Introduction to Radiochemistry

Lecture 11

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Nuclear Reactions

Introduction

- Momentum Correction
- Coulomb Correction
- Excitation Function for:
  - Neutrons/Charged Particles
  - EM reactions
- Fission (Natural, Artificial)
- Fusion (Natural, Artificial)

- Summary

Induced Reactions

Our aim is to calculate an approximate estimate of the energy needed for initiate a nuclear reaction in the case where the projectile is absorbed by the target nucleus.

If $Q<0$, energy has to be provided for starting the reaction.

We will estimate two corrections:

1) **Momentum Correction**: needed for fulfilling the conservations laws.

2) **Coulomb Correction**: additional energy needed to overcome the electric repulsion among nuclei.

We will use the approximations:

1) Non-relativistic motion

2) Target nucleus at rest ($v=0$)

Momentum Correction

- Mass of the projectile: m
- Velocity of the projectile: v_i
- Mass of the target at rest: M
- Final mass of the compound nucleus: (M+m)

Initial kinetic energy: $E_i=(1/2)mv^2_i$
Final kinetic energy: $Ef=(1/2)(M+m)v^2_f$

From momentum conservation: $mv_i = (m+M)v_f \Rightarrow v_f = \frac{m+M}{m} v_i$

Squaring the last equation and comparing it with $v_i = \frac{2E_i}{m}$ we obtain:

$$\boxed{\frac{E_f}{E_i} = \frac{m}{m+M}}$$

which is the fraction of initial energy required to fulfill the momentum conservation law.

Momentum Correction

Now we can invert the last relation and calculate the fraction still available to the reaction using: $E_f/E_i + X = 1$.

$$X = \frac{M}{M+m} E_i$$

The fraction X must be at least equal to Q in order to just start the reaction. We can therefore write the **threshold energy** as:

$$\boxed{E_{th} = \frac{M+m}{M} |Q|}$$

This is the minimum amount of energy needed for inducing the reaction respecting the momentum conservation law (momentum correction).

Momentum Correction

Example:

$$^{14}N + \alpha \rightarrow ^{17}O + p + Q$$

with Q=-1.19 MeV.

Using the momentum correction formula:

$$E_{th} = \frac{14+4}{14} \cdot 1.19 = 1.5\mathrm{MeV}$$

So it is not sufficient to provide an energy Q: more energy is actually needed for fulfilling the momentum conservation law.
Still, we need a correction also for overcoming the Coulomb repulsion.

Coulomb Correction

The total Coulomb energy of the projectile+target system when they are in "contact" is:

$$E_C = \frac{1}{4\pi\epsilon_0} \frac{e^2 Z_1 Z_2}{(R_1 + R_2)}$$

Using the appropriate unit conversions (see notes) and $R=r_0A^{1/3}$ we obtain:

$$E_C = \frac{1.44 \cdot Z_1 Z_2}{r_0(A_1^{1/3} + A_2^{1/3})}$$

which in the case of the last reaction considered is:

$$E_C = \frac{1.44 \cdot 2 \cdot 7}{1.4 \cdot (14^{1/3} + 4^{1/3})} \approx 3.6\text{MeV}$$

Putting everything together

Now we combine the two corrections together.
First, we need at least the energy needed for overcoming the Coulomb barrier. Then, we correct for the momentum conservation.

Let's apply the corrections in the case of the alpha-Nitrogen reaction:

$$E_{th} = E_{Coulomb} \cdot \frac{M_N + M_\alpha}{M_N}$$

Using the numbers calculated before, E_{th} = 4.6 MeV.

Conclusion:

Despite Q=-1.19 MeV, we need about 4.6 MeV in order to induce the reaction.
This reaction was studied by Rutherford with alpha-emitting isotopes.
<u>Question</u>: Was indeed possible for him to reach 4.6 MeV? Why?

Excitation Function

- The nuclear cross section as a function of the energy is called **excitation function.**

- The EF. is different depending on the projectile used:

    - Neutrons

    - Heavy charged particles (protons, light ions,..)

