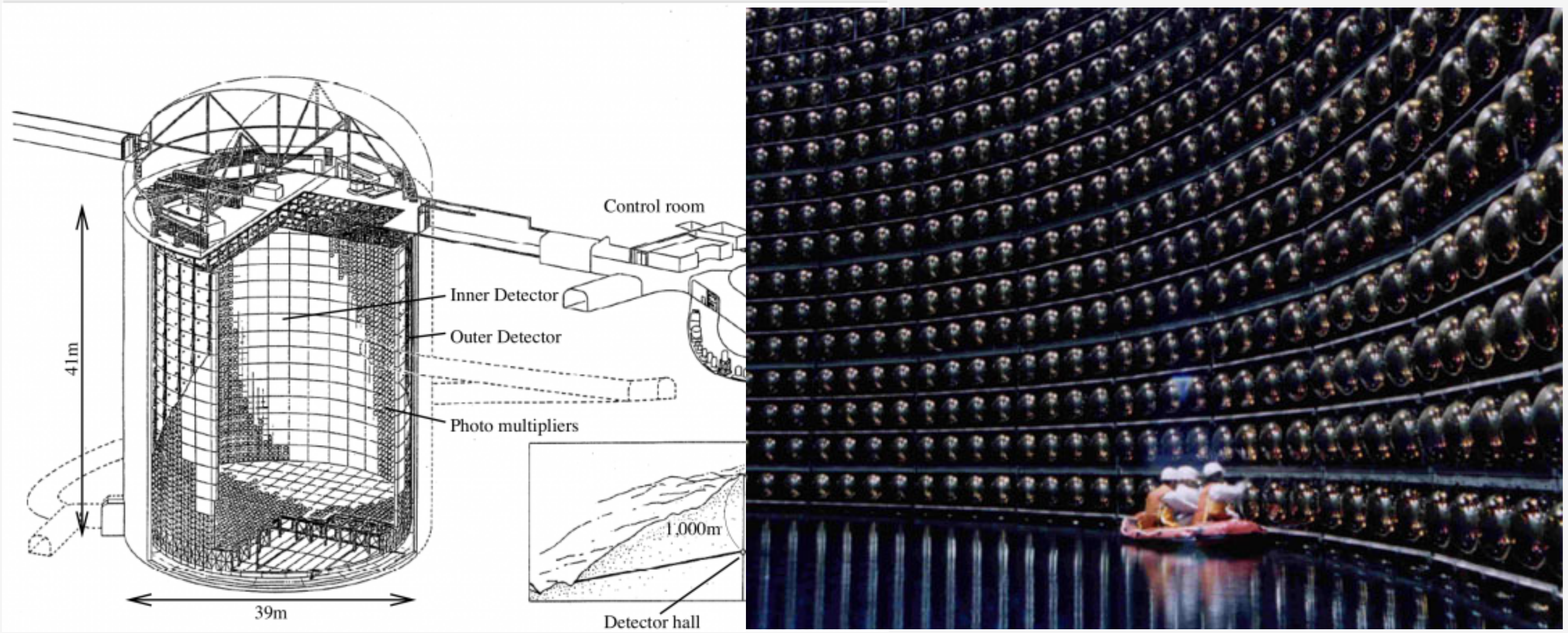


Approximate formula:

$$\frac{d^2N}{dEdx} = 370\ \sin^2\theta_C\ \text{eV}^{-1}\ \text{cm}^{-1}$$

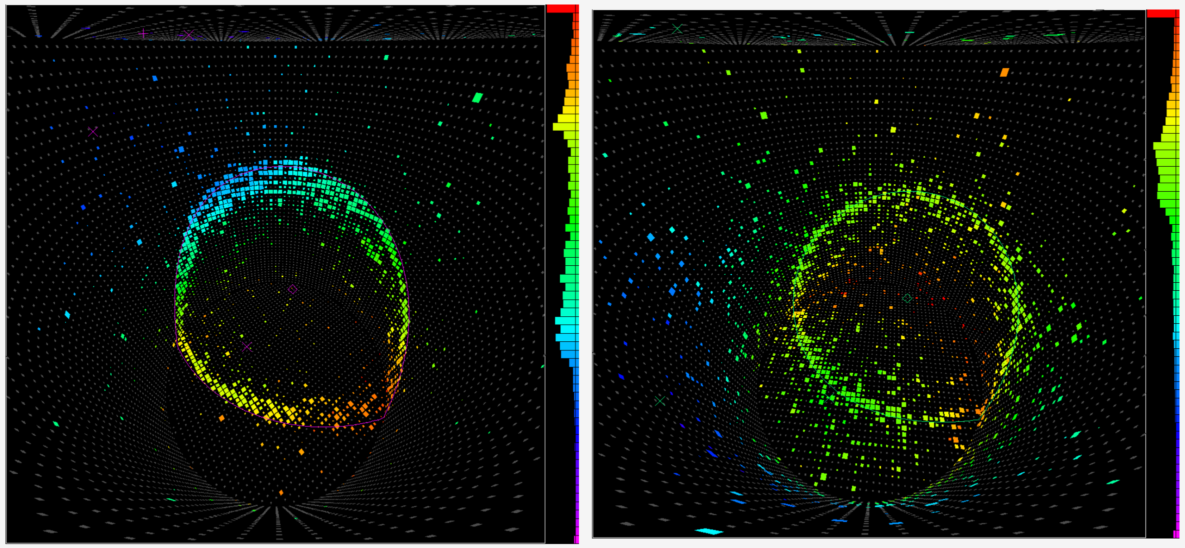
Cerenkov Radiation: Applications

Cerenkov radiation from relativistic electrons produced in beta decays of fission by-products in the core of a nuclear reactor.



The Super-Kamiokande Neutrino Detector is the world-largest Cerenkov detector. It is based on a 50000-tons ultra-pure water tank with dimensions of 40x40m. 11000 18inch PMTs are used for detecting the Cerenkov radiation from electrons and muons.

Muon (left) and electron (right) Cerenkov rings detected at Super-Kamiokande. Note that the electron ring is "fuzzier" than the muon one. This happens because the electron has multiple interactions in the medium and many overlapping Cerenkov cones are produced. This is an efficient method for discriminating between electrons and muons.

