Introduction to Radiochemistry

Lecture 4

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Nuclear Properties

Binding Energy

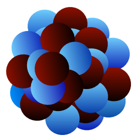
- Let us denote with $m(Z,N)$ the mass of the nucleus $^{A}_{Z}El_{N}$

- For every bound system the mass of the system is smaller than the mass of the separate constituents, if measured separately

$$m(Z,N) < Zm_p + Nm_n$$

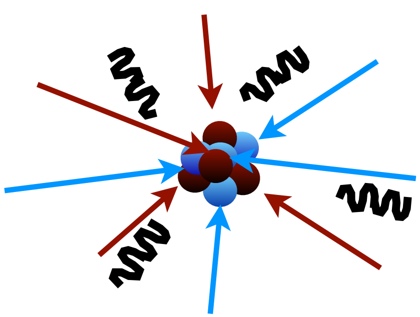
- Thus we can define a positive quantity, called binding energy, as

$$\boxed{BE(Z,N)} = Zm_pc^2 + Nm_nc^2 - m(Z,N)c^2 > 0$$



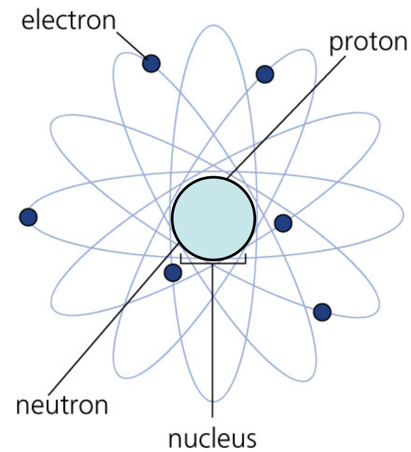
Conceptually: energy needed to separate all the nucleons in the nucleus

Another way of seeing it is:
suppose we assemble the nucleus from Z protons and N neutrons, initially at infinite separation, then the binding energy is the amount of energy given off when the nucleus is assembled.



Binding Energy

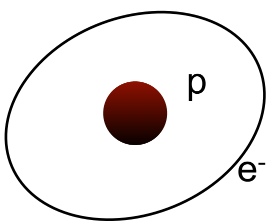
- The binding energy is a quantity that can be defined also for an atom



The binding energy of Z electrons in an atom is

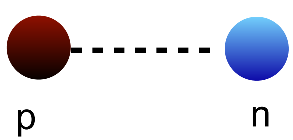
$$BE_{elec}(Z) = Zm_e c^2 + m(Z,N)c^2 - m_{atom}c^2$$

Consider the simplest atom (Hydrogen)



The atomic binding energy is 13.6 eV

Consider the simplest nucleus (deuteron)



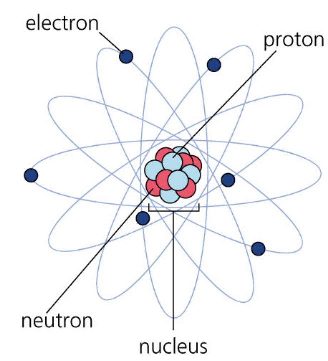
The nuclear binding energy is 2.22 MeV

- How do we know the binding energy of a nucleus? We need an operative way to define the mass of the nucleus. What exactly is $m(Z,N)$?

Binding Energy

- BE are measured from **masses of the atoms**, since they are much better determined than nuclear masses

$$BE(Z,N) = Zm_pc^2 + Nm_nc^2 - BE_{elec}(Z) + Zm_ec^2 - m_{atom}c^2$$



- Atomic masses are referred to the Hydrogen atom $m_{atom}c^2 = Zm_Hc^2$

$$m_Hc^2 = m_ec^2 + m_pc^2 - BE_{elec} \quad \times Z$$

$$\rightarrow \quad Zm_Hc^2 = Zm_ec^2 + Zm_pc^2 - ZBE_{elec}$$

$$ZBE_{elec} \simeq BE_{elec}(Z)$$

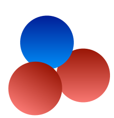
$$\boxed{BE(Z,N) = Zm_Hc^2 + Nm_nc^2 - m_{atom}c^2}$$

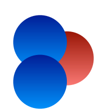
Binding Energy

Symbol	BE (MeV)	BE/A (MeV)
2	2.22	1.11
3	8.48	2.83
3	7.72	2.57
4	28.3	7.07

Why are binding energies of A=3 nuclei slightly different?

The nuclear force does not know about the difference of p and n, but the Coulomb does.