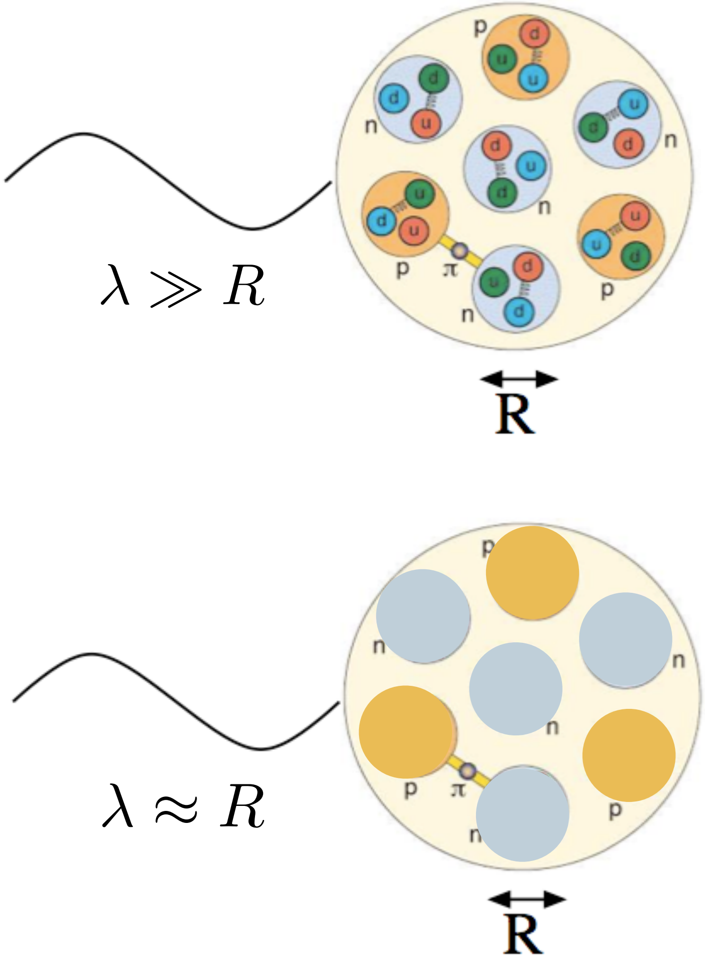
    - Electromagnetic Probes (photons, electrons, positrons)

- The energy of the probe is connected to its deBroglie wavelength: probing the nucleus at different energies gives a picture of it at different length scales:

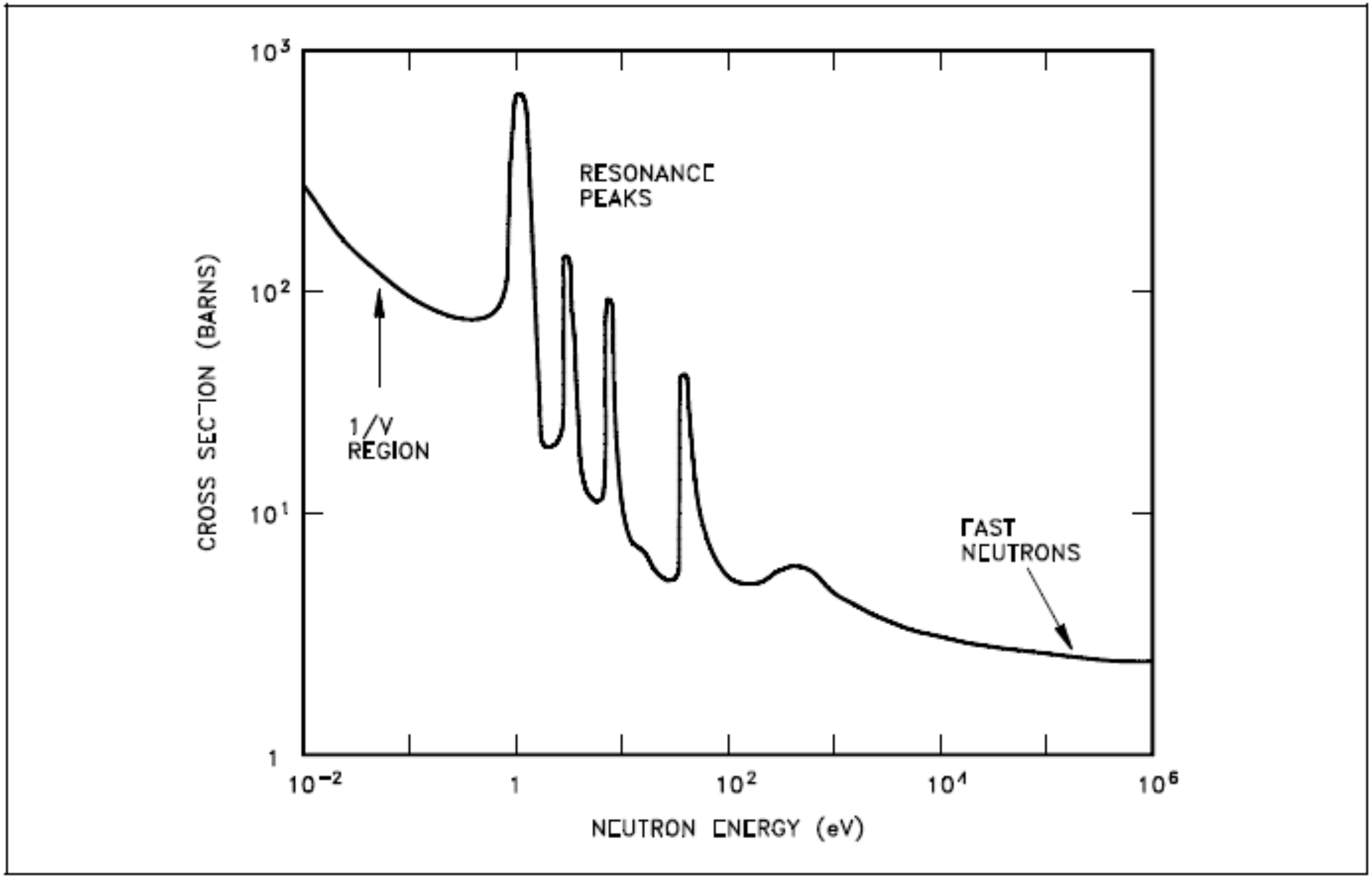
$$\lambda = \frac{h}{p}$$

Low energy probes: will "see" the nucleus as a whole
Higher energy probes: will "see" the nucleus' substructure.

Excitation Function: Resolution of the Probe



Neutrons



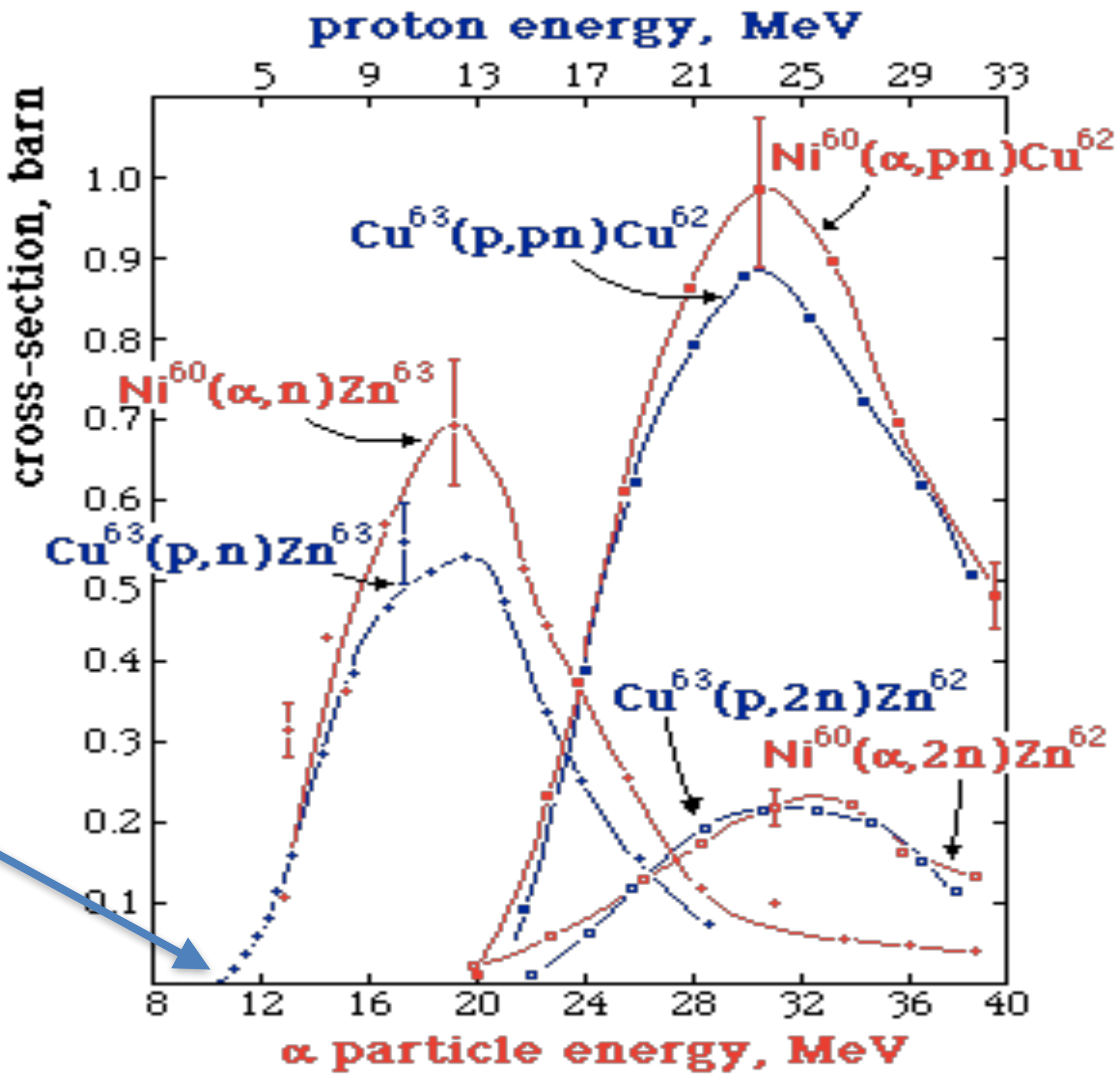
CROSS SECTION (BARNS)
10³
10²
10¹
1
10⁻²
1
10²
10⁴
10⁶
NEUTRON ENERGY (eV)
RESONANCE PEAKS
1/V REGION
FAST NEUTRONS

