Introduction to Radiochemistry

Lecture 3

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Notation, Constants and Useful Relations

The Nucleus

# of nucleons (mass number) = Z+N

$$^{A}_{Z}X_{N}$$

# of protons = total charge

# of neutrons = A-Z

Element's chemistry decided by Z (= # of electrons for neutral atoms)

Example:

- Oxygen: ^{16}O ; Z=8; N=8 ; A=16
- Net nuclear charge is +8
- 8 extranuclear electrons in the neutral atom of oxygen

Nomenclature (I)

Isotopes :

Same Z, but different N

Example: ^{40}Ca, ^{42}Ca, ^{44}Ca

"Isotope" is often used instead of "nuclide"

Same Z implies same chemistry

Different N implies different stability properties:

- Some of them might be radioactive
- Key to radiochemical analysis

Isobares:

Same A

Example : ^{42}Ca, ^{42}Ti, ^{42}Cr

Isotones:

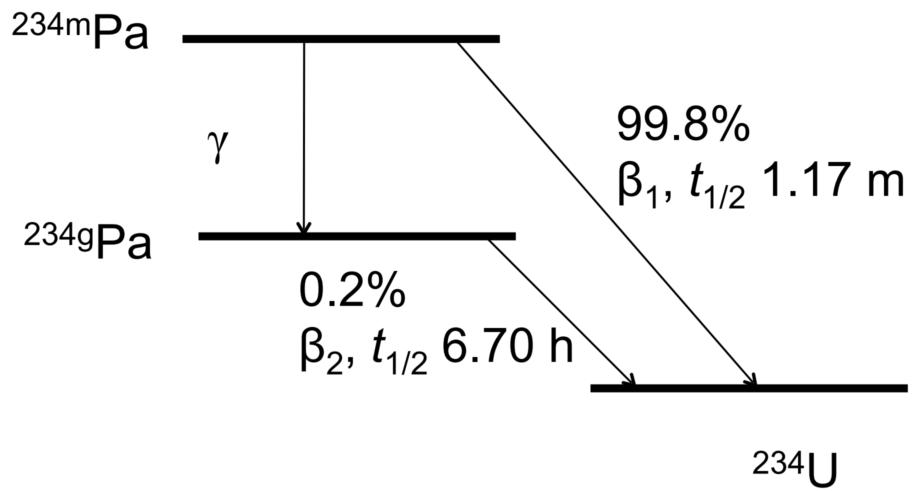
Same N

Example: ^{40}Ca, ^{42}Ti, ^{44}Cr

Nomenclature (II)

Isomers

Nuclei can be found in excited states. This can happen after a collision with another particle or after a radioactive decay. Z and A remain the same.