$^{3}\mathrm{He}$ 

$^{3}\mathrm{H}$ 

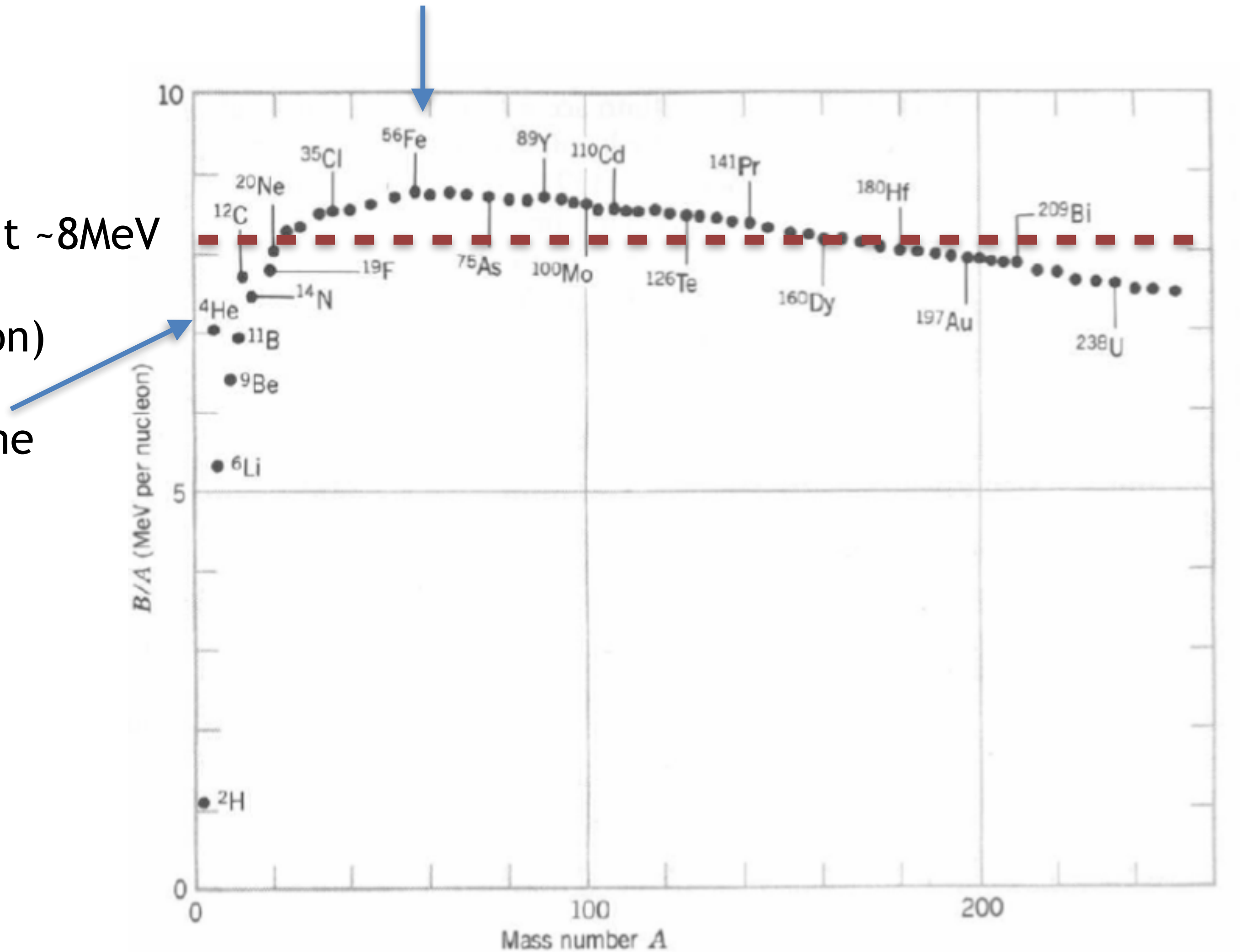
Binding Energy per Nucleon

OBSERVATIONS:

1) B/A is almost constant ~8MeV

2) Maximum at A=56 (Iron)

3) Local maxima for some light nuclei



Binding Energy per Nucleon

The fact that BE is roughly constant leads to the **saturation of nuclear force.** On the contrary, if every nucleon would interact with all the others, we would expect a behaviour of BE like $A(A-1) \sim A^2$

So the conclusion is that nucleons "feel" only the nearest neighbours.
The situation is similar to molecules bound together by the van der Waals force.

When B/A is constant, it is like the cohesive strength of a drop of liquid.

This observation lead to the **liquid drop model** (see later) for the nucleus

Maximum of the Binding Energy per Nucleon

- The maximum at A=56 is crucial for the synthesis of elements and for the nuclear power production

Fusion: $d + d \rightarrow {}^{4}\text{He}$

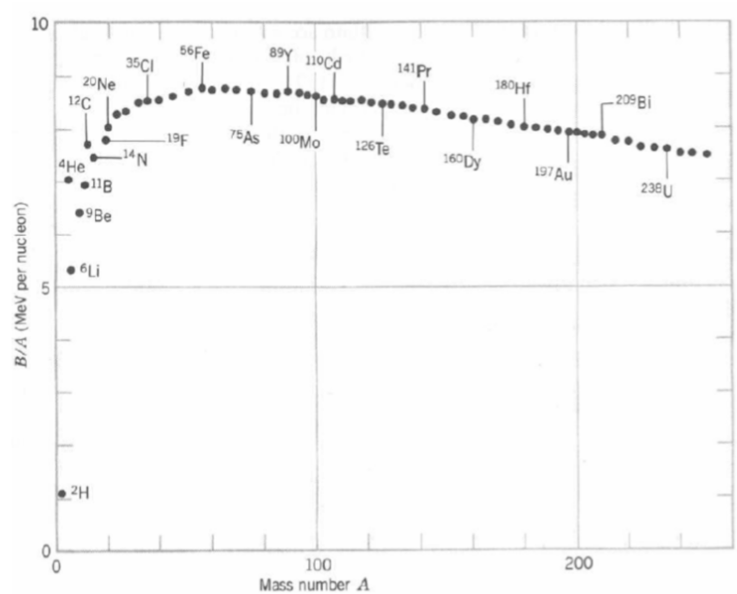
(2+2) MeV ⟶ 28 MeV Gain 24 MeV!

Released energy!

$$^{56}\text{Fe} + {}^{56}\text{Fe} \rightarrow A = 112$$

(9+9) MeV/A ⟶ 8 MeV/A Need energy!

Nuclides only up to A=56 can be formed by fusion in normal stars. Heavier elements can be formed in other contexts where extra energy is available.



Maximum of the Binding Energy per Nucleon

Fission:

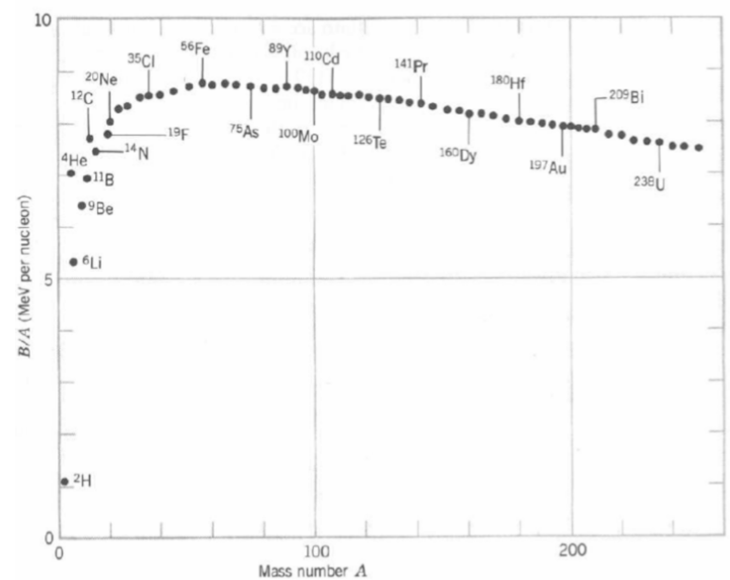
For A>56 one can gain energy from the separation of a heavy nucleus into two lighter ones

$^{235}_{92}\mathrm{U} \longrightarrow$ separates into two approximately equal parts

7.5 MeV/A $\rightarrow$ ~2*8.3 MeV/A ~ 16.6 MeV/N

Released energy!

