Introduction to Radiochemistry

Lecture 8

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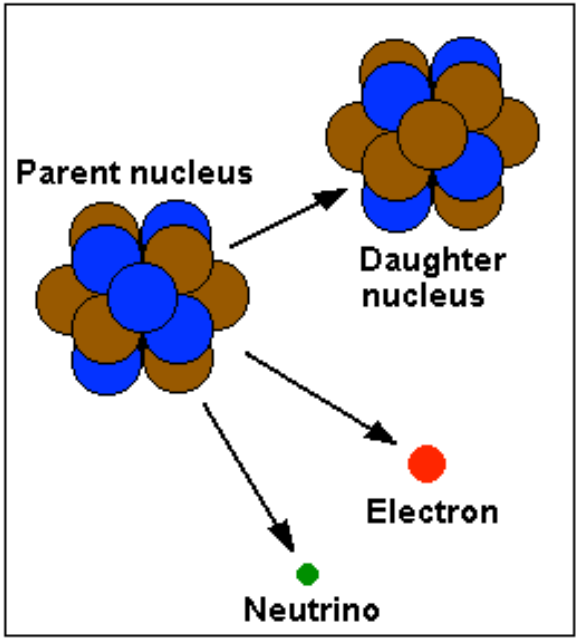
Radioactive Decays