- $^{234m}Pa^* \rightarrow {}^{234g}Pa + \gamma$

Isodiaphors: Same neutron excess: Z-N

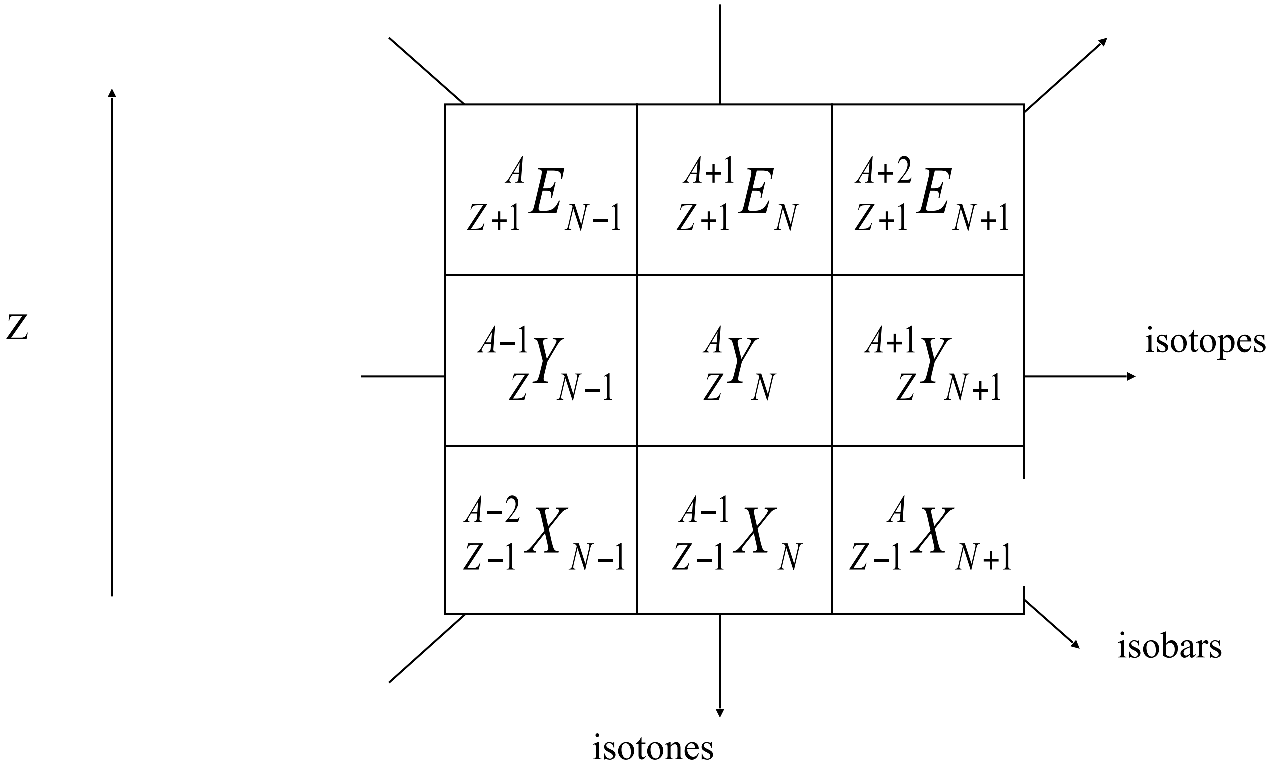
Classification of Nuclides

- Stable nuclei: 264; Example : ^{16}O
- Primary natural radionuclides: 26; very long half-lives; Example: ^{238}U with $t_{1/2} = 4.47 \times 10^9$ y
- Secondary natural radionuclides: 38;

  Example: ^{226}Ra $t_{1/2} = 1600$ y decay of ^{238}U
- Induced natural radionuclides: 10;

  cosmic rays; ^{3}H $t_{1/2} = 12.3$ y; $^{14}N(n,t)^{12}C$
- Artificial radionuclides: 2-4000, ^{60}Co, ^{137}Cs...