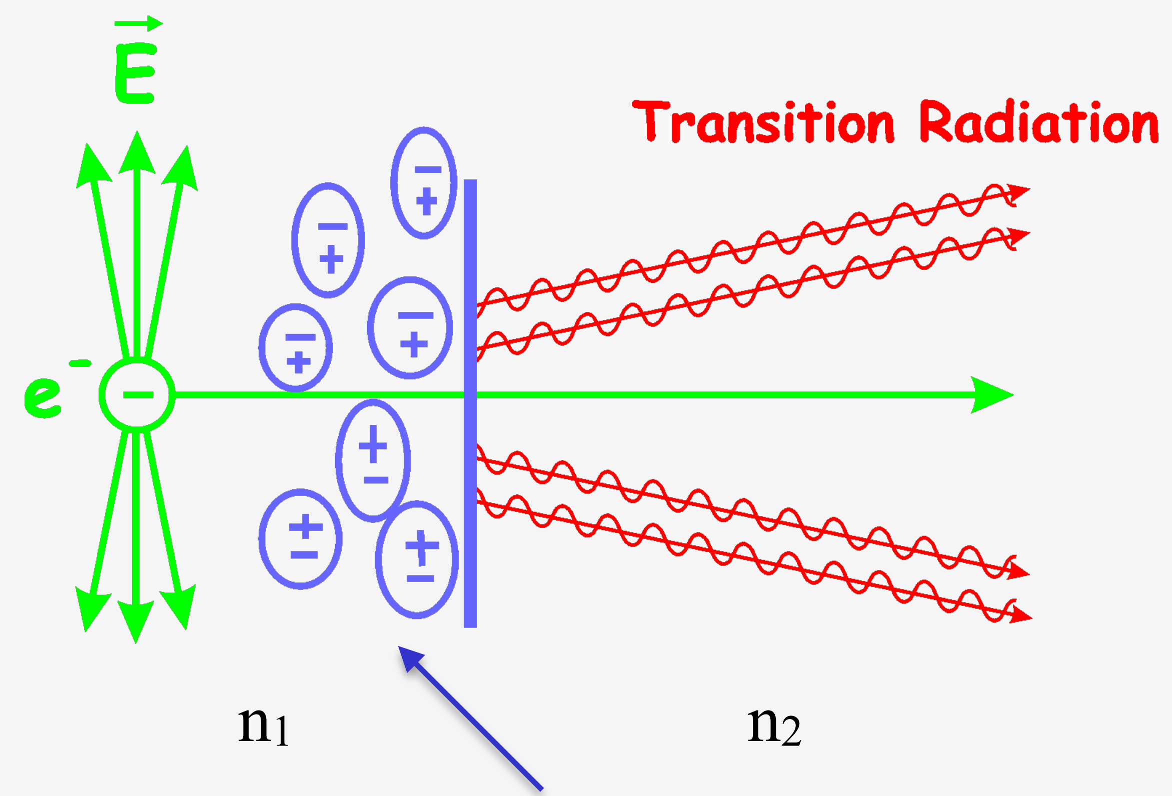


Transition Radiation

Transition Radiation is emitted when a relativistic particle (high "gamma factor") crosses the boundary between two media with different index of refraction.

Such radiation effect was predicted in 1946 by Ginzburg and Frank but the experimental confirmation happened in the 70ies.

It can be understood as a "rearrangement" of the particle's electric field, since it is different in the two media: the difference "evaporates" as transition radiation.



The radiation is produced by oscillating dipoles created by the EM fields of the incoming particle.

Energy flux per angle of Transition radiation from a medium (dielectric constant = epsilon) to vacuum (epsilon=1)

$$\frac{dW}{d\theta} = \frac{2e^2\beta^2}{\pi c^3} \frac{\sin^2\theta \cos^2\theta}{(1-\beta^2\cos^2\theta)} d\omega$$

$$\times \left[\frac{(\epsilon - 1)\left(1 - \beta^2 - \beta\sqrt{\epsilon - \sin^2\theta}\right)}{\left(1 - \beta\sqrt{\epsilon - \sin^2\theta}\right)\left(\epsilon \cos\theta + \sqrt{\epsilon - \sin^2\theta}\right)} \right]^2$$

Electrons: Summary

Energy loss by radiation: electrons are the most affected (small mass).

1) Electrons experience large deflections when traversing materials

2) The range is poorly defined (large range straggling)

3) The energy loss mechanisms are (mainly):
   a) Ionization
   b) Bremsstrahlung
   c) Cerenkov Radiation

4) Transition Radiation

Interaction of Photons with Matter

- Photoelectric Effect
- Compton Scattering
- Pair Production
- Attenuation

Photons / Photoelectric Effect

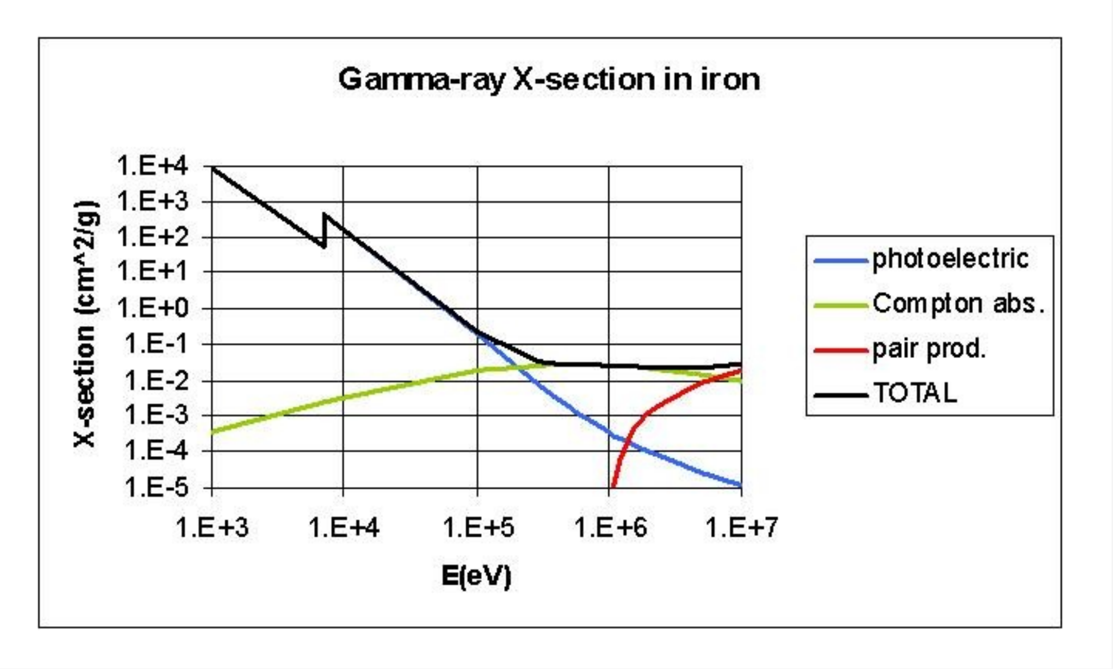
Photons (X- and γ-rays) interact with matter via 3 processes:
1) Photoelectric effect, 2) Compton scattering and 3) pair production.

1) Photoelectric Effect: This process consists of the absorption of the photon and the emission of an atomic electron. This effect can happen only if the electron is bound to an atom, otherwise energy-momentum conservation will prohibit it.
The emitted electron energy E_e is:

$$E_e = E_\gamma - B_e$$

where E_γ is the photon energy and B_e the electron binding energy. The photoelectric effect is significant for low energy photons (<100keV) and the probability is approximately proportional to Z^4/E^3_γ

The photoelectric effect probability shows jumps at particular photon energies when E becomes large enough to liberate electrons from the next deepest atomic shell.

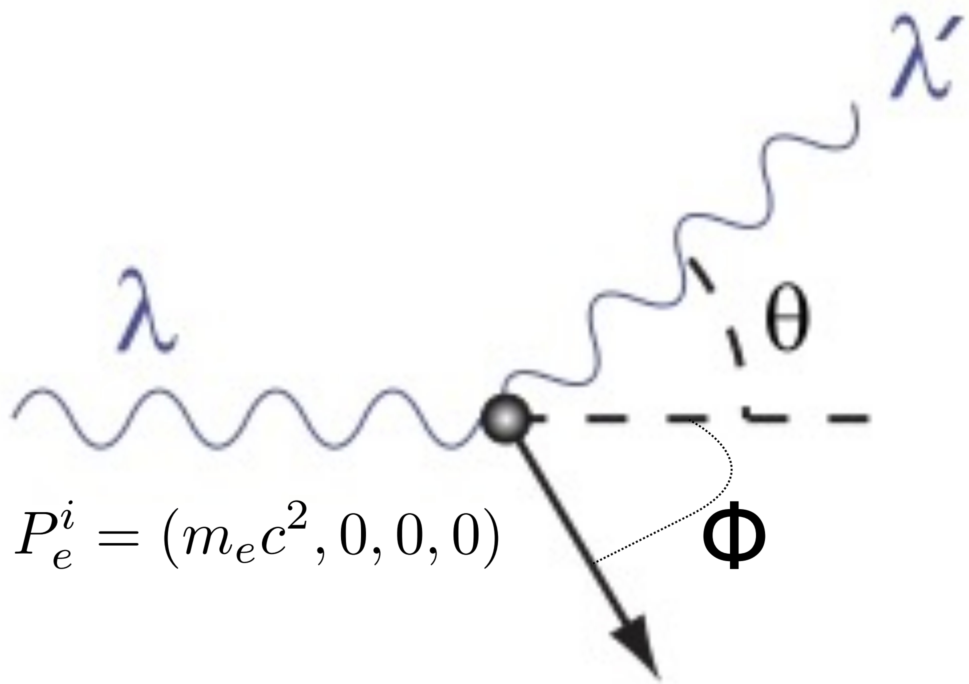


Compton Scattering (I)