Protons/Charged Particles

- Much smaller CS wr to neutrons.

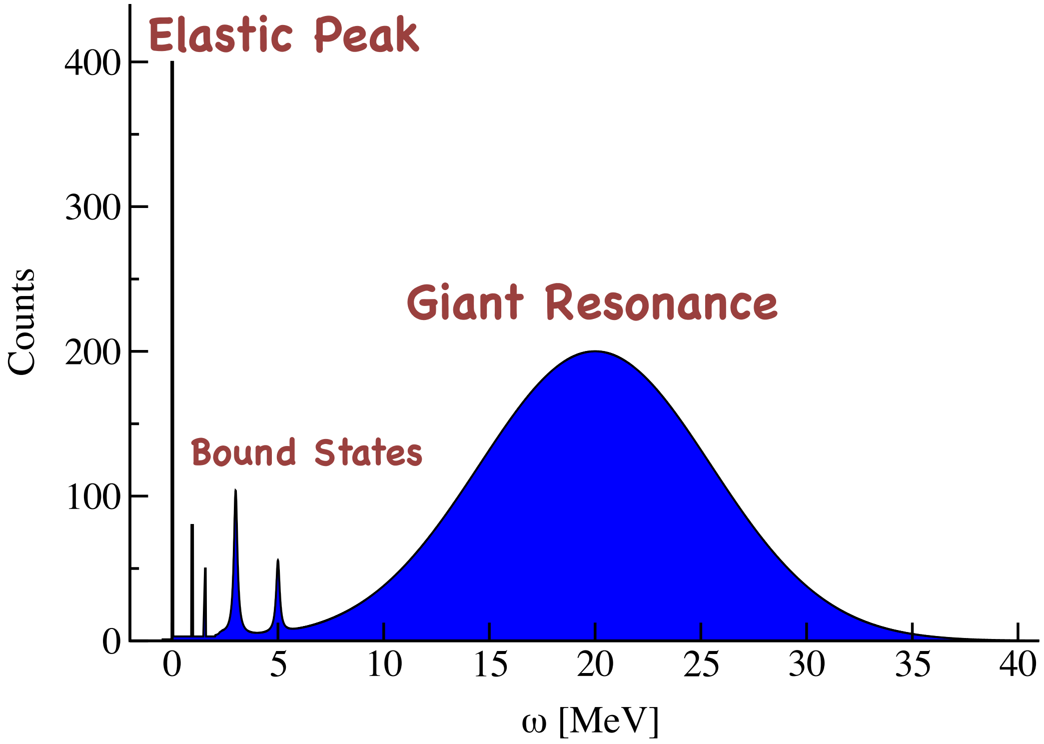
- Threshold Energy

- Different Channels for the final state (n,pn,nn,etc..).



Electromagnetic Reactions

- Reactions induced by photons/electrons/positrons. Cross sections in the mb range.
- Very interesting theoretically: EM part fully known —> study of the strong part of the transition amplitude.
- Response characterized by the Giant Resonance



For heavy nuclei (A>20):

Interpretation in terms of density fluctuation of p and n distributions.

$$E_{GDR} \approx \sqrt{\frac{4ZN}{A^2}}\frac{76.5\mathrm{MeV}}{A^{1/3}}$$

For light (A<20) nuclei:

Interpretation in terms of p-n coherent dipole motion.

$$E_{GDR} \propto \frac{1}{A^{1/6}}$$

Nuclear Reaction Mechanisms

1) Compound Nucleus Formation: This mechanism happens when the projectile is assimilated into the target nucleus. After the formation, the new nucleus might stabilize into its ground state. If enough energy is present, one or more nucleons can be emitted in the stabilization process. The emission of nucleons during the stabilization phase is called **evaporation**.