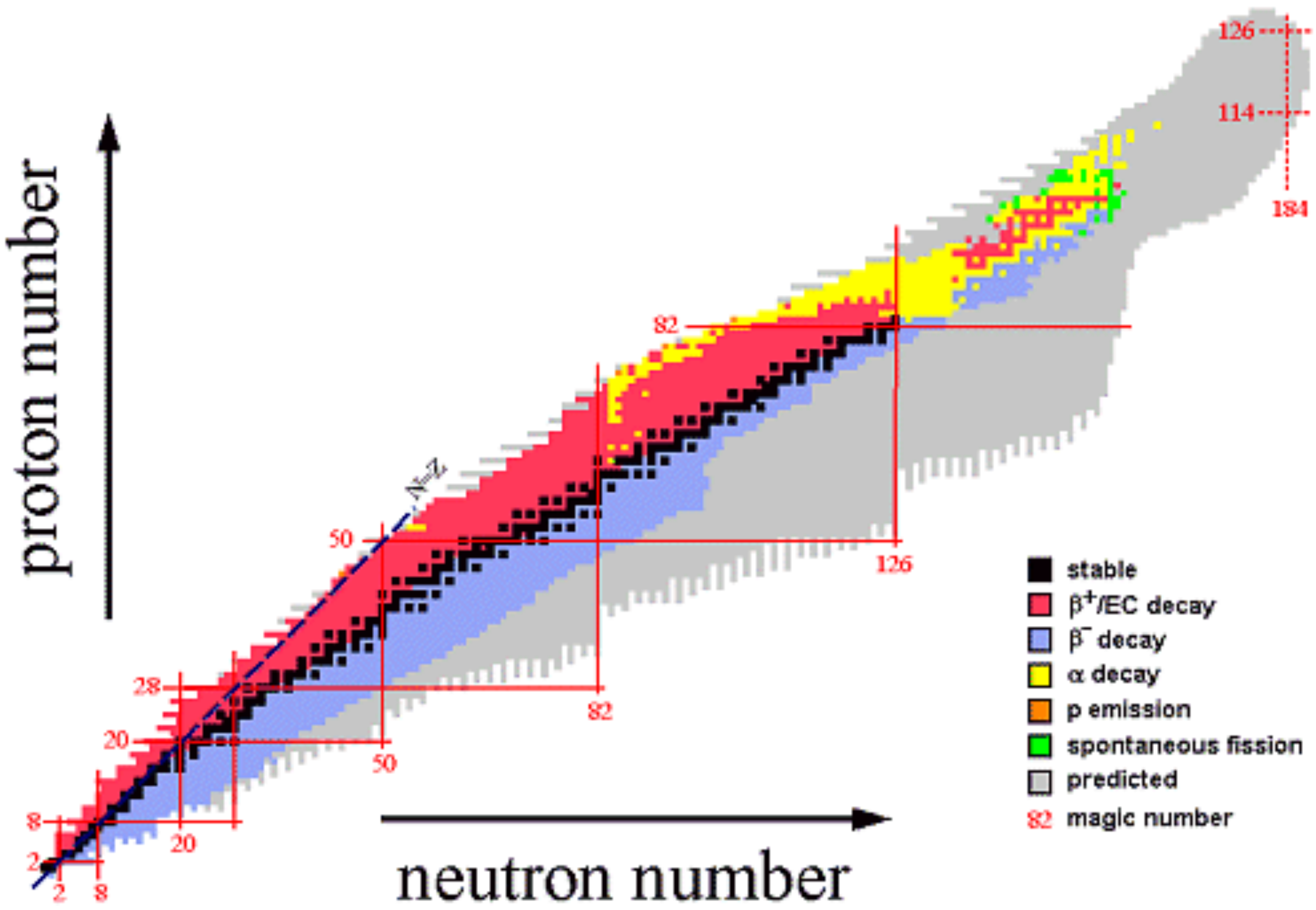
Basic "block" of a Nuclear Chart



isodiaphors: same neutron excess, N - Z

N

The Nuclear Chart



Mass units

- **Universal mass unit**: 1 u = 1.6605×10^{-27} kg; defined as 1/12 of the mass of the ^{12}C atom, which is also defined to be exactly 12 u.

- The absolute mass of a ^{12}C atom is obtained

$$12/N_A = 12/6.022\,137 \times 10^{23} = 1.992\,648 \times 10^{-23} \text{ kg}$$

This is the nucleus + 6 extranuclear electrons

- Atomic masses are expressed in units of **u** relative to the ^{12}C standard.

- 1u = 931.5 MeV

Mass of a nucleus

m_e = 0.000594u = 9.1094x10^{-31}kg
Mass of a nucleus = atomic mass - sum of electron masses

<u>Example:</u>
Mass of ^{12}C = 1.992 648 x 10^{-26} Kg - 6x 9.1094x10^{-31} kg = 1.992101x10-26 kg

In the last calculation, the binding energy of the electrons is not included since it is insignificant wrt the other numbers.

Atomic Mass (Weight) I

Consider an element with multiple isotopes:

Isotope #1: n_1 atoms
Isotope #2: n_2 atoms
...etc.