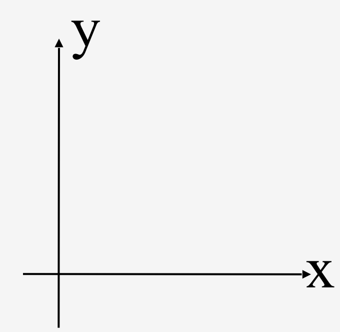
2) <u>Compton Scattering</u>: This process is the scattering of a photon on a nearly free electron:

$$P_\gamma^f = \left(E_\gamma, \frac{E_\gamma}{c}\cos\theta, \frac{E_\gamma}{c}\sin\theta, 0\right)$$

$$P_\gamma^i = \left(E_\gamma, \frac{E_\gamma}{c}, 0, 0\right)$$



$$P_e^i = (m_e c^2, 0, 0, 0)$$



$$P_e^f = (m_e c^2, p_e^f\cos\phi, p_e^f\sin\phi, 0)$$

Compton Scattering (II)

We want to calculate the final energy of the photon after the scattering. We perform the calculation with a fully covariant approach, so we can take advantage from the fact that we can choose the most convenient reference frame. We choose the frame where the electron is initially at rest.

4-Momentum conservation (valid in all the reference frames): $P^i_\gamma + P^i_e = P^f_\gamma + P^f_e$

$$P^i_\gamma + P^i_e = P^f_\gamma + P^f_e \Rightarrow (P^i_\gamma + P^i_e - P^f_\gamma)^2 = P^f_e P^f_e = m_e^2 c^4$$

$$\underbrace{P^i_\gamma P^i_\gamma}_{=0} + 2P^i_\gamma P^i_e - 2P^i_\gamma P^f_\gamma + \underbrace{P^i_e P^i_e}_{m_e^2 c^4} - 2P^i_e P^f_\gamma + \underbrace{P^f_\gamma P^f_\gamma}_{=0} = \underbrace{P^f_e P^f_e}_{m_e^2 c^4}$$

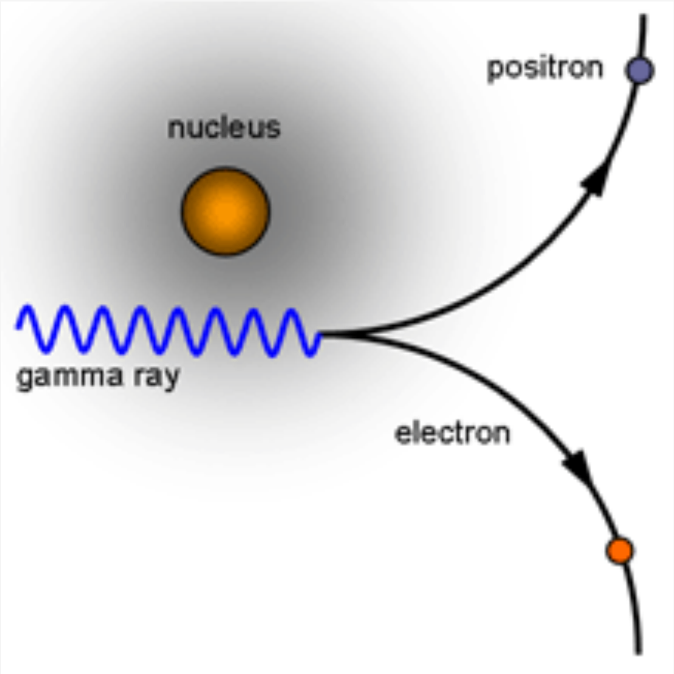
$$\left.\begin{aligned} &P^i_\gamma P^i_e - P^i_\gamma P^f_\gamma - P^i_e P^f_\gamma = 0 \\ &P^i_\gamma P^i_e = E^i_\gamma m_e c^2 \\ &P^i_\gamma P^f_\gamma = E^i_\gamma E^f_\gamma (1 - \cos\theta) \\ &P^i_e P^f_\gamma = E^f_\gamma m_e c^2 \end{aligned}\right\} \Rightarrow \quad E^f_\gamma = \frac{E^i_\gamma}{1 + \left(\frac{E^i_\gamma}{m_e c^2}\right)(1 - \cos\theta)}$$

Note that for ϑ =0, E^f=E^i for the "backscattering" case (ϑ =180°), $E^f_\gamma = E^i_\gamma / (1 + 2E^i_\gamma / m_e c^2)$

The probability (cross-section) for this process is given by the Klein-Nishina formula as a function of the photon scattering angle. [Z. Phys. 52 (11-12): 853, 869 (1929)].

Pair Production

A photon with enough energy can create an electron-positron pair. In this process, the photon disappears and the pair is created. According to energy-momentum conservation, this process cannot happen in vacuum. If a Coulomb field of e.g. a nucleus in a material is present, then the conservation law can be satisfied and the process can happen.



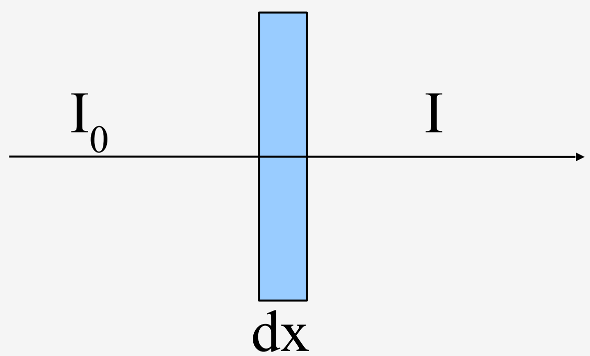
The minimum energy for a photon being able to convert in a pair is

$$E_\gamma \geq 2m_e c^2 = 1.022MeV$$

Above a photon energy of ~5MeV, pair production becomes the dominant process in photon-matter interaction.

Photon Attenuation

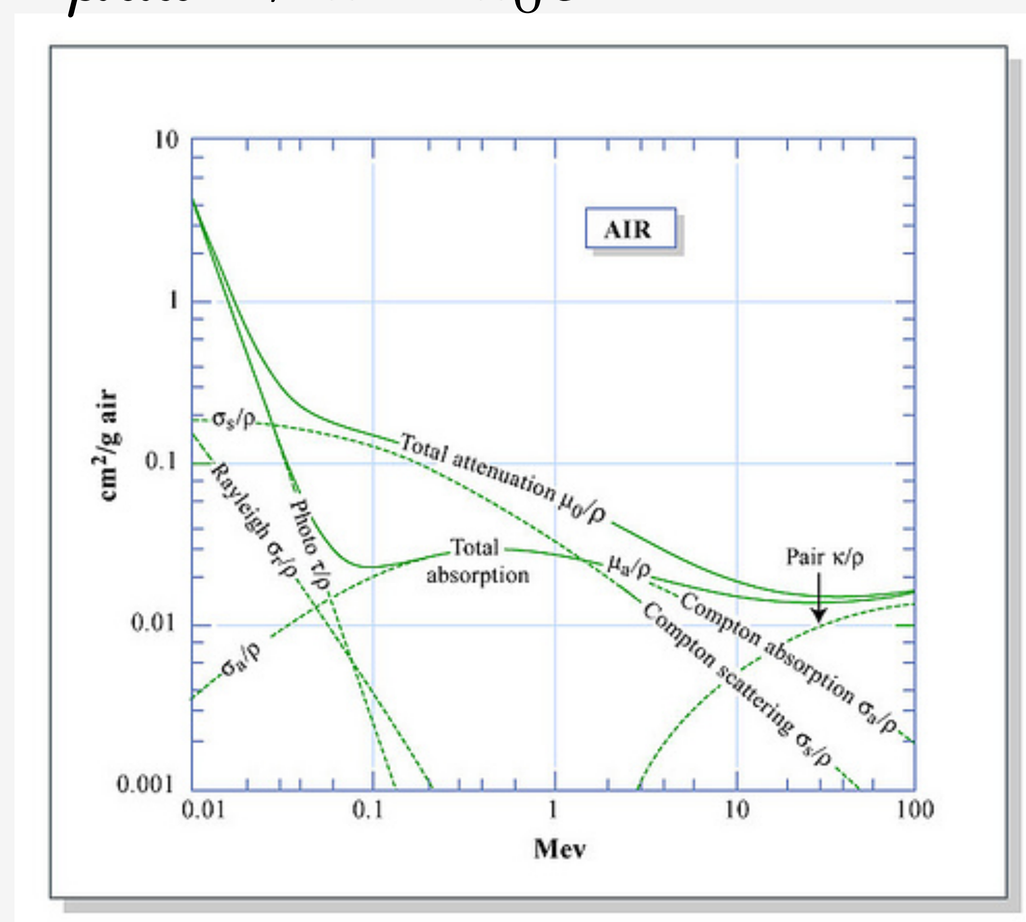
Charged massive particles deposit all their energy if they stop in a material. On the contrary, photons experience **attenuation**. If a beam of photons impinges on a material, they can be absorbed by photoelectric effect or pair production or scattered by Compton effect. This means that some photons will never reach the detector and are effectively lost.
Considering a photon beam with intensity I traversing a material of thickness x:



$$\frac{dI}{I} = -\mu dx \Rightarrow I = I_0 e^{-\mu x}$$

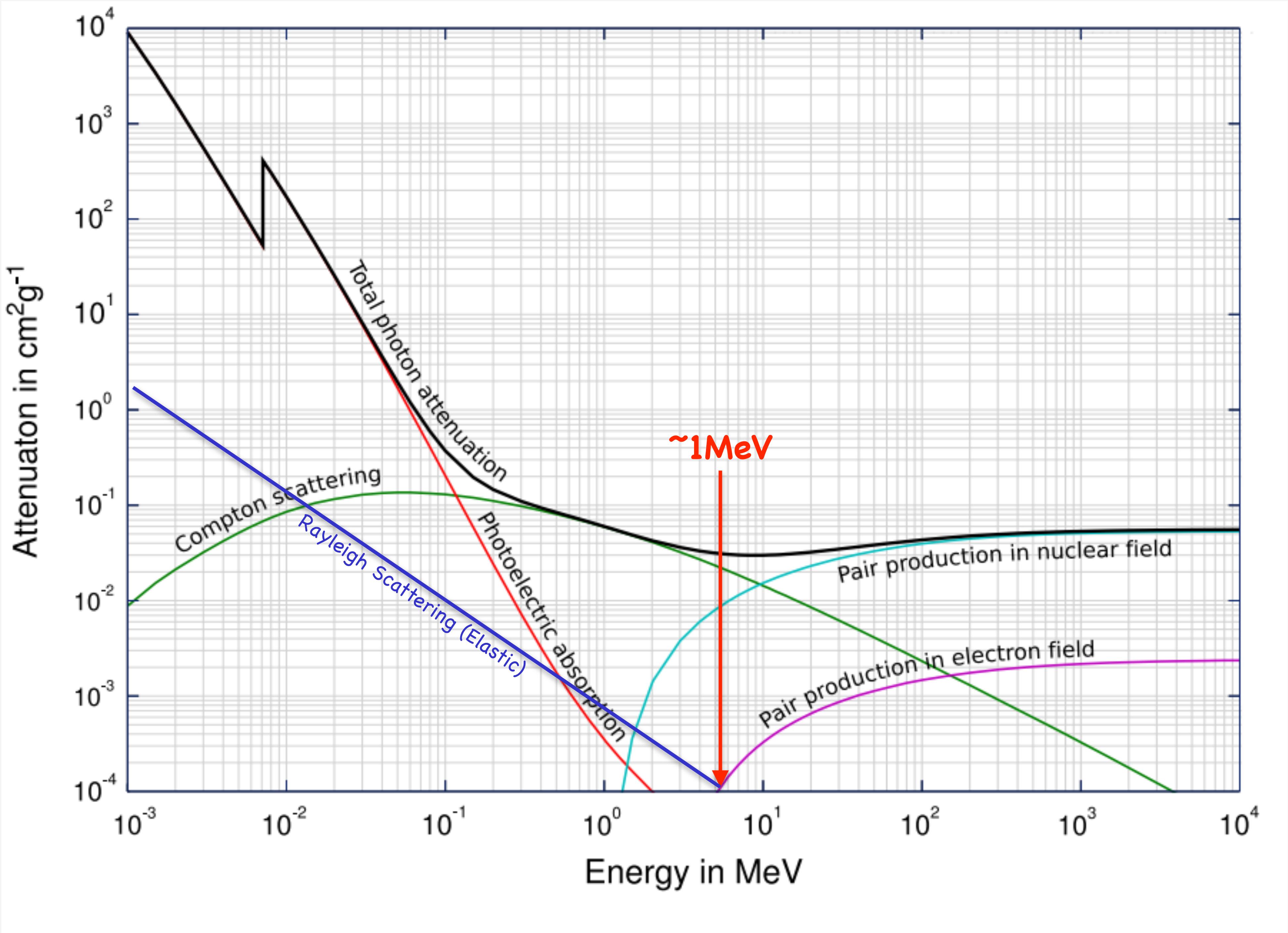
The fractional loss of beam intensity dI/I is proportional to the thickness x and this leads to an <u>exponential attenuation</u> through matter.
The total attenuation coefficient is the sum of three contributions coming from photoelectric effect (τ), Compton scattering (σ) and pair production (κ):

$$\mu = \tau + \sigma + \kappa \text{ cm}^{-1}$$



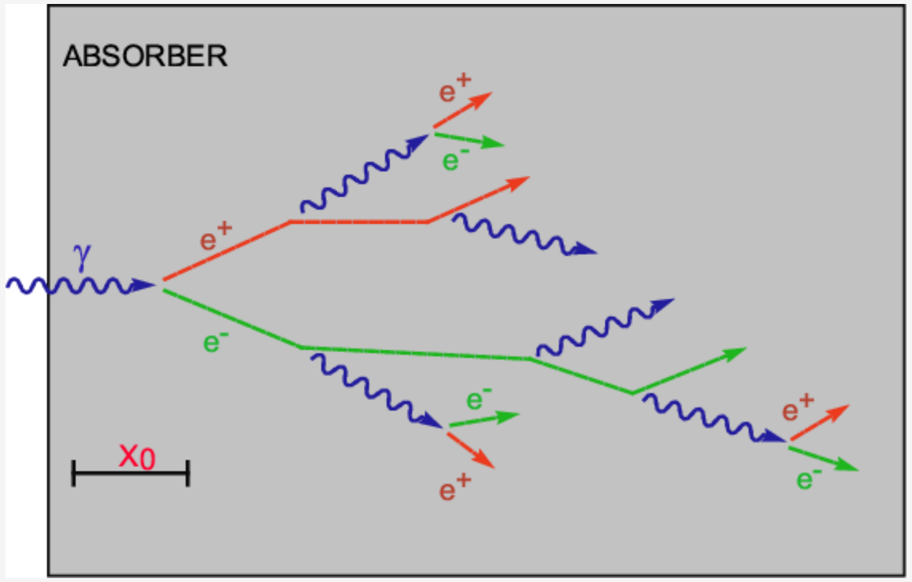
<u>NOTE:</u> Remembering cancer treatment, now you can compare the exponential attenuation of a photon beam in a tissue versus the peak-like energy deposit of charged particles, appreciating its higher efficiency in delivering the deposit (dose) in a small region.

