Introduction to Radiochemistry

Lecture 10

Luca Doria
SFU & TRIUMF

Fall 2014

Nuclear Reactions
and
Cross Sections

Introduction

- Introduction and Nomenclature

- Types of Nuclear Reactions

- Cross Section

- Electron Scattering and Form Factors

- Summary

Reaction Types

Reactions

- Elastic
- Inelastic

Elastic

Target and Projectile remain unchanged at the end of the reaction.

Example: p-C elastic scattering:

$$^{12}\mathrm{C}(p,p)^{12}\mathrm{C}$$

Inelastic

In this case, the projectile looses energy and a change in the target is induced.

Example: p-C inelastic scattering:

$$^{12}\mathrm{C}(p,p)^{12}\mathrm{C}^*$$

Reaction Types

If we consider nuclear reactions where both projectile and target are nuclei, many subcases arise. In the following table we show them by means of examples:

Reaction	Type
^{59}Co(d,d)^{59}Co	Elastic
^{59}Co(d,d')^{59}Co*	Inelastic
^{59}Co(d,γ)^{61}Ni	Radiative Capture
^{59}Co(d,p)^{60}Co	Stripping
^{59}Co(d,n)^{60}Ni	Stripping
^{59}Co(d,^{3}He)^{58}Fe	Pickup
^{59}Co(d,α)^{57}Fe	Pickup

The projectile is captured with a gamma ray emission.

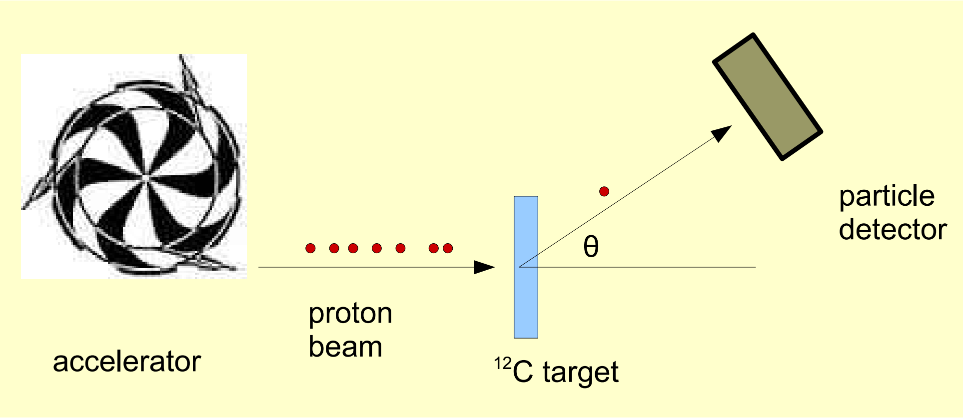
One (or more) nucleon(s) is(are) stripped from the projectile.

One (ore more) nucleon(s) is(are) stripped from the target.

Cross Section

Most of the information about the nucleus is obtained with scattering experiments.

A concrete situation is where we send a beam of particles (or nuclei) with an accelerator against a target nucleus and then measure the properties of the particles (or nuclei) emerging after the reaction.



The probability for the reaction to occur is called cross section and it is indicated with the greek letter σ .
Referring to the figure and the reaction $^{12}\mathrm{C}(p,p)^{12}\mathrm{C}$ we can ask: what is the probability for a proton to scatter against Carbon at an angle θ where our detector is located?

Cross Section

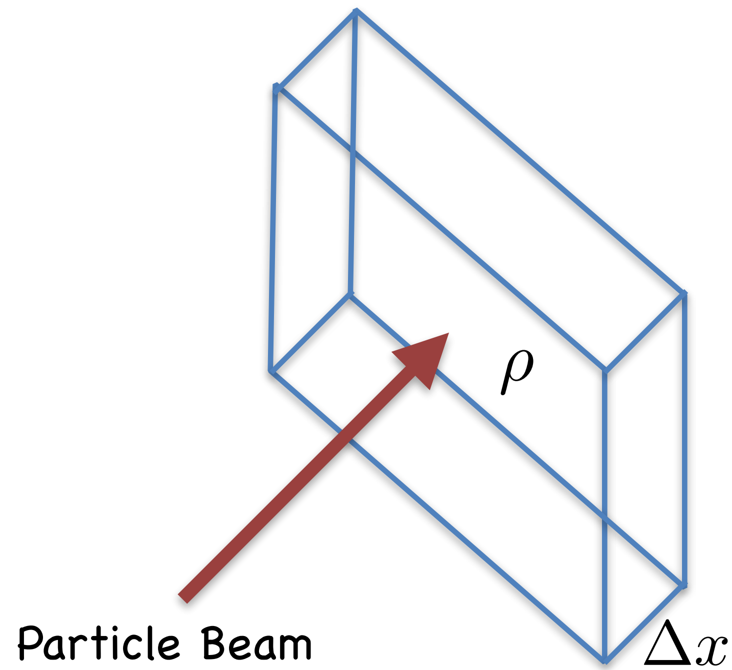
ρ : Number density of target nuclei (#/cm^3)

Δx : Thickness of the target.