The **atomic fraction** is defined as: $x_1 = \frac{n_1}{\sum n_i}; x_2 = \frac{n_2}{\sum n_i}; \ldots$

The **isotopic ratio** is: $\frac{x_1}{x_2} = \frac{n_1}{n_2}$

The **atomic mass** (weight) of an element is:

$$M = x_1 M_1 + x_2 M_2 + \ldots = \sum x_i M_i$$

Atomic Mass (Weight) II

- $m_n = 1.6749 \times 10^{-27}$ kg = 1.008665 u
- Mass of the hydrogen atom 1.007 825 u
- The atomic mass of a nuclide should be close to the number of nucleons, i.e the mass number A.
- Cl 35.453 u; Cu 63.54 u

Example: Atomic mass of ^{nat}Cl

- ^{nat}Cl: 2 isotopes
- ^{35}Cl 75.77% 34.9689 u
- ^{37}Cl 24.23% 35.453 u
- Average atomic mass of Cl is:

  0.7577 x 34.9689 u + 0.2423 x 35.453 u = 36.9659 u

Units of Energy

- Mass and energy are interchangeable:

$$E = mc^2$$

  where energy usually expressed in MeV (*)
- 1 eV = 1.602×10^{-19} J = 1.60219×10^{-12} erg
- 1 MeV = 1.602×10^{-13} J = 1.60219×10^{-6} erg
- 1 u = 931.5 MeV/c^2

(*) 1eV is the energy of a (unit-charge) electron accelerated by a 1V potential.

Reaction's Notation

Short notation for describing a reaction (due to Walther Bothe).

Example: the reaction:

$$^{27}Al + {}^{4}He \longrightarrow {}^{30}P + n$$

is rewritten as:

$$^{27}Al({}^{4}He,n){}^{30}P$$

First element: the target
Parenthesis: (incident particle,lightest product(s))
Last element: heavier product

Binding Energy

The **binding energy E_b** of a nucleus is the amount of energy needed for removing all the nucleons from it. It is calculated as the difference between the sum of the masses of the constituent nucleons and the mass of the nucleus:

$$E_b(Z,N) = \{Zm_p + Nm_n - M(Z,N)\}c^2$$

Q-value

The **Q-value** of a nuclear reaction is the amount of energy released or absorbed by the reaction. Q is positive if the reaction produces energy and negative if requires energy to happen.

Example: $^{14}N + {}^{4}He \longrightarrow {}^{17}O + p + \mathbf{Q}$ (Q=-1.19)

DeBroglie Wavelenght and birth of Quantum Mechanics

L.DeBroglie (1924): Wave-particle duality.
Every particle (not only waves!) with momentum p has a wavelength:

$$\lambda = \frac{h}{p}$$

E. Schroedinger (1925): First equation describing the new “matter waves”

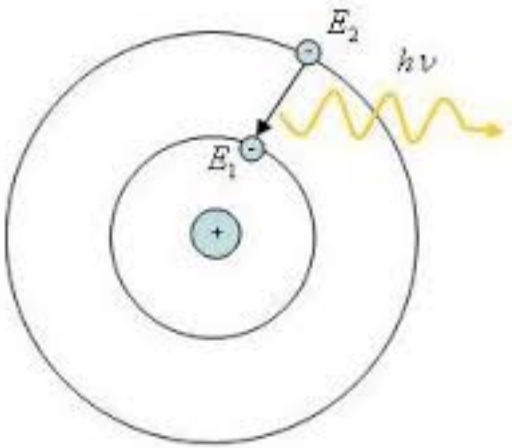
$$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$$

h is the Planck constant ($h=6.62606957\times10^{-34}$ m^2kg/s)