This is the basic idea of every nuclear reactor operation as well as of nuclear bombs.



- Nuclei like $^{4}\mathrm{He}, ^{12}\mathrm{C}, ^{16}\mathrm{O}, ^{20}\mathrm{Ne}, ^{24}\mathrm{Mg}$ are well bound systems with Z=N=magic number, explained by shell model $\rightarrow$ in a few Lectures

Summarizing on BE/A:

- The nuclear binding energy per nucleon BE/A has important features which point to properties of the nuclear force and nuclear structure:

  1) BE/A is roughly constant —> saturation.

     Liquid-drop like model.

  2) The maximum at A=56 divides the curve in two regions: it the lower one, fusion releases energy, while in the upper one, fission releases energy.

     In the stars, nuclei are fused starting from hydrogen and heavy nuclei up to A=56 are created. Where are the heavier nuclei coming from? **Nuclear Astrophysics.**

  3) There are local maxima which show particularly bound nuclei. Similar properties in atoms.

     Does this point towards a **shell structure**?

Summarizing on the Chart of Nuclides:

- For the light elements: N=Z
- With increasing Z for achieving nuclear stability, the N/Z ratio increases from 1 to ~1.5 (at Bi).
- Pairing of nucleons is not a sufficient criterion, but a certain N/Z must also exist.
- At high-Z, a new mode of decay appears (α-decay) in addition to β-decay.
- Nuclei far from the valley of stability (see later):
  - high N/Z (neutron-rich): β-decay for lowering N
  - low N/Z (proton-rich): β^+-decay for lowering Z

For better understanding all the collected facts, we need a more quantitative description of the nucleus, ie we need:

Nuclear Models

Collective Models

- Try to describe the nucleus as a whole
- Identify collective variables.

Microscopic Models

- Try to describe the nucleus using variables relative to the single nucleons.

The Liquid Drop Model

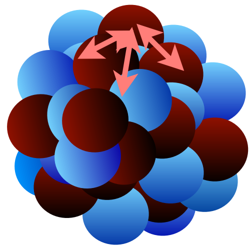
- Tries to construct a formula for the binding energy as a function of A and Z.
- The first idea is to assume a linear dependence from A, such that B/A = constant.
- Add correction terms inspired from the liquid drop idea and phenomenology.
- First proposed by von Weizsaecker.

Step 1: the BE is proportional to A. The prop. constant is the "volume energy", since it is proportional to the size of the nucleus.

$$BE(Z,N) = a_1 A \qquad \left(V = \frac{4\pi}{3}R^3 \simeq \frac{4\pi}{3}r_0^3 A \right)$$

Step 2: Viewing the nucleus as a liquid drop, the nuclei at the "surface" of it will be less bound, so we need a correction proportional to the surface:

$$BE(Z,N) = a_1 A - a_2 A^{2/3}$$



The constant a_2 is the "surface energy".

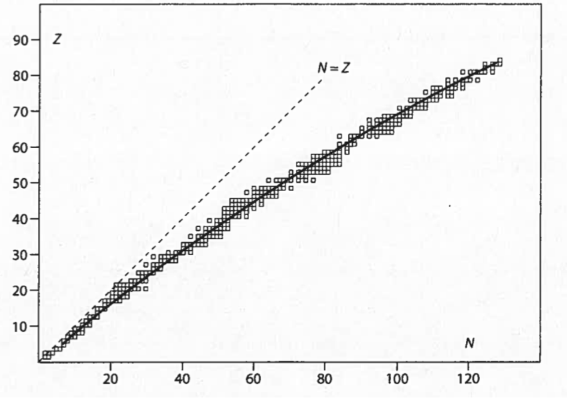
The Liquid Drop Model

Step 3: Coulomb energy: Nuclei with high Z tend to be less bound because of the Coulomb repulsion. We can add a correction term inspired by the Coulomb potential formula. It will be proportional to Z^2 and inversely proportional to the radius. Since the volume goes like A, the radius will go like $A^{1/3}$:

$$BE(Z,N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}}$$

Step 4: Nuclei with Z=N are more bound. Too high or too low Z/N ratios are disfavoured especially for light nuclei. We can envision a $(Z-N)^2$ correction. We want to allow more neutrons for heavier nuclei. All in all we can add:

$$BE(Z,N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A}$$



The last tem is called asymmetry energy.

The Liquid Drop Model

Step 5: Pairing energy. Nuclei are more stable when they have an even number of protons and and even number of neutrons. Nuclei with odd-Z/even-N and even-Z/odd-N are more stable than odd-Z/odd-N ones.
In nature, there are 167 stable even/even nuclei and only 4 with odd/odd configuration. These considerations lead to the inclusion of the last term:

$$BE(Z,N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A} - a_5 \Delta$$

with:

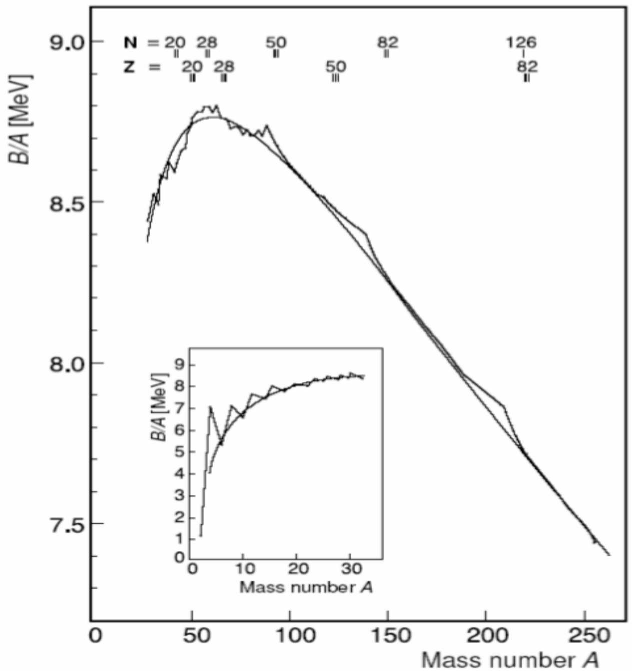
	$+\delta$	even-Z/even-N
$\Delta =$	0	odd-Z/even-N (or viceversa)
	$-\delta$	odd-Z/odd-N