Also gamma rays might be emitted during the evaporation process.

Example:

$$^{16}O +^{16} O \rightarrow^{32} S + p + n + \text{gammas}$$

In the case of gamma emission, the term **radiative capture** is used.

Example:

$$p +^{12} C \rightarrow^{13} N + \gamma$$

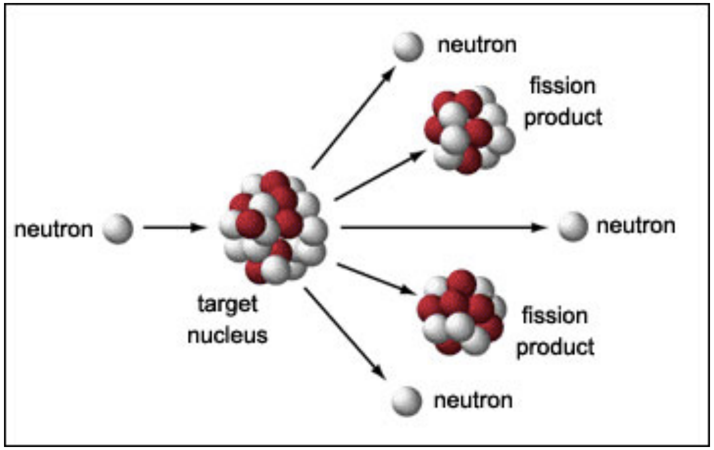
2) Direct Reactions: In direct reactions, the projectile is not assimilated in the target but it interacts at the "periphery" of it.

Neutron Induced Nuclear Fission

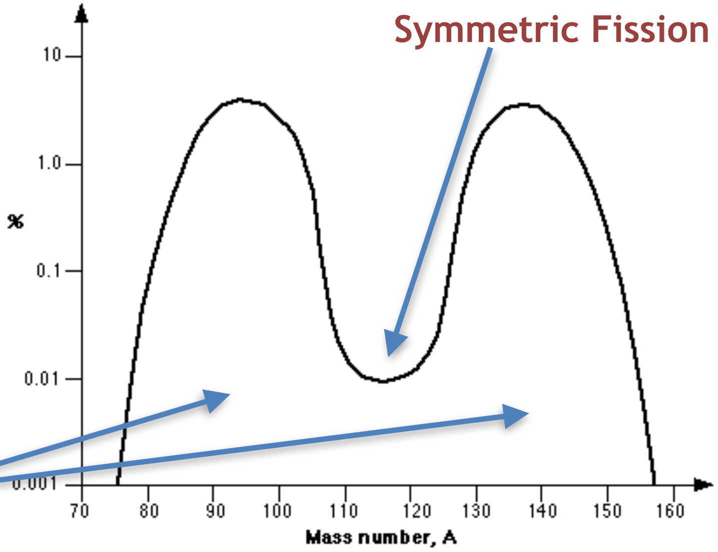
In high-mass nuclei, the addition of a neutron might be enough for tilting the balance between attractive and repulsive forces in favour of the latter. The Coulomb force is able to split the nucleus in two fragments with considerable kinetic energy.

After the fission, the fragments further decay emitting nucleons and gamma rays until a stable state is reached.

The fission does not always result in the same fission fragments (see figure).



Distribution of fission products from Uranium-235



Symmetric Fission

Asymmetric Fission

A Natural Nuclear Reactor

- Discovered in 1972.
- ^{235}U fraction lower than normal average
- Rocks were about 1.7×10^9 y old: extrapolating back in time, ^{235}U should have been about 3% in the past: concentration close to that of a nuclear reactor.
- Rocks with very high U content.
- High percentage of U fission products