-As seen by the beam, the number of target nuclei per cm^2 is: $\rho\Delta x$

-If each nucleus has a cross section σ then the total area covered by the nuclei is: $\rho\Delta x \cdot \sigma$.

- The last expression gives the fraction of area covered by nuclei per cm^2



Cross Section

If we imagine that each time a beam particle hits a nucleus a reaction occurs, w can write:

$$\frac{\text{reaction rate}}{\text{beam rate}} = \sigma \rho \Delta x$$

i.e. the ratio of reaction rate and beam rate is equal to the fraction of total area covered by the nuclei.
We can rewrite the last equation as:

$$R = I_0 \sigma \underline{\rho \Delta x}$$

Reaction Rate

Beam Intensity

Cross Section

# of nuclei per cm^2

Cross Section

$$\boxed{R = I_0 \sigma \rho \Delta x}$$

We can consider the last equation as the definition for the cross section. σ is not the geometrical cross section of the nucleus πR^2 but expresses the probability for a specific reaction to occur.

Cross section unit: the **barn (b)**.

$$\boxed{1 \text{ b} = 10^{-28} \text{ m}^2}$$

The name comes from the first experiments where a surprisingly large cross section was measured: "It is as big as a barn !".

Cross Section

Different reactions on the same nucleus have different cross sections.
Example:

$$n +^{235} U \to \text{fission} \qquad \sigma \approx 1000b$$

$$\nu +^{235} U \to^{235} Np \qquad \sigma \approx 10^{-16}b$$

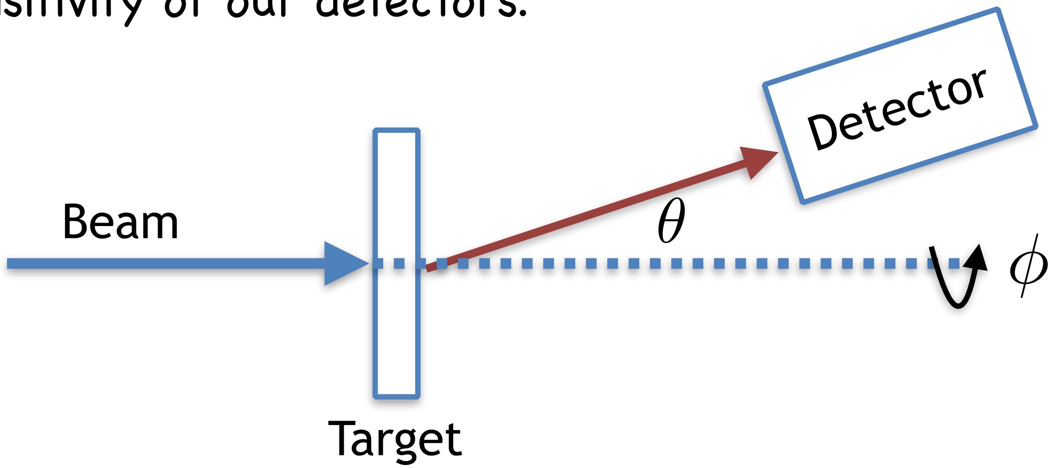
Just for comparison, let's calculate the geometric cross section of Uranium:

$$\sigma_g = \pi R^2 = \pi r_0 (A^{1/3})^2 \approx 1.7b$$

The fission cross section is much larger than the geometric area of the nucleus!
This shows how unstable ^{235}U is against (low energy)neutron scattering.

Differential Cross Section

Up to now, we discussed the **total cross section** but in reality what is really measured is almost always a **differential cross section**.
This means that usually we cannot measure all the particles emerging from a reaction at all the angles and all the energies. This is mainly due to the limited area and sensitivity of our detectors:



If the detector subtends a solid angle $\Delta\Omega$ the cross section is:

$$\boxed{R = I_0 \frac{d\sigma}{d\Omega}(\theta, \phi) \Delta\Omega \cdot \rho \Delta x}$$