Summary: Photon Interactions in Matter

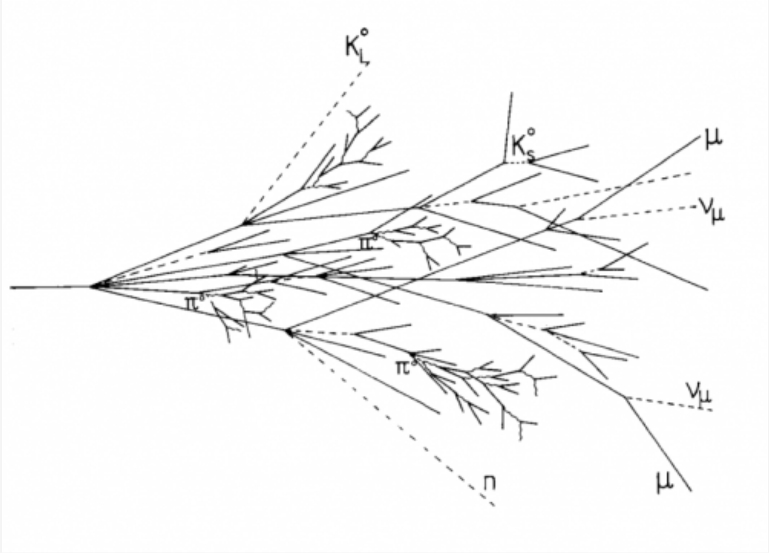


Particle Showers (I)

Particle showers develop when a very high-energy particle interacts with matter. When the energy is high, more particles can be produced. These secondary particles can still have enough energy to produce again other particles. This process, called shower continues until particles do not have enough energy to produce new ones and eventually loose all the energy by ionization or collisions.



Electromagnetic Showers:
If a high-energy electron, positron or photon enters a material, it generates a shower through pair creation and brehmsstrahlung, giving rise to an exponential increase of particles with lower and lower energies. The energy of the secondary particles decreases until no energy is left for further conversion.



Hadronic Showers:
Hadronic particles (e.g. protons and in general particles interacting with the strong force) at high-energy can create showers of other hadronic particles. Cross sections for such processes are small, since interactions with atomic nuclei are needed and they are rarer than atomic interactions.

Particle Showers (II)

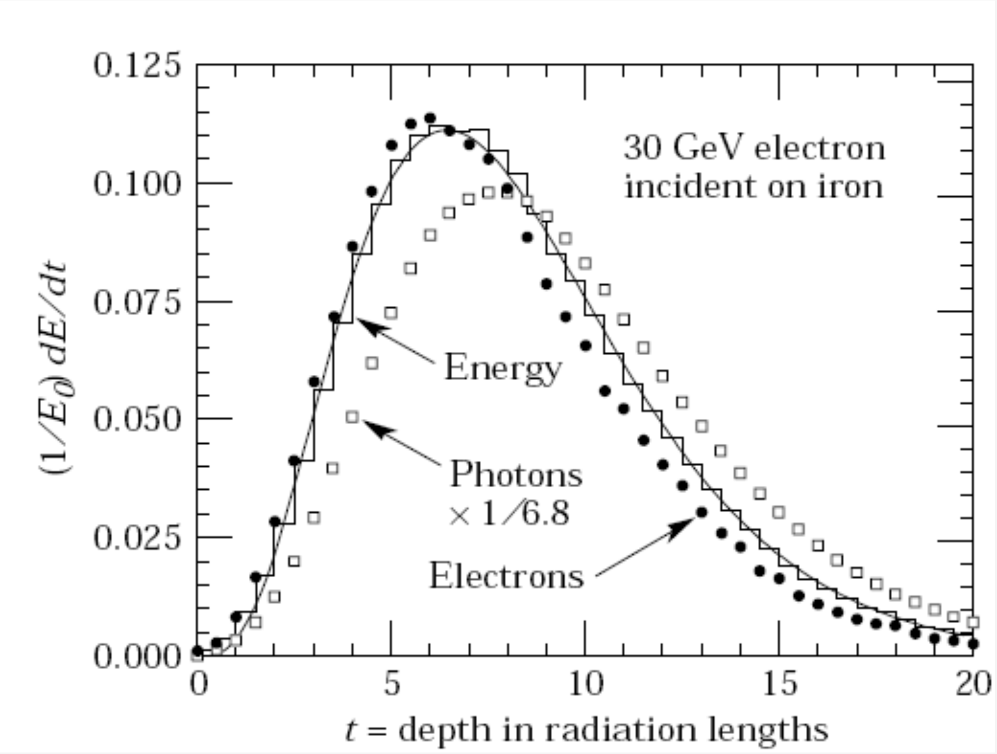
Longitudinal Development: The depth of the shower in a material is mainly defined by the high-energy part of the cascade. The description of showers is more convenient introducing scaled variables:

$$t = x / X_0 \qquad y = E / E_c$$

In this way the length is in units of radiation lengths and the energy is in units of critical energy.
The longitudinal profile of the energy deposition in an electromagnetic cascade is well approximated by:

$$\frac{dE}{dt} = E_0 b \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

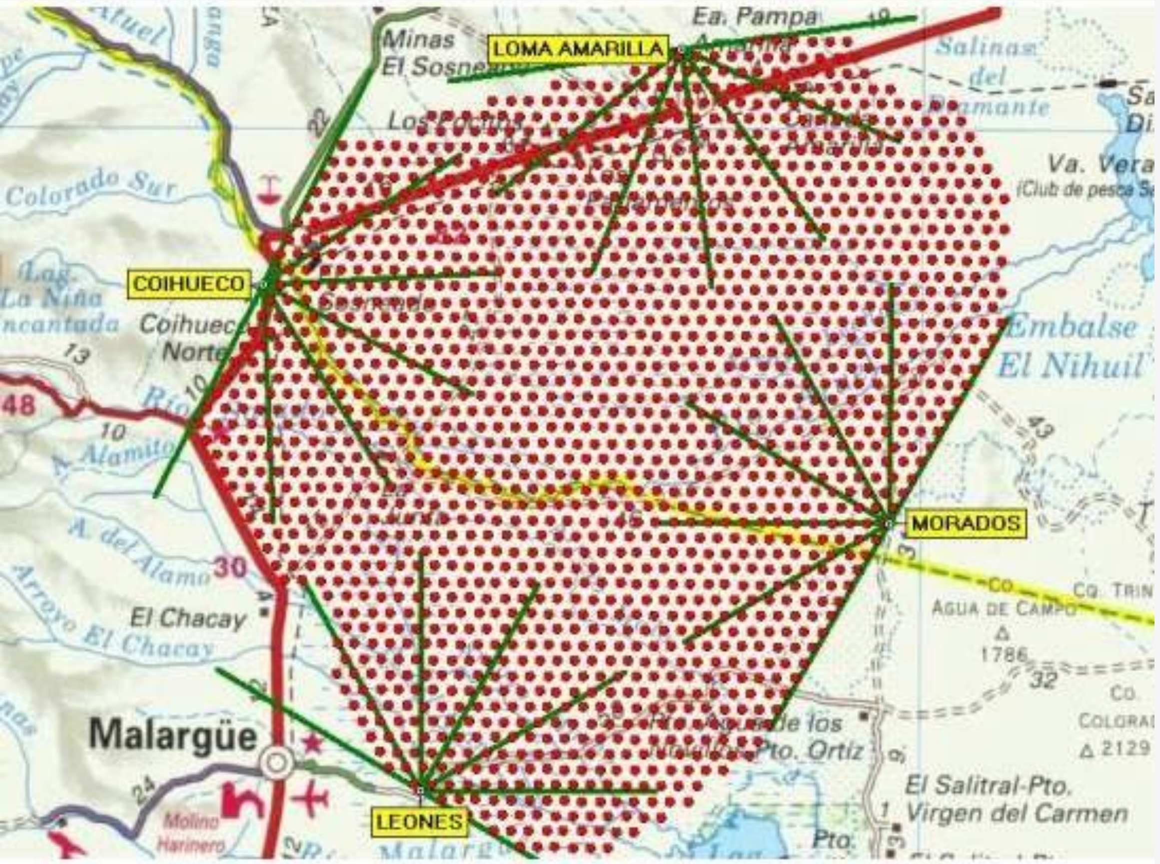
Since the cascade is a strongly fluctuating process, the last formula works well only when the average depth is needed.