Introduction

- General Properties
- Alpha Decay
- Beta Decay
- Gamma Decay



Beta decay

Chart of Nuclides / Decays

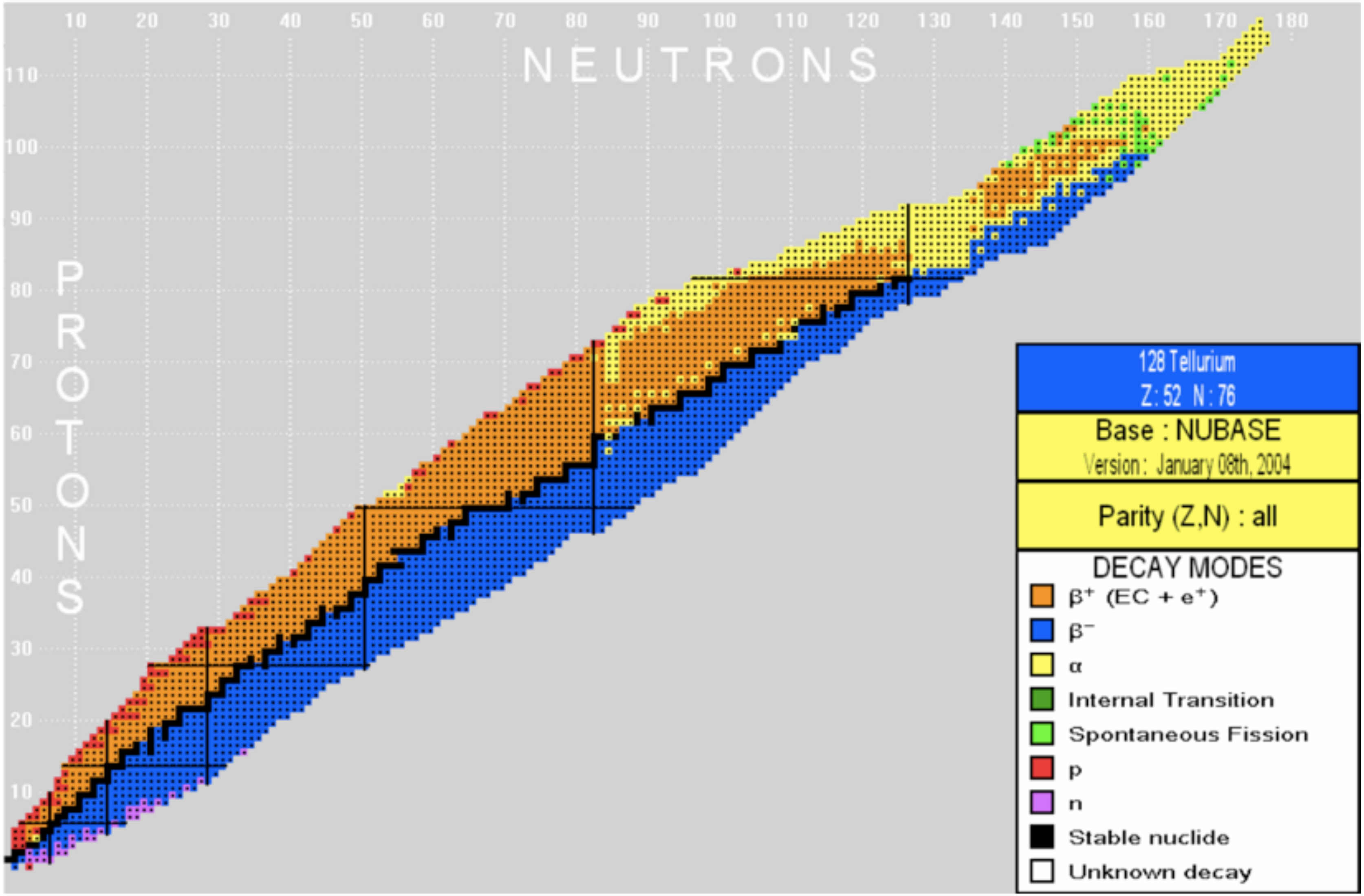
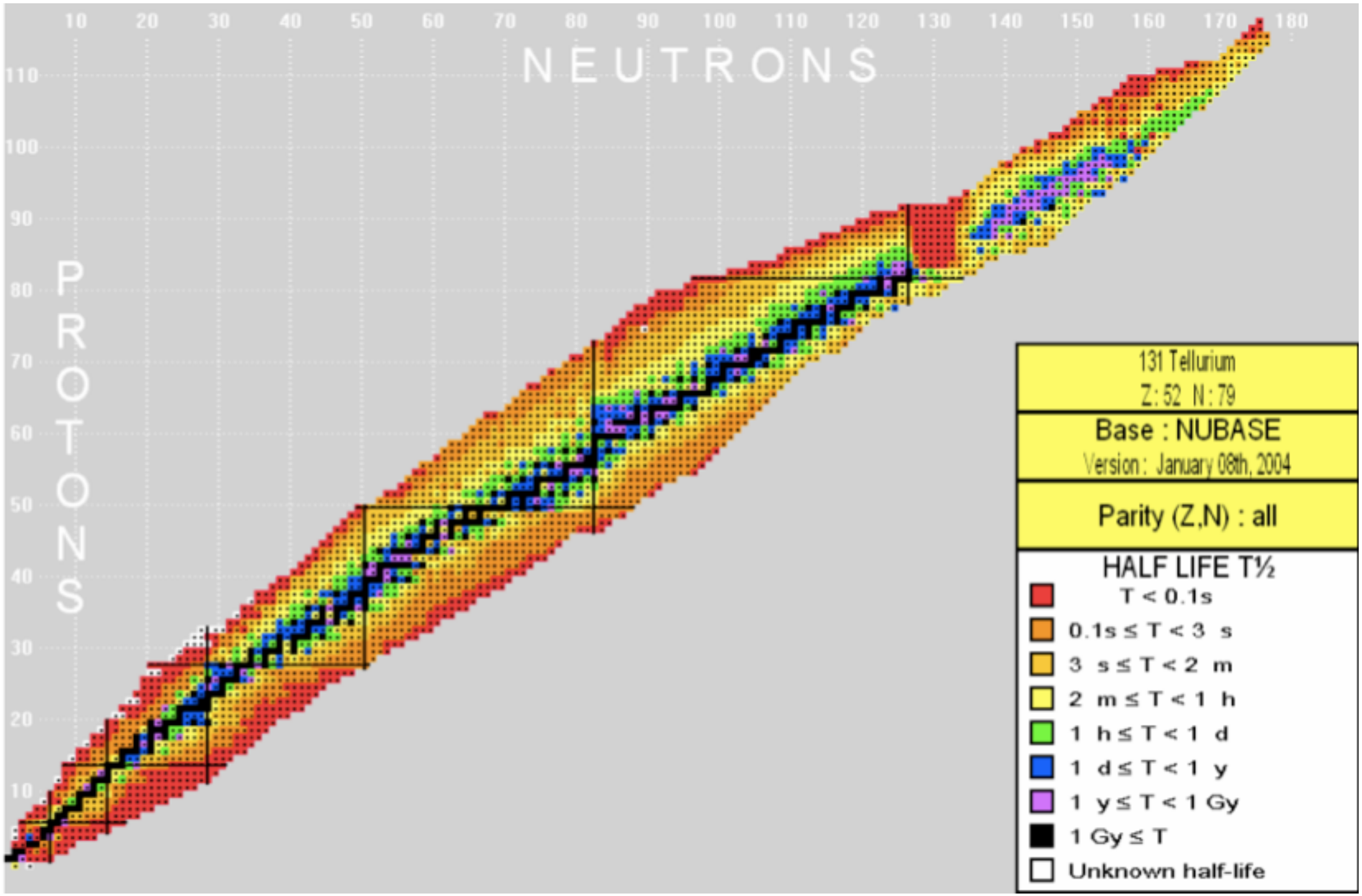
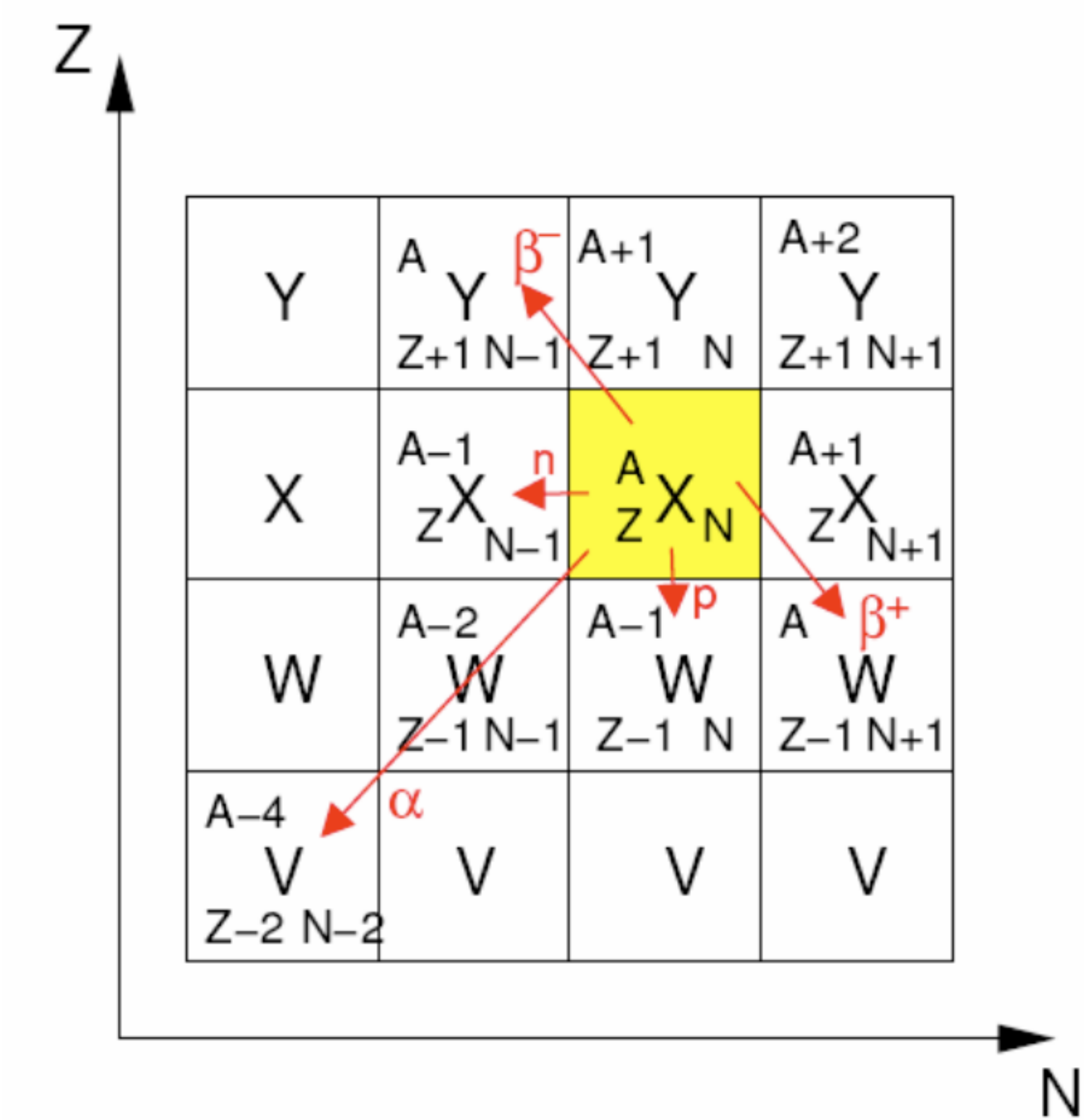