..back to Rutherford's Experiment

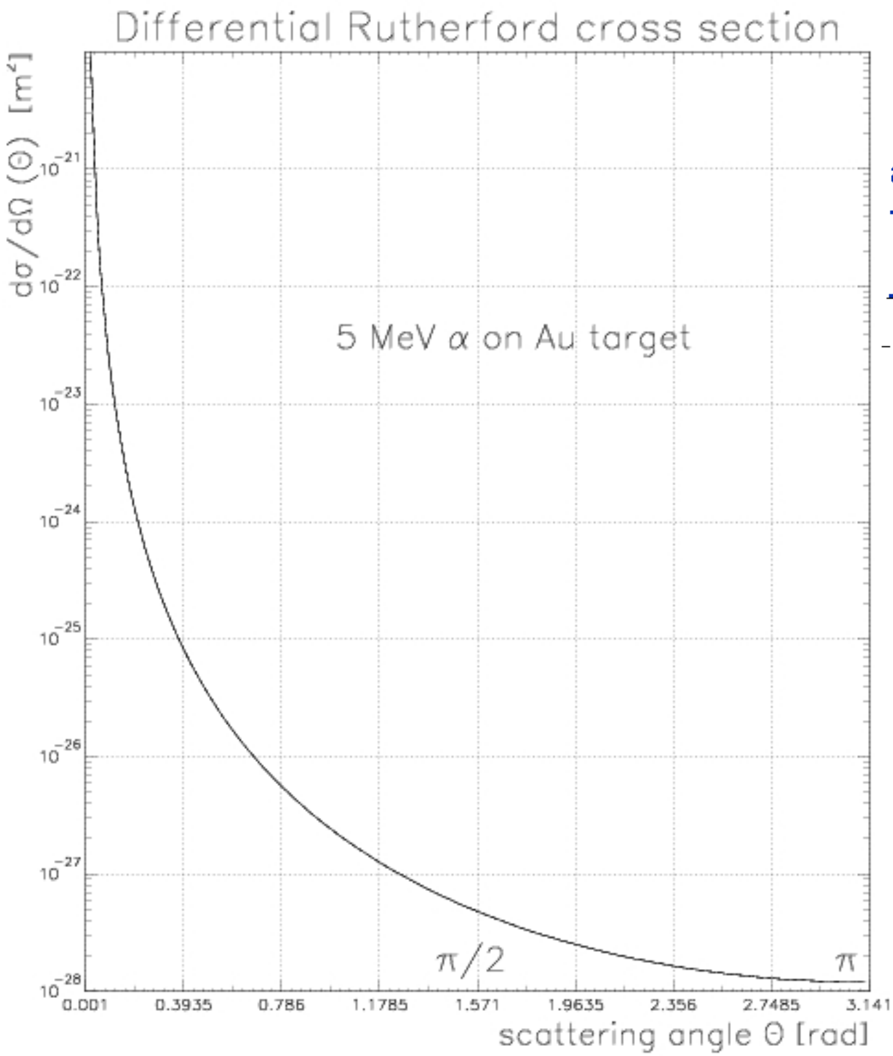
By alpha-scattering, Rutherford deduced that the atom was made by a dense object in its centre. At that time, the structure of the nucleus was not resolved. Assuming the nucleus as a point with positive charge e, its electric potential is the one of a point-like charge V~1/r .

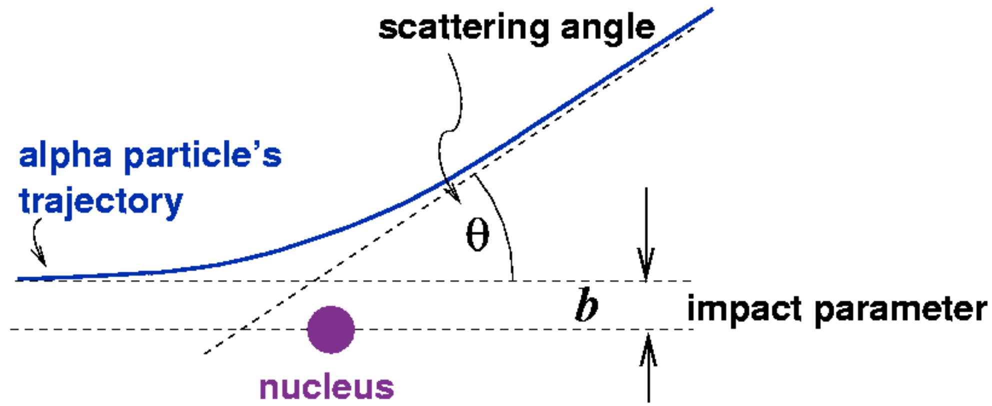
The differential cross-section for such a configuration can be calculated and the result is:

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{4p^2}\frac{1}{\sin^4(\theta/2)}}$$

This is the distribution Rutherford found as a function of θ.
Note that the differential cross section does not depend on ϕ since the problem is symmetric wrt the beam axis.