DeBroglie Wavelenght

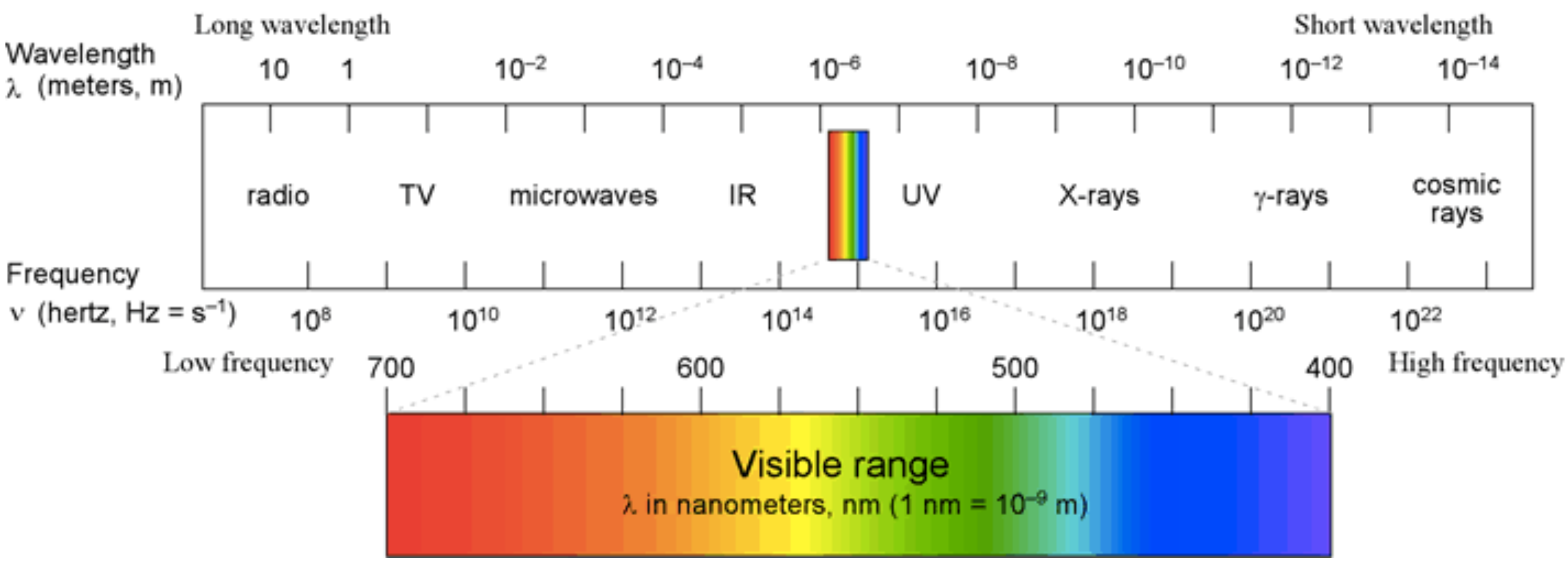
What is the wavelength associated to a level transition?

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar c}{pc} \longrightarrow \frac{2\pi\hbar c}{E}$$



- The limit is ultra-relativistic (v~c) or (equivalently) we assumed m=0.
- $\hbar = h/2\pi$ (read: h "bar")

Electromagnetic Spectrum



Electric Potential

Electrostatic Potential between two particles with a and b elementary charges separated by a distance R:

$$V = \frac{1}{4\pi\epsilon_0} \frac{(ae)(be)}{R}$$

Introducing the fine structure constant: $\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \approx \frac{1}{137}$

We can rewrite the potential (in MeV) as:

$$V = (\alpha \hbar c) \frac{ab}{R} = 1.44[\mathrm{MeV} \cdot \mathrm{fm}] \frac{ab}{R[\mathrm{fm}]}$$

Summary

- Building Blocks of Matter

- Definitions:
  - Isotopes, Isotones, Isobars
  - Mass, Energy, Units

- Organization of Nuclides in the nuclear chart

- Binding Energy and Q-values.
- DeBroglie wavelenght and other relations useful in nuclear science calculations.