Chart of Nuclides / Lifetimes



Types of Decay



Alpha Decay

- Studied by Ernest Rutherford, who found out that some natural radioactive nuclei, like uranium and radon were emitting positively charged particles that were the least penetrating ones, in the sense that they were stopped and absorbed earlier ($100 \mu m$) in materials

- They were later identified like the nuclei of helium atoms

- They are very ionizing and are stopped by just a sheet of paper

- We can write the alpha decay equation like

$$^{A}_{Z}\mathrm{El}_{N} \rightarrow ^{A-4}_{Z-2}\mathrm{El}_{N-2} + ^{4}_{2}\mathrm{He}_{2} + Q_{\alpha}$$

positive number —> exothermic spontaneous alpha decay

Example $^{238}\mathrm{U} \rightarrow ^{234}\mathrm{Th}^{2-} + ^{4}\mathrm{He}^{2+} + Q_{\alpha}$

- The alpha decay lowers the Coulomb energy of the nucleus, increasing his stability

- It is the dominant decay in nuclei with A>=210 and the energy of the emitted alpha ranges from 4-9 MeV

Alpha Decay

Observation: the energy range of the alpha particles is rather small (4-9 MeV) with respect to the variation of the lifetimes of alpha-decaying nuclei which extends over many orders of magnitude.

Empirically, it is observed that the alpha decay probability per unit time W is related to the alpha energy E with this relationship:

$$\log W = a - \frac{b}{\sqrt{E}}$$

Element	E_α (MeV)	Half-life
^{206}Po	5.22	8.8 days
^{208}Po	5.11	2.9 years
^{210}Po	5.31	138 days
^{212}Po	8.78	0.3 μs
^{214}Po	7.68	164 μs
^{216}Po	6.78	0.15 s