The Liquid Drop Model

$$BE(Z,N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A} - a_5 \Delta$$

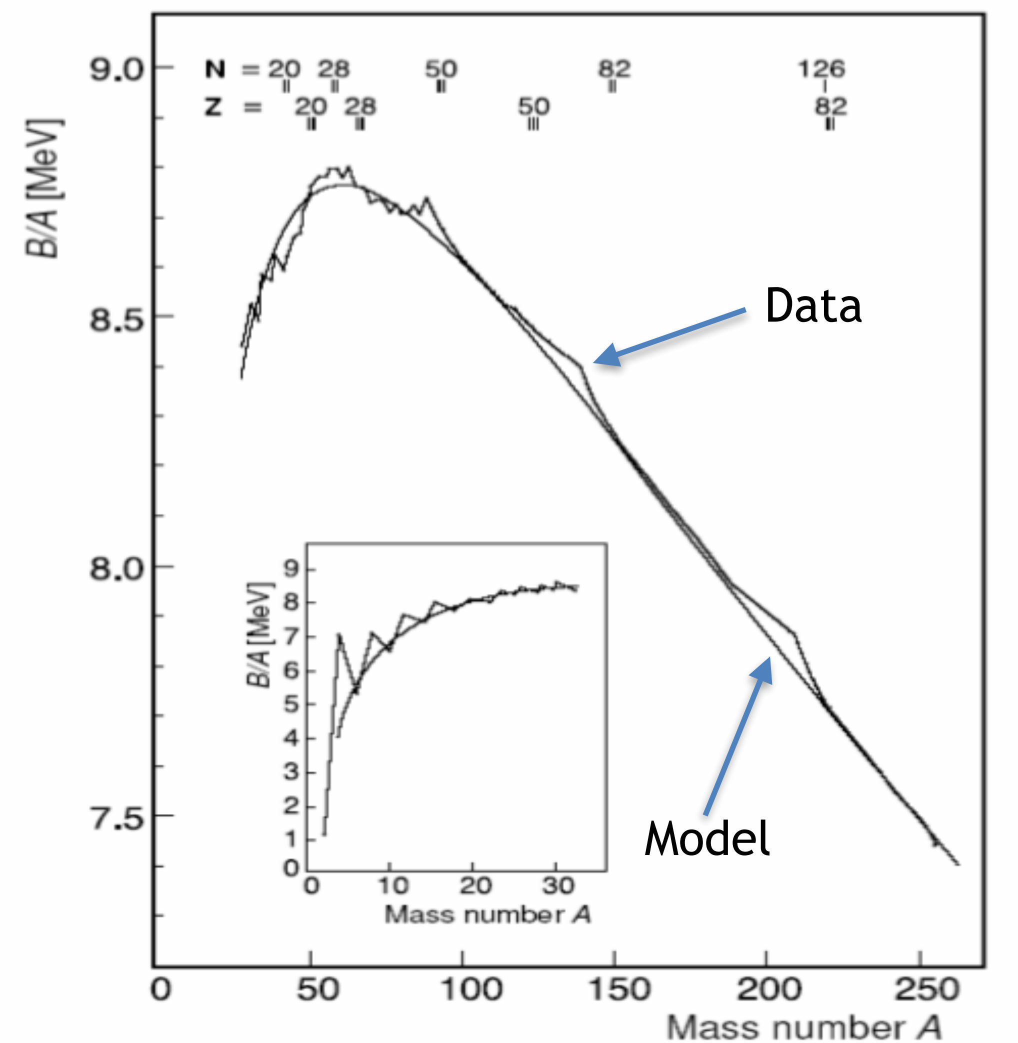
A fit to a set of nuclides data gives:

a1 = 15.67 MeV
a2 = 17.23 MeV
a3 = 0.714 MeV
a4 = 23.29 MeV
δ = 25/A MeV



Fits might yield slightly different results depending on the dataset. The pairing parameter is the most difficult to determine.

The Liquid Drop Model



- The formula reproduces the overall trend
- There is a relative large deviation from the data in the light nuclei region
- The large binding of some light nuclei will be explained by the shell model