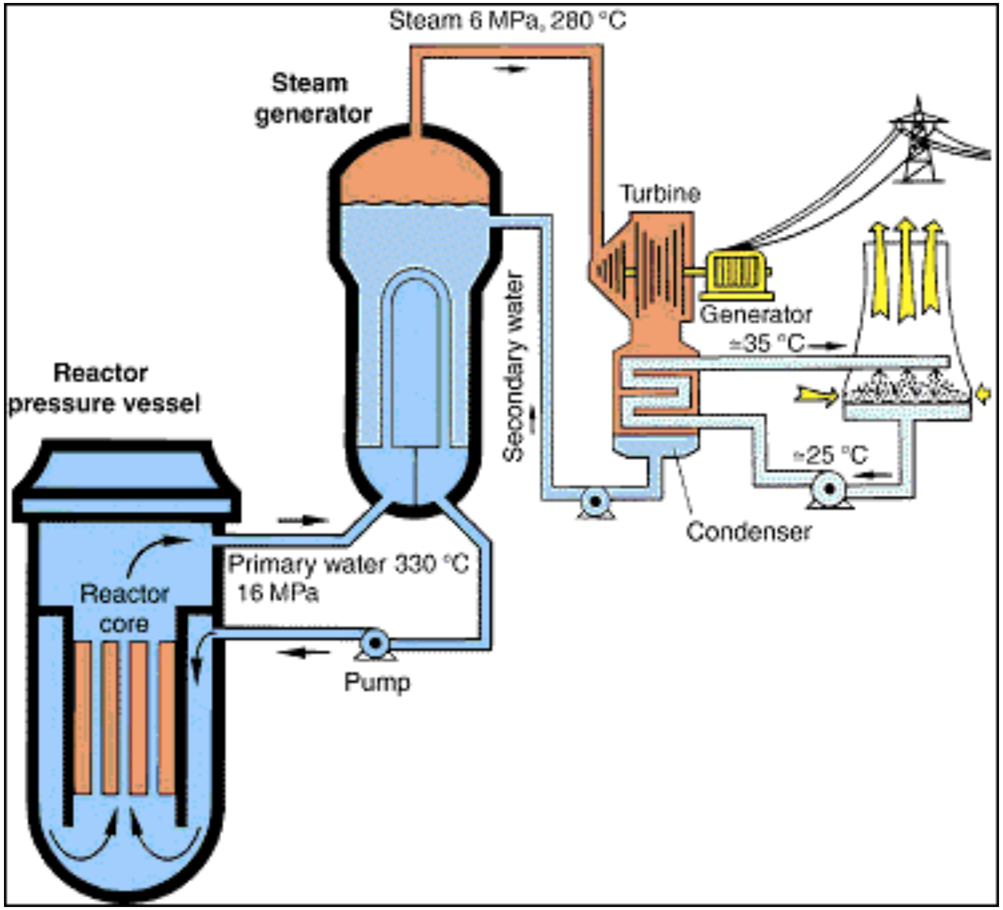
Conclusion: The Oklo mine is a

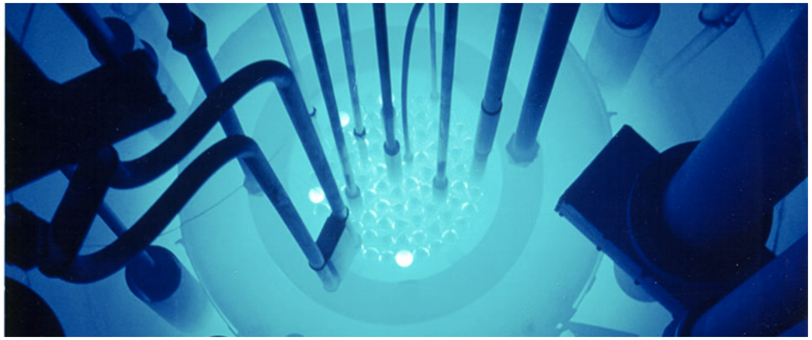
fossil nuclear reactor !

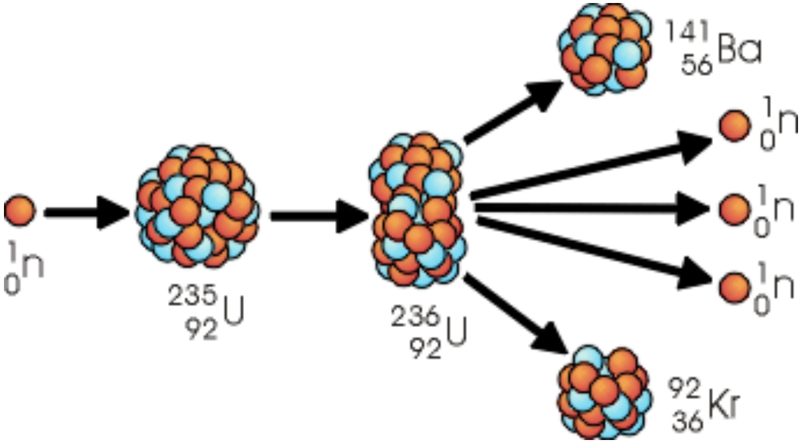
- The trigger for the fission could come from cosmic rays inducing neutron emissions.
- Clay and other high-water content rocks are prensent: they acted as neutron moderator.