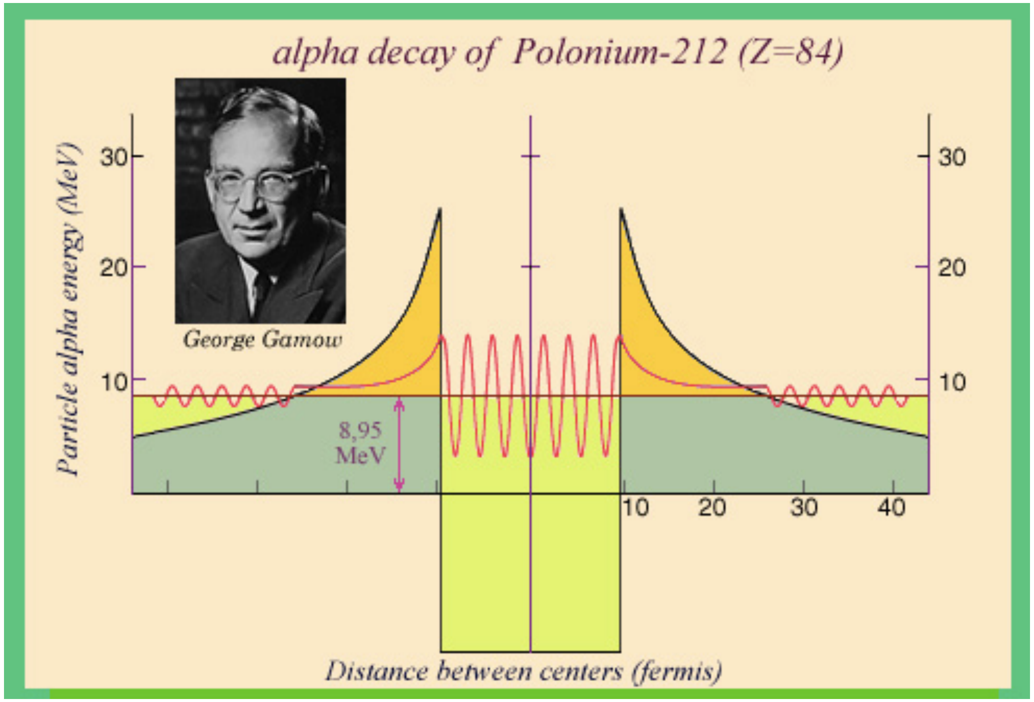
The last relation is known as **Geiger-Nuttal law**.

Quantum Tunnelling and alpha Decay

Classically, a particle cannot overcome a potential barrier bigger than its kinetic energy but quantum mechanically the process might happen with a certain probability.

The two competing forces: attractive nuclear force and Coulomb repulsion create a potential barrier. In principle, the alpha is bound in the nucleus but it can escape by tunnel effect.

Even if the alpha has energies below the Coulomb "barrier", it can "tunnel" through it allowing the decay.



Quantum Tunnelling and alpha Decay

For estimating the alpha energy we can consider the opposite problem of bringing an alpha very close to the nucleus at $r = R_{nucleus} \approx r_0 A^{1/3}$
Outside the nucleus we have a repulsive Coulomb potential:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Ze \cdot 2e}{r} = \frac{2.88 \cdot Z}{r(\mathrm{fm})}\mathrm{MeV}$$

Let's estimate the alpha energy using the above formula for Z=92 and A=238, i.e. the case of Uranium 238:

$$E \approx 35\mathrm{MeV}$$

The energy is too high! A purely classical explanation of the decay does not work. Only the tunnel effect can account for the observed energies.

Quantum Tunnelling and alpha Decay

We can do an approximate quantum treatment of the problem with a square-well confining potential with height V.
In the limiting case where V>>E, we have for the tunnelling probability:

$$W \approx e^{-\frac{2b}{\hbar}\sqrt{2m(V-E)}}$$

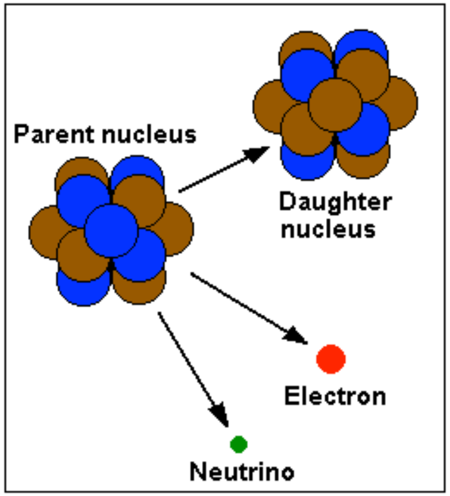
W is better known as transmission coefficient.
You can notice that the last result describes exactly the behaviour of the Geiger-Nuttal empirical law: the logarithm of the probability is proportional to the square root of the energy.