Residuals of the Liquid Drop Model

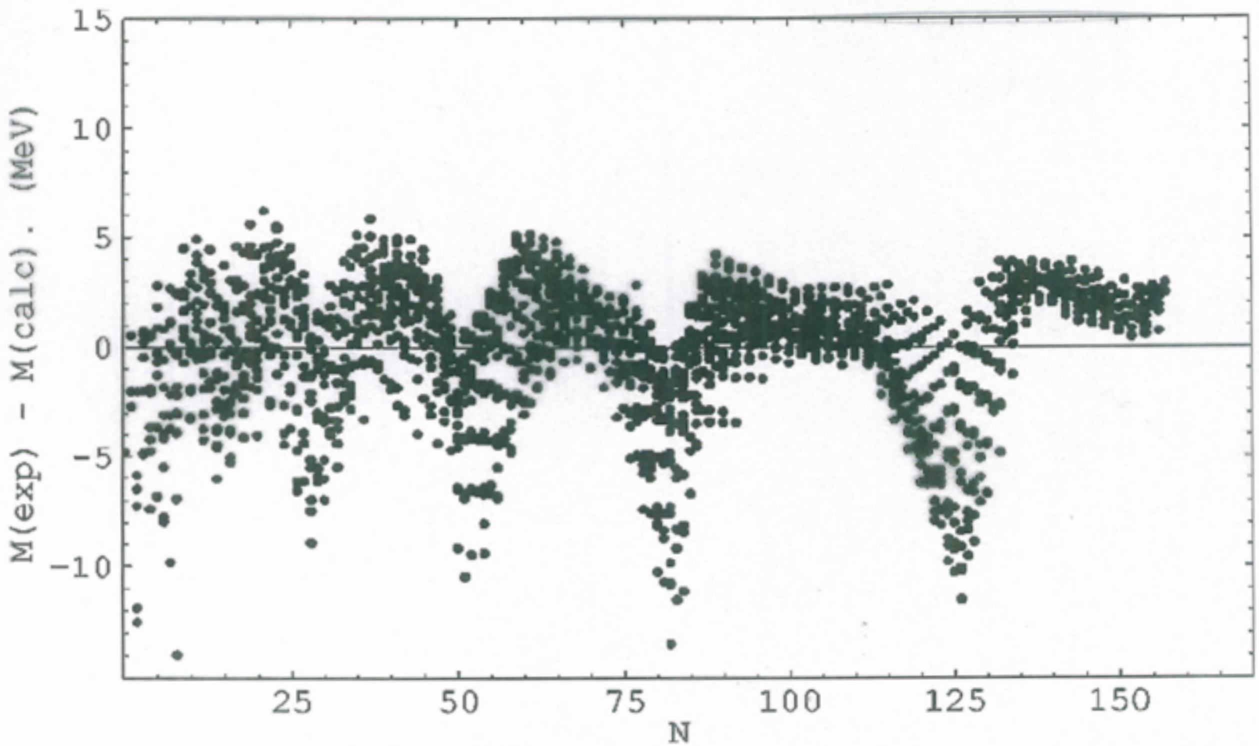
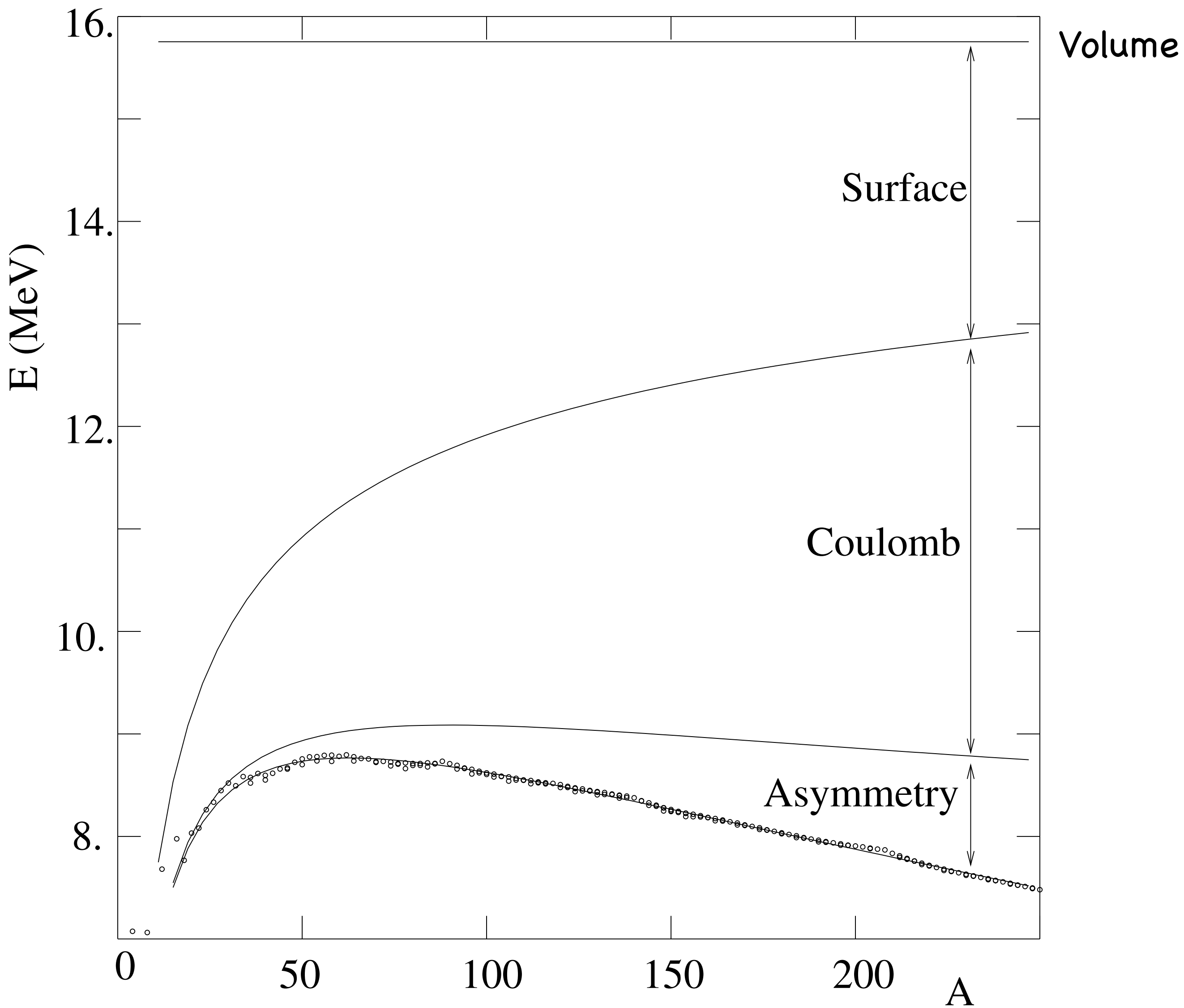


FIG. 8. Deviations from experiment of the von Weizsäcker mass formula (9), shown as a function of neutron number N.

Evidence of MAGIC NUMBERS ?