Simulation of a cascade induced by a 30GeV electron on Iron.

Transverse Development: For electromagnetic showers, the radius of the shower scales in good approximation with the following formula: $R_M = X_0 E_s / E_r$
R_M is called "Moliere Radius", where E_s~21MeV and E_r is the energy at which the energy loss by ionization divided by the radiation length is equal to the initial electron (or photon) energy. The Moliere radius defines a cylinder containing approximately 90% of the deposited energy.

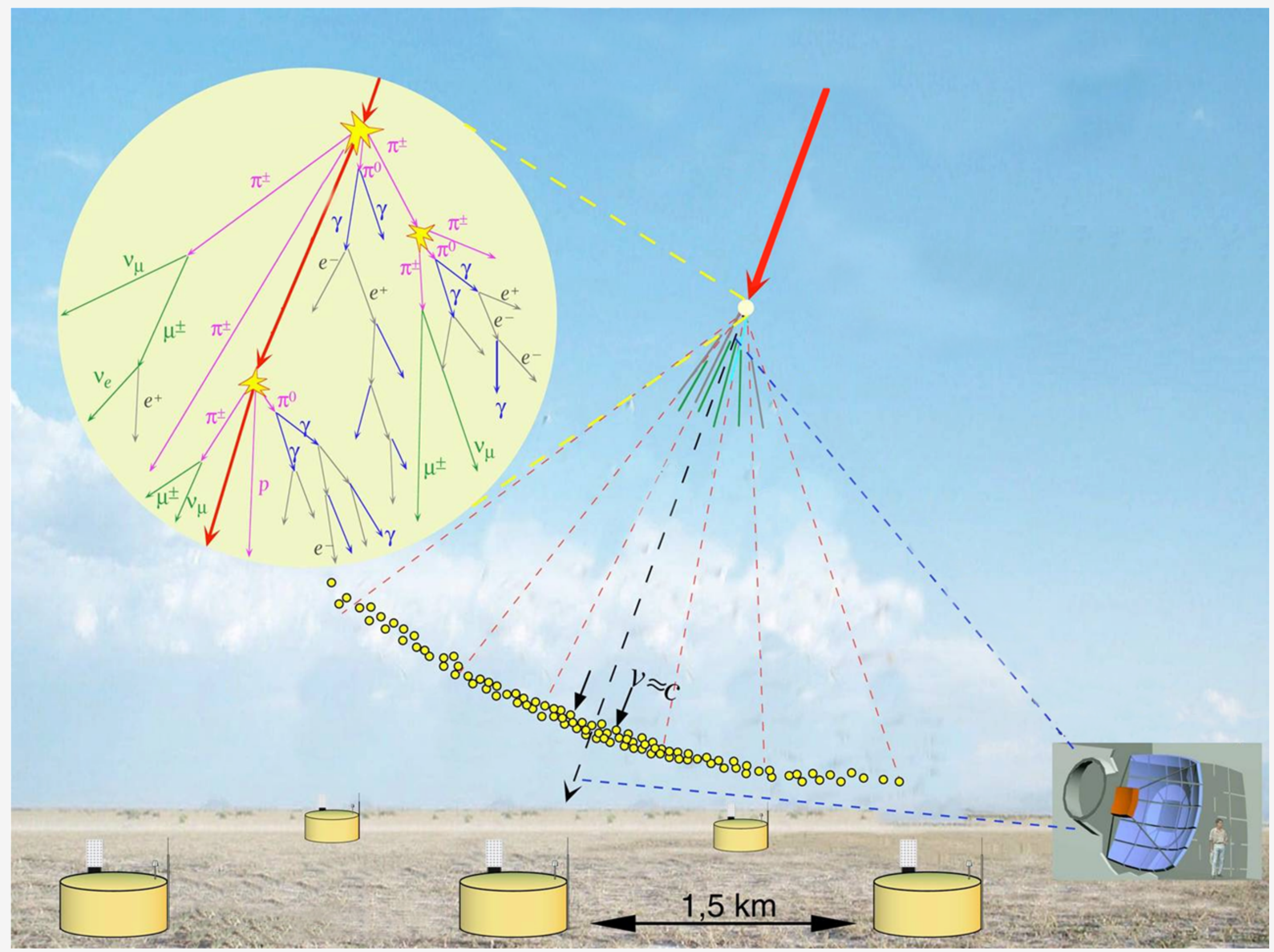
Particle Showers (III)



The observatory is located in Argentina and is composed of many detectors spaced 1.5 Km from each other and covering a surface of about 6000 km^2.

In the case of high-energy cosmic rays, showers can extend even on several kilometers.

The Pierre Auger Observatory was designed to measure direction and energy of such very large showers.



Neutrons (I)

Neutrons are heavy particles, but charge-neutral, therefore they do not interact electromagnetically and the Bethe-Bloch formula cannot be applied. They can still interact with atomic nuclei, but the cross sections are small: neutron radiation is very penetrating.

Common sources of neutrons are:

- Nuclear Reactors (Fission and Fusion)
- Spallation Sources (Accelerator based)
- Radioactive decays of heavy nuclei
- Secondary interactions (within showers)

In applications, neutrons are classified according to their energy range:

- High Energy: >10MeV (e.g. from nuclear fission, spallation or showers)
- Medium Energy: ~1MeV
- Fast: ~50keV
- Epithermal: 0.5eV - 50keV
- Thermal: <0.5eV
- Cold: ~meV
- Ultra-Cold: <<meV

Neutrons (II)

Neutron nuclear interactions can be elastic, inelastic or capture reactions.

Elastic Interaction

The neutron can scatter off a nucleus leaving its state unchanged. The average energy loss of a neutron after elastic scattering is approximately:

$$E = \frac{2E_0 A}{(A+1)^2}$$

From the formula, it is clear that light nuclei are the most effective targets for slowing neutrons. Hydrogen in particular is the best neutron moderator : $E = E_0/2$.

As an example, consider the number of collision needed for moderating a 2MeV neutron to 0.025eV (thermal):