..back to Rutherford's Experiment





The cross section can be written as a function of the impact parameter. In this way, Rutherford was able to estimate the size of the nucleus.

$$\frac{1}{2}mv^2 = \frac{e^2}{4\pi\epsilon_0}\frac{1}{b} \Rightarrow$$

$$b = \frac{2e^2}{4\pi\epsilon_0}\frac{1}{mv^2}$$

Cross Section of Extended Objects

If an object is not point-like but extended (like the nucleus), the cross section becomes (in the case where the projectile has no structure, ex: an electron):

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} \cdot Z^2|F(q)|^2}$$

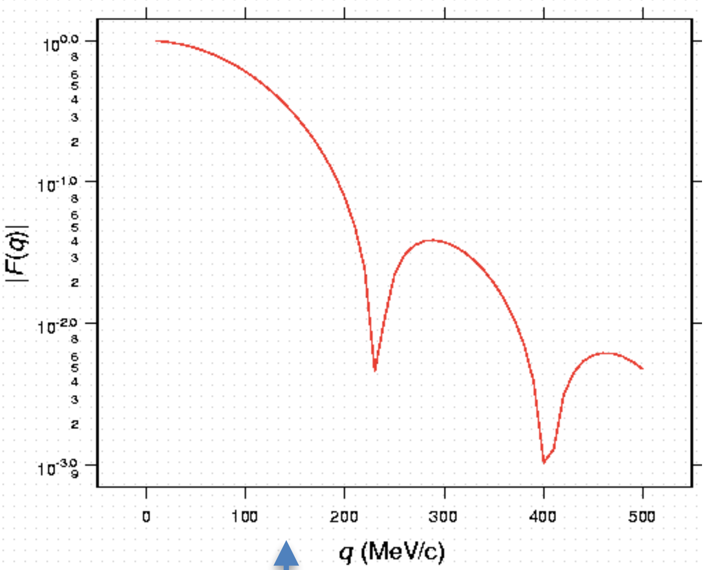
Rutherford's cross section (point-like object)

F(q) is the Form Factor: encodes the details of the inner structure as a function of the momentum q.

The form factor is the **Fourier transform of the charge distribution** of the nucleus:

$$F(q) = \int \rho(r) e^{iqr} d^3r$$

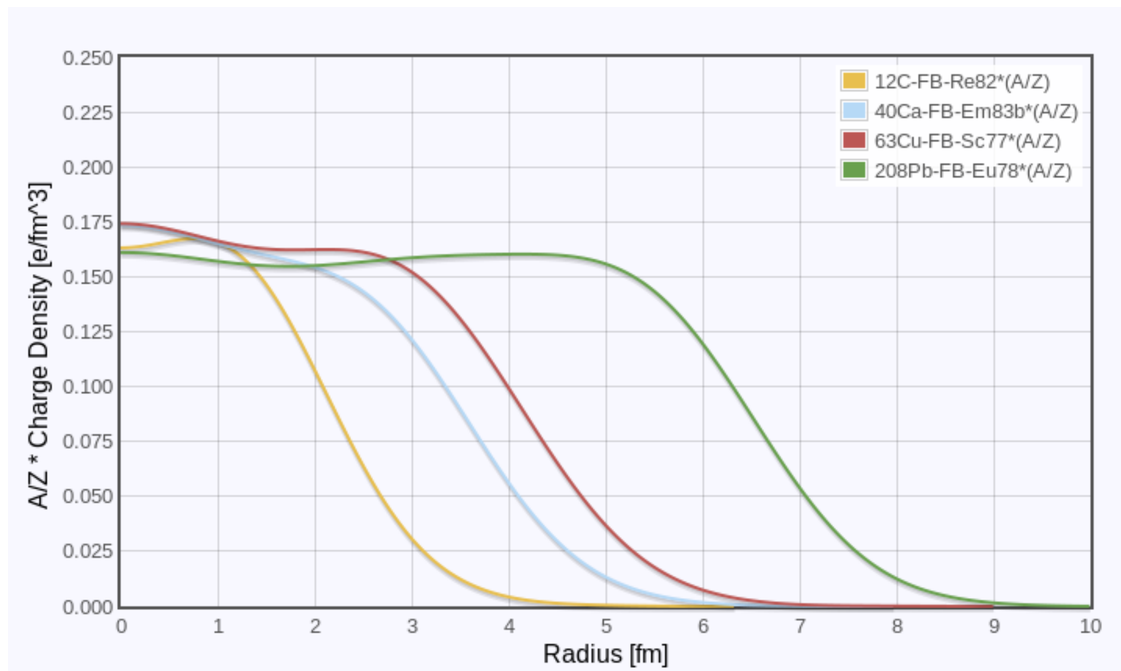
Cross Section of Extended Objects



Charge density in momentum and space coordinates are connected via the Fourier transform.
In large nuclei the charge density can have nontrivial distributions but overall the constant density approximation is good.

Measured

Calculated (FT):



Summary

- Types of reaction.

- Cross Section.

- Rutherford experiment and cross section for point-like objects.

- Extended objects and electron scattering

- Form Factors and charge distribution.