Liquid Drop Model: Separate Contributions



The Semi-Empirical Mass Formula

- Now substituting the formula for the liquid-drop binding energy

$$BE(Z,N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A} - a_5 \Delta$$

the definition of binding energy:

$$BE(Z,N) = Z m_p c^2 + N m_n c^2 - m(Z,N)c^2 > 0$$

we can get the formula for the mass of the nucleus, using N=A-Z

$$\boxed{m(Z,A)c^2 = Z m_p c^2 + (A-Z) m_n c^2 - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} + a_5 \Delta}$$

This is known as von Weiszaecker semi-empirical mass formula

There exists much more sophisticated mass formulas that include shell effects

The Valley of Stability

Considering the mass formula we just obtained, we want to calculate what is the Z of the most stable nucleus for a given nuclear mass A.
In other words, we want to calculate the Z of the most stable isobar.
The condition is:

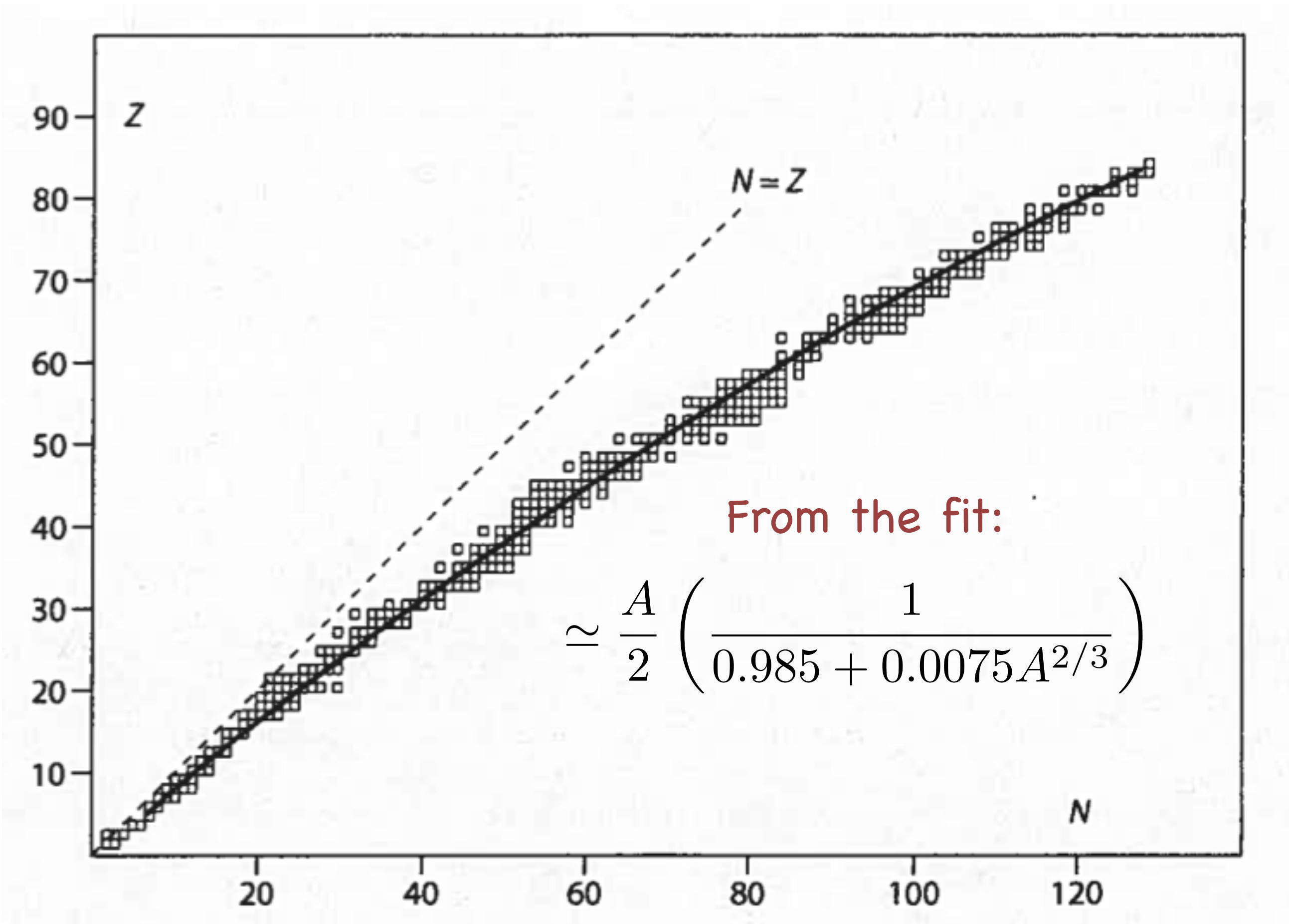
$$\left.\frac{\partial m(Z,A)}{\partial Z}\right|_{A=const} = 0$$

The calculation gives:

$$Z = \frac{A}{2} \frac{(m_n - m_p)c^2 + 4a_4}{a_3 A^{2/3} + 4a_4}$$

- **The formula gives the location of the valley of stability given A.**
- **For small A, Z=A/2 and therefore, Z=N.**
- **In general, the minimum is at Z<A/2 —> N grows faster with A.**