An alpha particle is rater heavy and its emission induces a recoil on the parent nucleus. The **recoil energy** is:

$$E_{recoil} = \frac{m_\alpha}{m_N} E\alpha$$

Beta Decay

- The beta decay is a type of radioactive decay that can take place inside a nucleus. It is the most common decay type, since it has been detected in isotopes of almost all elements



Beta decay

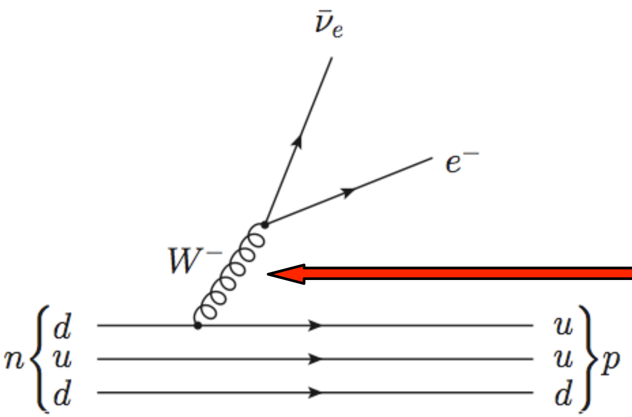
- There are two types of beta decay: β^+, β^-

- The beta decay is:

$$A(Z,N) \rightarrow A(Z+1,N-1) + e^- + \bar{\nu}_e$$

and occurs when a neutron decays into a proton inside the nucleus: $n \rightarrow p + e^- + \bar{\nu}_e$

- At the quark level:



The weak force is responsible for it!

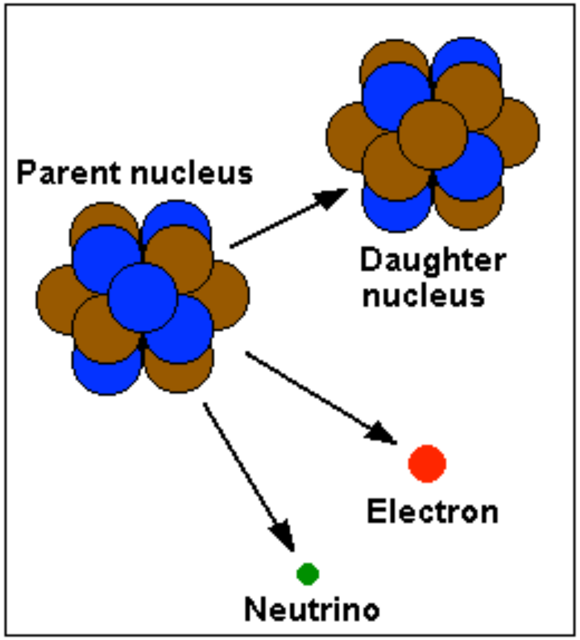
Beta Decay

- The weak force is responsible for the beta decays.
  Since the mediators of the weak force are the $W^{+/-}, Z^0$ bosons, with masses

$$m_{W^{\pm}}c^2 = (80.8 \pm 2.7)\ GeV$$
$$m_{Z^0}c^2 = (92.9 \pm 1.6)\ GeV$$

and we have seen that the range of the force is inverse proportional to the mass of the exchanged particle (using the uncertainty principle), one gets that the range of the weak force is extremely small