- No other natural nuclear reactor is found up to now.

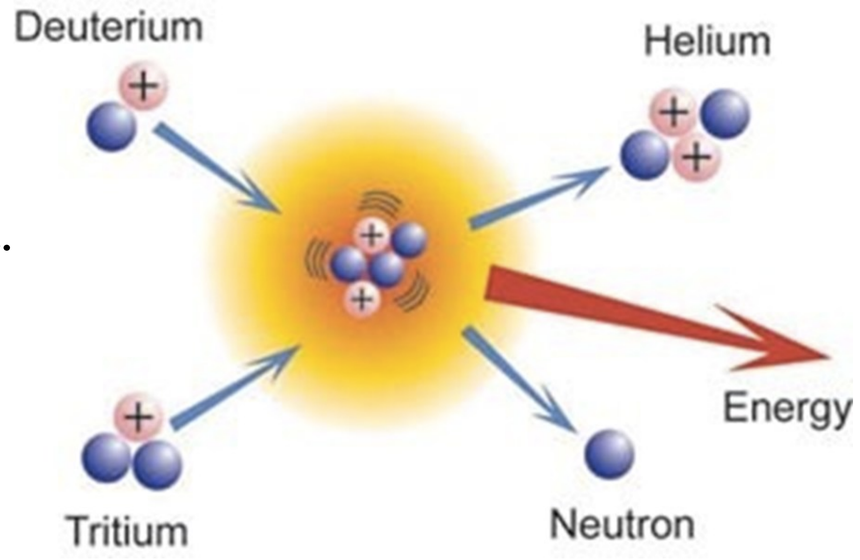
Nuclear Reactors







Nuclear Fusion



- Energetically useful reactions happen for low-Z nuclei.

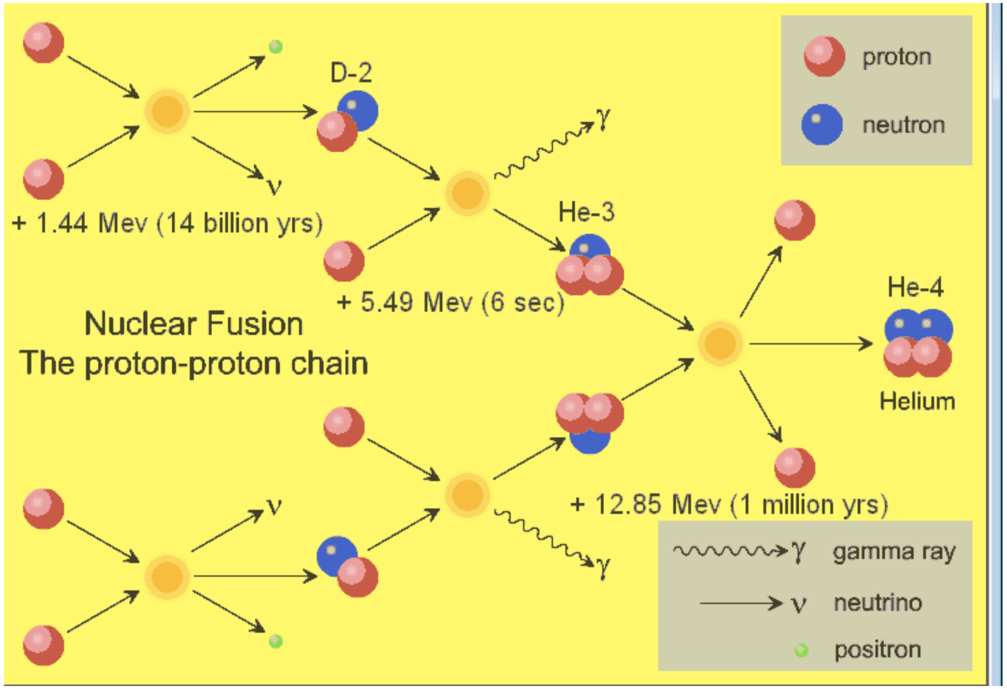
- Fusion chain reactions are not achievable. Energy is needed in order to bring the nuclei close enough to overcome the Coulomb barrier and fuse them.