The Valley of Stability



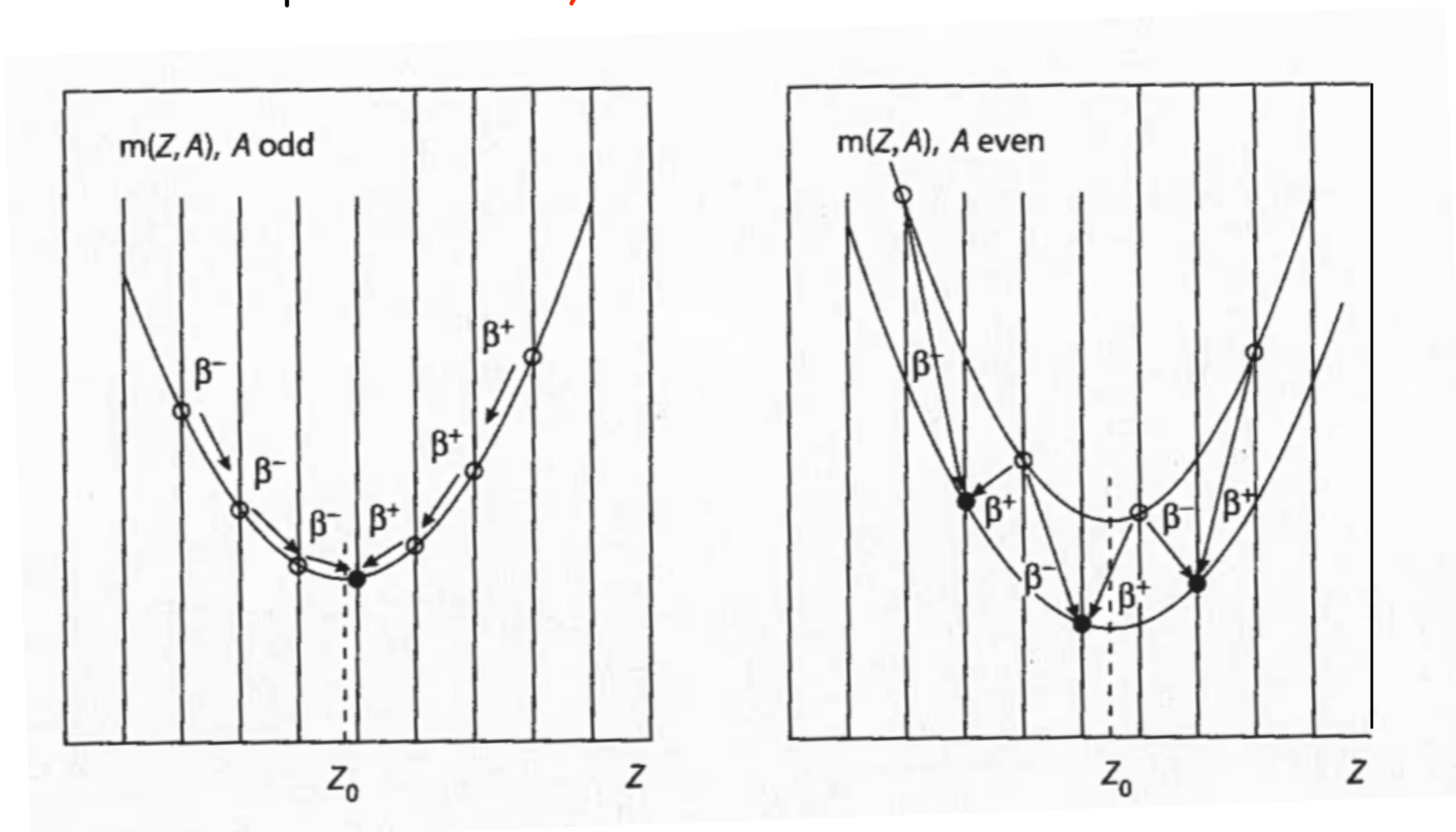
From the fit:

$$\simeq \frac{A}{2}\left(\frac{1}{0.985 + 0.0075A^{2/3}}\right)$$

Chart of Stable Nuclei

The Valley of Stability

- For A=odd all the data points fall on one parabola
- For A=even, the data points fall on two parabolas, with the points alternating between the upper and lower parabola. Why? The PAIRING TERM!