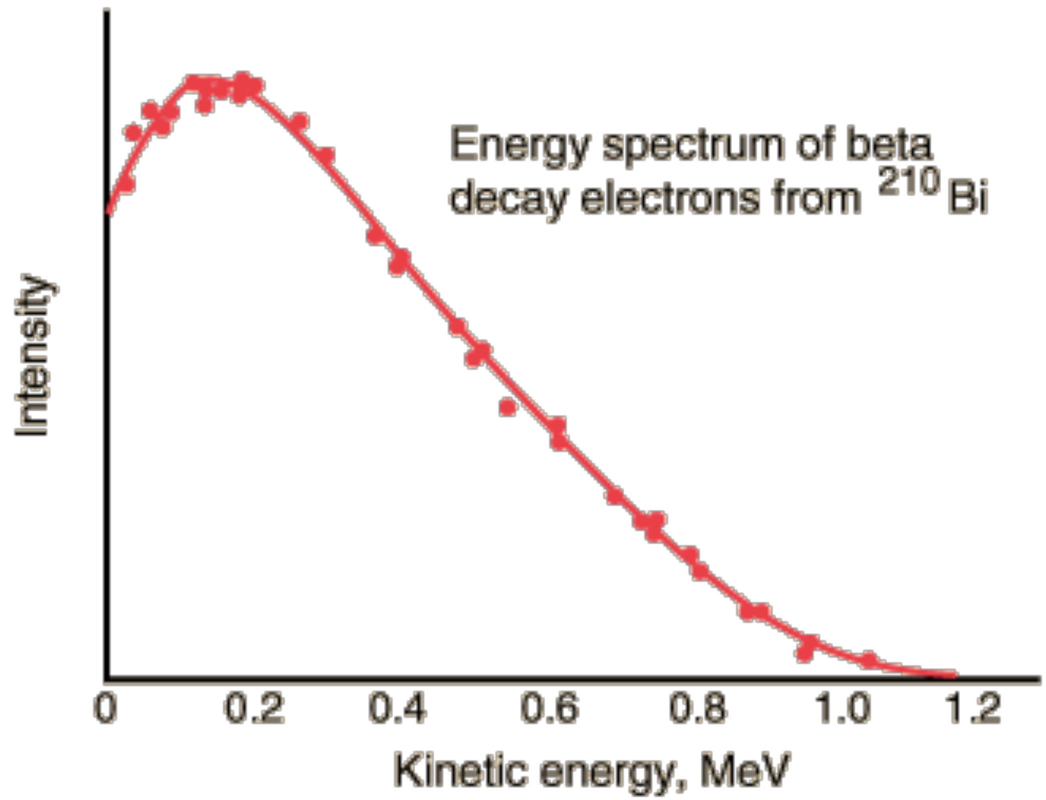
range ~ 10^{-3} fm



Beta decay

- Since the beta decay is quite common in nuclei, it was observed before physicists knew about the neutrino. What one could measure were the electrons.
If the decay goes from a parent nucleus that has a certain energy, to a daughter nucleus with a different energy and there is only one emitted particle (the electron), then the spectrum of this particle should be discrete, but is is not! (—> neutrino).

Beta Decay



If more than one particle is emitted, then the spectrum can be continuos.
The observed electron and positron spectra were indeed continuous.
This brought Pauli to postulate the existence of another particle (1931) that was emitted during the decay. Such particle was later named neutrino by Fermi.

Beta Decay: Energy Balance

- Q-value $Q = m_i c^2 - m_f c^2$; $Q = T_f - T_i$ due to conservation of energy $m_i c^2 + T_i = m_f c^2 + T_f$

$$Q_{\beta^-} = T_f - T_i = T_e + T_\nu + \cancel{T_N}^{\,0} - 0 > 0$$

Sum of electron and neutrino kinetic energy

- Energy balance

$$m(Z,A)c^2 = [m(Z+1,A) + m_e + \cancel{m_\nu}]c^2 + T_e + T_\nu + \cancel{T_N}^{\,0}$$

$$T_e + T_\nu = m(Z,A)c^2 - m(Z+1,A)c^2 - m_e c^2$$

Using $BE_{elec}(Z) = Z m_e c^2 + m(Z,N)c^2 - m_{atom}^{{}^A_Z X_N} c^2$

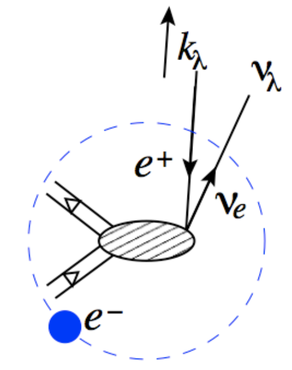
we obtain:

$$\begin{aligned} T_e + T_\nu &= m_{atom}^{{}^A_Z X_N} c^2 - \cancel{Z m_e} c^2 + \cancel{BE_{elec}(Z)} \\ &- m_{atom}^{{}^A_{Z+1} X_{N-1}} c^2 + (\cancel{Z+1}) m_e c^2 - \cancel{BE_{elec}(Z+1)} - \cancel{m_e} c^2 \end{aligned}$$