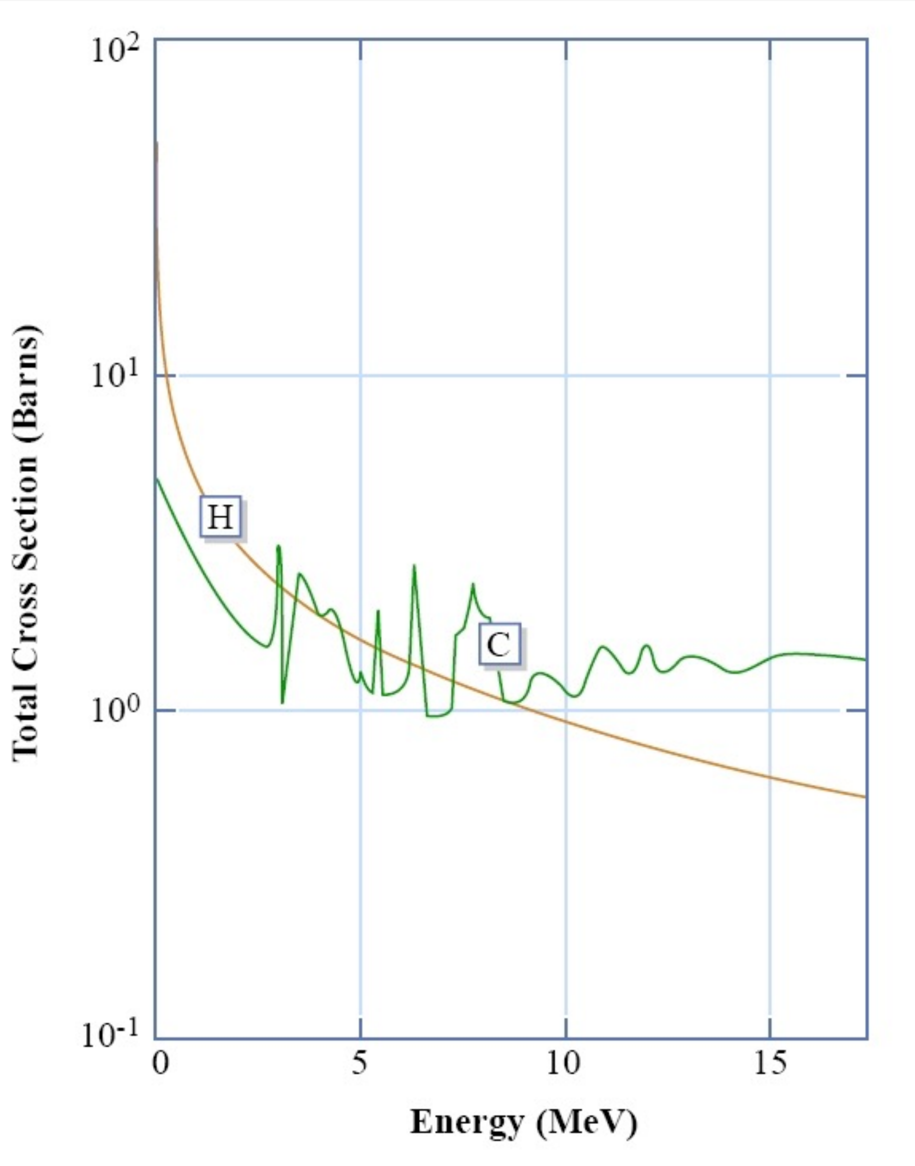
Nucleus	A	Collisions
H	1	27
D	2	31
He	4	48
Be	9	92
C	12	119
U	238	2175

NOTE: In nuclear reactors heavy water is often used as moderator. This means that the moderator is effectively deuterium. Hydrogen would be a better moderator: why deuterium is used instead?

Neutrons (III)

Inelastic Interactions

These interactions are more complex because the nuclear structure is involved. Generally, after an inelastic nN reaction, the nucleus is found in an excited state. De-excitation follows with emission of additional photons. In the figure, the neutron crosssection for H and C are displayed. Note the complicated features of the C crosssection due to its nuclear structure.



Nuclear Capture

Neutrons can be captured by nuclei and the consequence of this strongly depends from the specific nucleus. Cases can be classified from the kind of particles in the final state:

- Charged: emission of p,d,alpha,...
- Neutral: emission of more (>1) neutrons
- Fission: the nucleus divides in two or more fragments

Neutrons (IV)

Applications (Example)

Thermal Neutrons have wavelengths similar to interatomic distances in many materials. This means that low energy neutrons can be used for diffraction experiments in condensed matter physics. Moreover:

- Neutrons are very penetrating (no EM interactions): a material can be studied well beyond its surface.
- Neutrons have a magnetic moment: neutron scattering can also be used to investigate magnetic properties of materials.

The combination of neutron scattering with other techniques (e.g. X-ray diffraction) is a powerful analysis tool.

ILL Neutron Reactor Source (Grenoble, France)

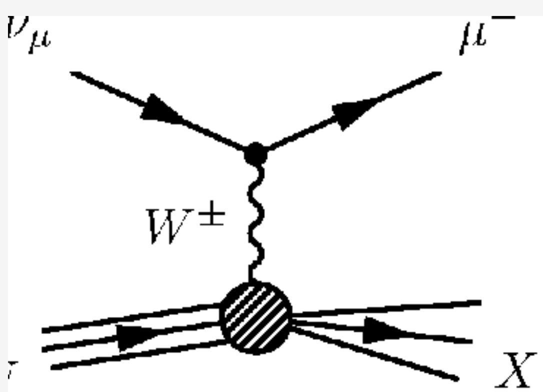
SNS Spallation Neutron Source (ORNL, USA)

Neutrinos

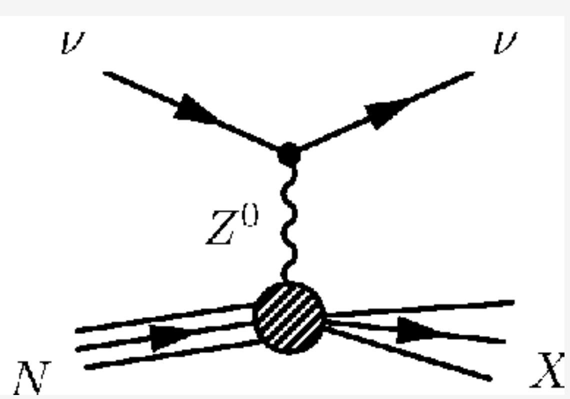
Neutrinos

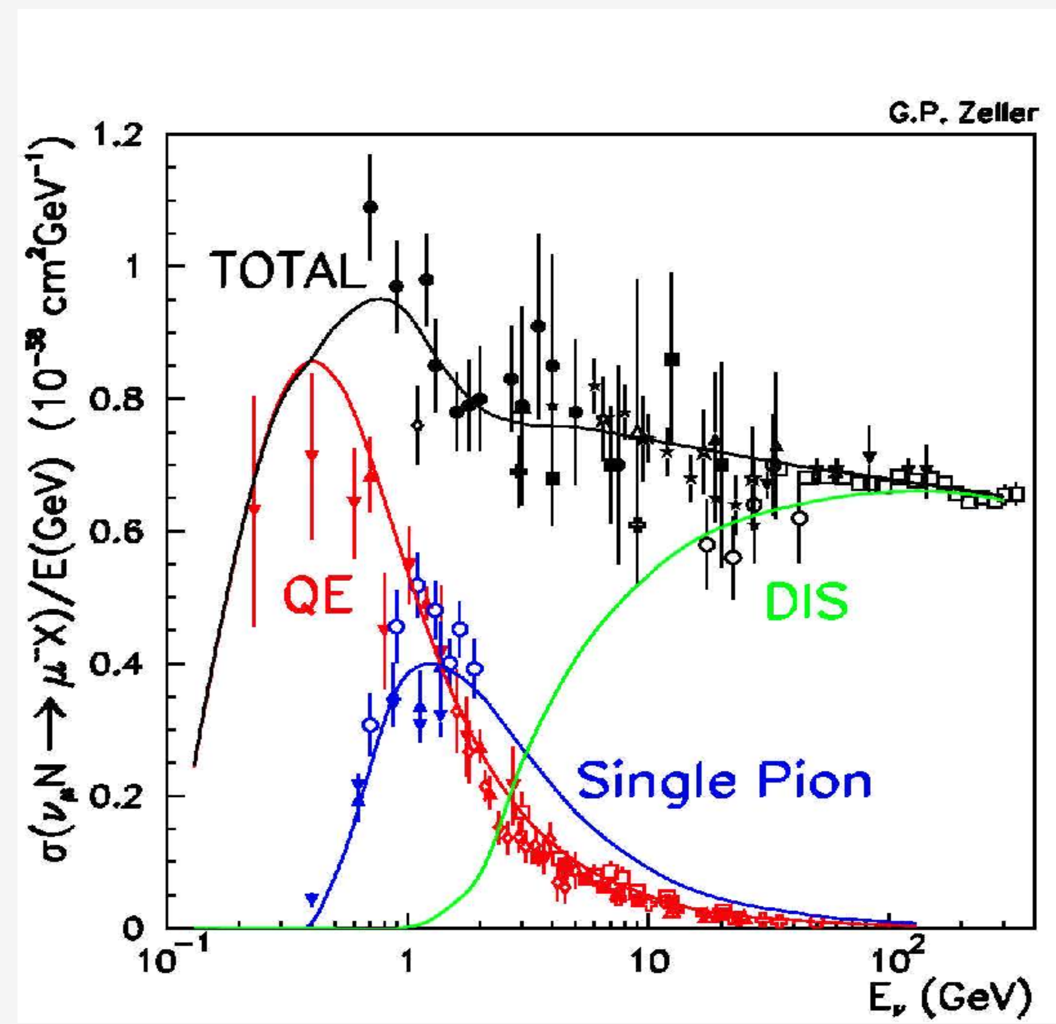
Neutrinos are (almost) massless and electrically neutral. They interact only through the weak force and therefore the interaction cross sections with other particles are extremely small. A neutrino can easily go through the planet Earth without experiencing any interaction. In materials, neutrinos can interact with atomic nuclei:

Charged Current Interaction



Neutral Current Interaction





Neutrino interactions with leptons and quarks are exactly described and calculable within the Standard Model of particle physics. In materials, quarks are bound in nucleons and nucleons are bound in nuclei: this makes the theoretical calculation of neutrino interactions an extremely hard problem. Such calculations are needed in modern neutrino oscillation experiments for a detailed understanding of the data.

Radiation-Matter Interactions: Summary

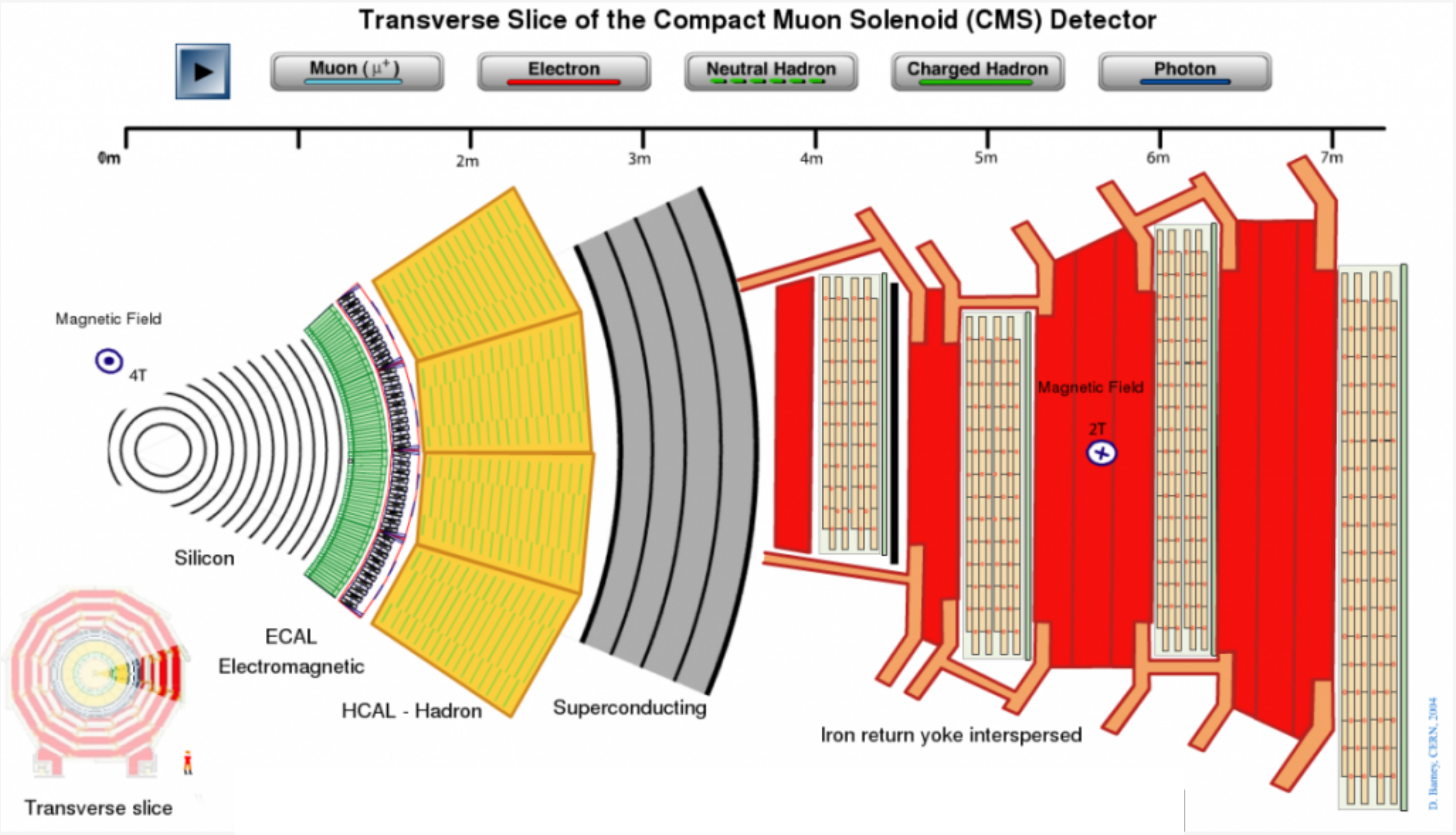
1) Charged massive particles
- Energy loss by Bethe formula
- Small range straggling
- Bragg peak

2) Electrons and Positrons
- Bremsstrahlung
- Cerenkov radiation

3) Photons
- Photoelectric effect
- Compton Scattering
- Pair Production

4) Showers
- Electromagnetic
- Hadronic

5) Neutrons and Neutrinos
- Nuclear interactions



A modern all-purpose particle physics detector (like CMS at LHC, in the above figure), measures a large number of particles including protons, photons, electrons, photons, etc... A detailed knowledge of the interaction properties of all these particles is crucial for designing detectors able to reconstruct complex events generated in the high-energy collisions realized at LHC.

Additional Slides

Fractional Kinetic Energy Loss

$$\begin{cases} MV_0 = MV + mv \\ \frac{1}{2}MV_0^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2 \end{cases}$$

Momentum Conservation [1]

Kinetic Energy (T) Conservation [2]

V_0 : Initial heavy charged particle velocity
V: Final heavy charged particle velocity
v_0 = 0 : electron at rest initially
v: final electron velocity
M: heavy charged particle mass
m: electron mass

Dividing the two conservation equations [1] and [2]:

$$\frac{V_0^2 - V^2}{V_0 - V_f} = \frac{v^2}{v} \Rightarrow V_0 + V = v \quad [3]$$

Substituting [3] back in [1]:

$$\frac{V}{V_0} = \frac{M-m}{M+m} \quad [4]$$

From [2] the difference between initial and final kinetic energies can be obtained. Dividing by the initial kinetic energy and eliminating v with the help of [1]:

$$\frac{\Delta T}{T} = \frac{\frac{1}{2}\frac{M^2}{m}(V_0 - V)^2}{\frac{1}{2}MV_0^2} = \frac{M}{m}\frac{(V_0 - V)^2}{V_0^2} = \frac{M}{m}\left(1 - \frac{2V}{V_0} + \frac{V^2}{V_0^2}\right) \quad [5]$$

Finally, substituting [4] in [5] :

$$\frac{\Delta T}{T} = \frac{4mM}{(M+m)^2} = \frac{4m}{M}\left(\frac{1}{1 + \frac{m^2}{M^2} + \frac{2}{M}}\right) \approx \frac{4m}{M} \qquad \text{if} \quad M \gg m$$

In the case of protons against electrons:

$$\frac{\Delta T}{T} \approx \frac{4 * 0.511\text{MeV}}{1000\text{MeV}} = 0.002044 \approx 0.2\%$$