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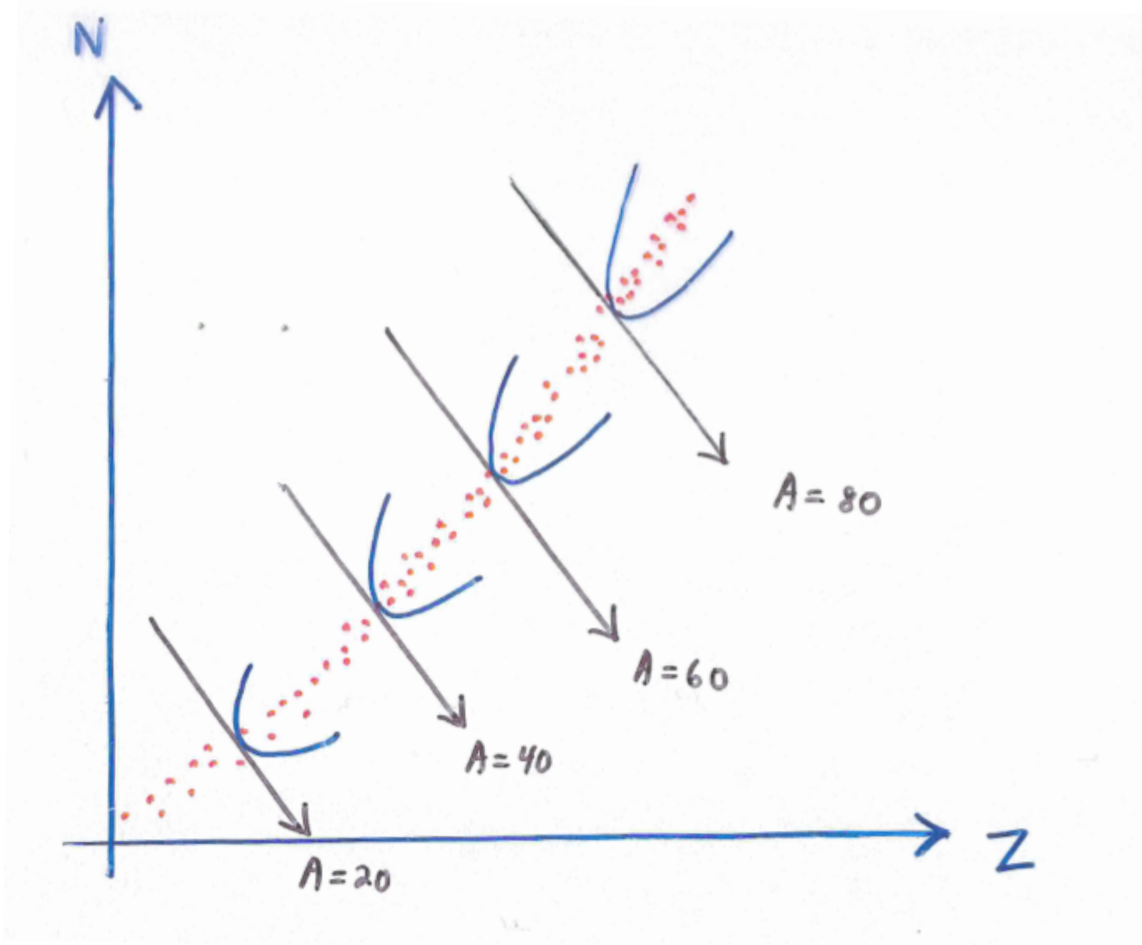
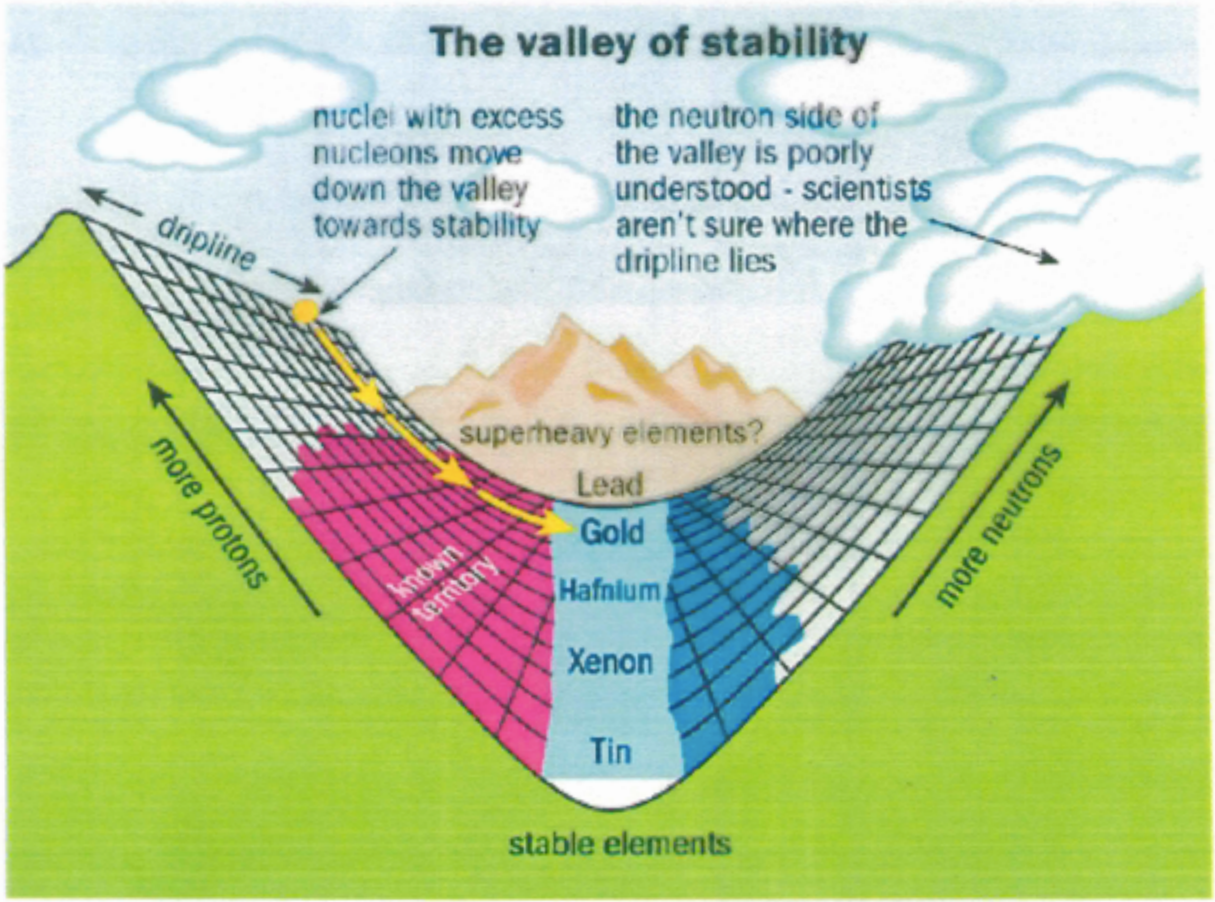
The Valley of Stability

If we plot a chart in 3D with Z,N and BE as coordinates, what we obtain is a kind of valley or canyon where the stable nuclei are at the bottom and the unstable ones on the sides.



The valley of stability

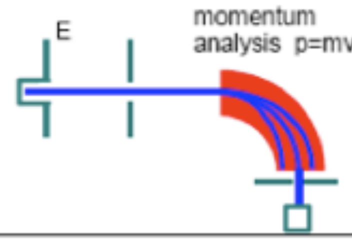


The "valley of stability" - new nuclear machines such as the Rare Isotope Accelerator will open up studies of nuclear phenomena using beams of short-lived isotopes, which form the high "walls" of the valley.

How to measure the Mass of a Nucleus?

Conventional mass spectrometry

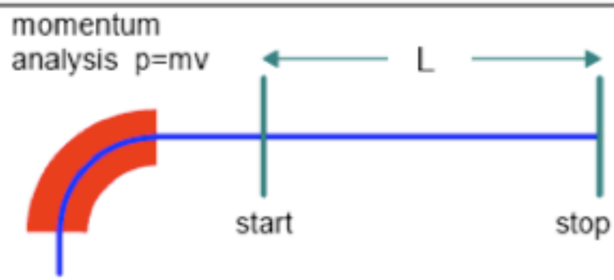
CERN-PS + ISOLDE
Chalk-River
St. Petersburg



Time-of-flight spectrometry

single turn: SPEG, TOFI

multi turn: cyclotrons CSS2, SARA,
storage ring ESR

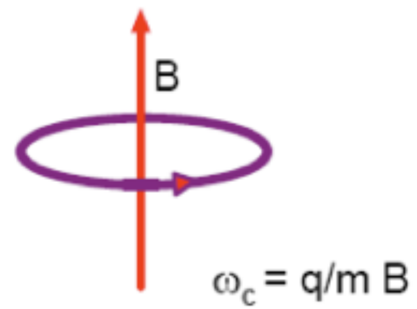


Frequency measurements

storage ring ESR/GSI,

RF transmission spectrometer MISTRAL/ISOLDE

Penning traps **ISOLTRAP/ISOLDE**
CPT/ANL
SHIPTRAP/GSI
JYFLTRAP/JYFL
LEBIT/NSCL
TITAN/ISAC



Summary

- Binding Energy
- Features of BE as a function of A
- Nuclear Models
- A collective model: The Weiszaecker Mass Formula
    - Explains a lot of nuclear features
    - Fairly simple
    - Inspired by a "liquid" analog system
- Valley of Stability: simple prediction for stable nuclei
- How to measure a mass (—> Binding energy)