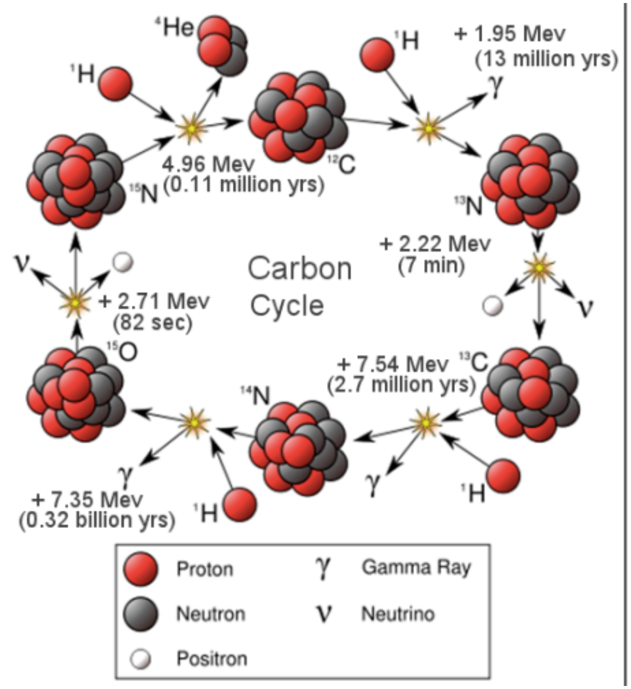
- Example (see picture):

$$^2H +^3 H \rightarrow^4 He + n + Q$$

- Topics discussed later:
  1) The previous reaction has application in the generation of neutrons by particle accelerators. Radiochemistry applications.
  2) Nuclear astrophysics reactions.
  3) Fusion for energy production.

Natural Fusion Reactors: Stars

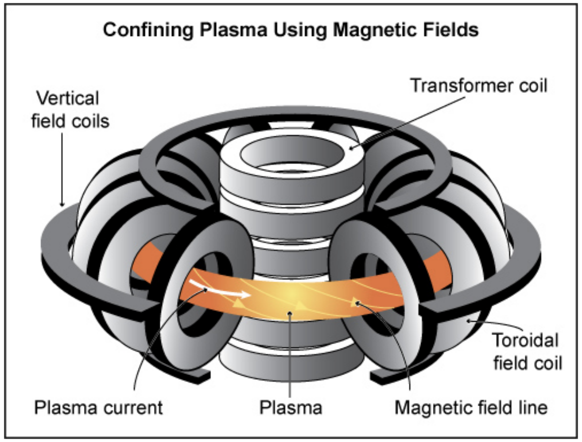


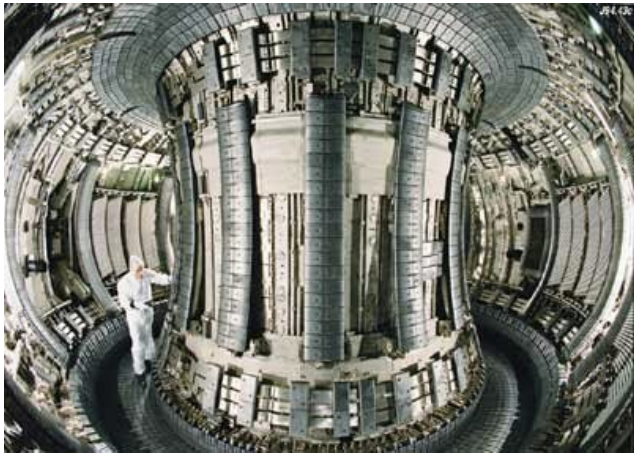


Fusion Reactors (among many kinds proposed)

TOKAMAK concept:

- Proposed in the 1950s in the URSS.
- Electromagnetic confinement of a plasma.
- Heating by radiofrequency
- Need of advanced materials for the fusion chamber
- Latest project: ITER
  - International Collaboration
  - Cadarache (France)
  - 500MW power
  - Will demonstrate the possibility to achieve a positive energy output.
  - Will operate at some point after 2020 (2027?).
  - D+T fusion with T-breeding concept.





Summary of Nuclear Reactions

1) **Elastic Scattering** (target and projectile remain the same)
2) **Inelastic Scattering** (target or projectile in excited/different state)
3) **Transfer Reaction** (1 or more nucleons exchanged: **stripping/pickup**)
4) **Fragmentation/Breakup/Knockout** (3 or more nuclei/nucleons in final state)
5) **Charge Exchange** (Z changes)
6) **Fusion** (+ Evaporation/Radiative emission)
7) **Fission**
8) **Electromagnetic Reactions**

1-5: Direct Reactions

6-7: Compound Reactions