$$Q_\beta = (m_{atom}^{{}^A_Z X_N} - m_{atom}^{{}^A_{Z+1} X_{N-1}})c^2$$

Energetically allowed if $m_{parent} > m_{daugter}$

Beta+ Decay



β^+ decay

- The β^+ decay is:

$$A(Z, N) \rightarrow A(Z-1, N+1) + e^+ + \nu_e$$

and occurs when a proton becomes a neutron inside the nucleus:

$$\text{energy} + p \rightarrow n + e^+ + \nu_e$$

- It cannot occur in isolation, because it requires energy.
- Q-value

$$Q_{\beta+} = (m_{atom}^{{}^A_Z X_N} - m_{atom}^{{}^A_{Z-1} X_{N+1}} - 2m_e)c^2$$

➡ EXERCISE: work out the energy balance for the β^+ decay

Energetically allowed if $m_{parent} > m_{daugter} + 2m_e$

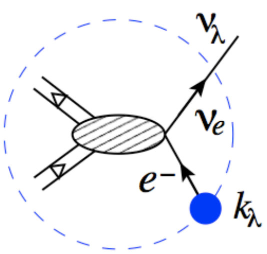
Electron Capture

- The electron capture is an alternative reaction to β^+ decay

$$e^- + A(Z,N) \rightarrow A(Z-1,N+1) + \nu_e$$

an atomic electron is captured by the proton inside the nucleus

$$e^- + p \rightarrow n + \nu_e$$



orbital e^-capture

- Q-value $Q_{EC} = (m_{atom}^{{}^A_Z X_N} - m_{atom}^{{}^{A}_{Z-1}X_{N+1}})c^2 - B_e$ ⟶ Electron ionization energy

Energetically allowed if $m_{parent} > m_{daugter} + B_e$

➡ EXERCISE: work out the energetics for the electron capture

- Electron capture can be favourable: ^{7}Be and ^{7}Li mass difference is 0.86 MeV/c^2: only EC can happen

- The captured electron is usually from the innermost (n=1 or K) electron shell, in which case it is called K-capture. Capture from higher shells is possible but less probable (i.e. L-capture from n=2 the shell or M-capture from the n=3 shell)

Electron Capture

Recall the K-shell hydrogenic wave function (n=1, l=0)

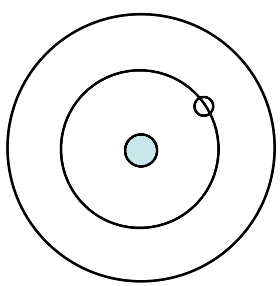
$$\phi_K(r) = 2(Z/a_0)^{3/2} e^{-Zr/a_0}, a_0 = 5.29\ 10^{-11} m$$

Because the nucleus is small with respect to the Bohr radius $\phi_K(r) \approx 2(Z/a_0)^{3/2}$

The probability density will be: $|\phi_K(r)|^2 = 4(Z/a_0)^3$

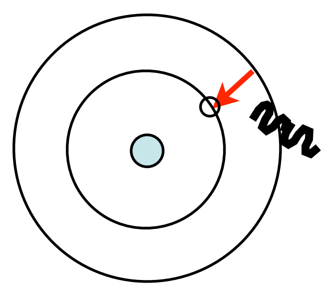
➡ The density of the K-shell electron near the nucleus increases with Z^3 and hence electron capture is more likely for heavy nuclei.

After electron capture



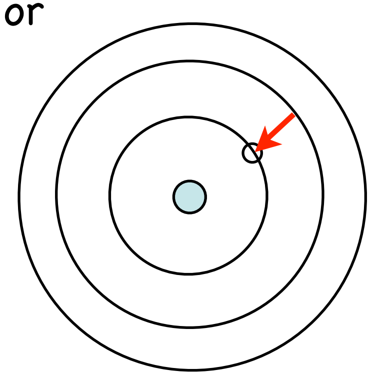
The electron is captured by the nucleus and a vacancy is left in the K-shell

Then either



An electron from a higher shell drops down to fill the vacancy, with the emission of a photon

or



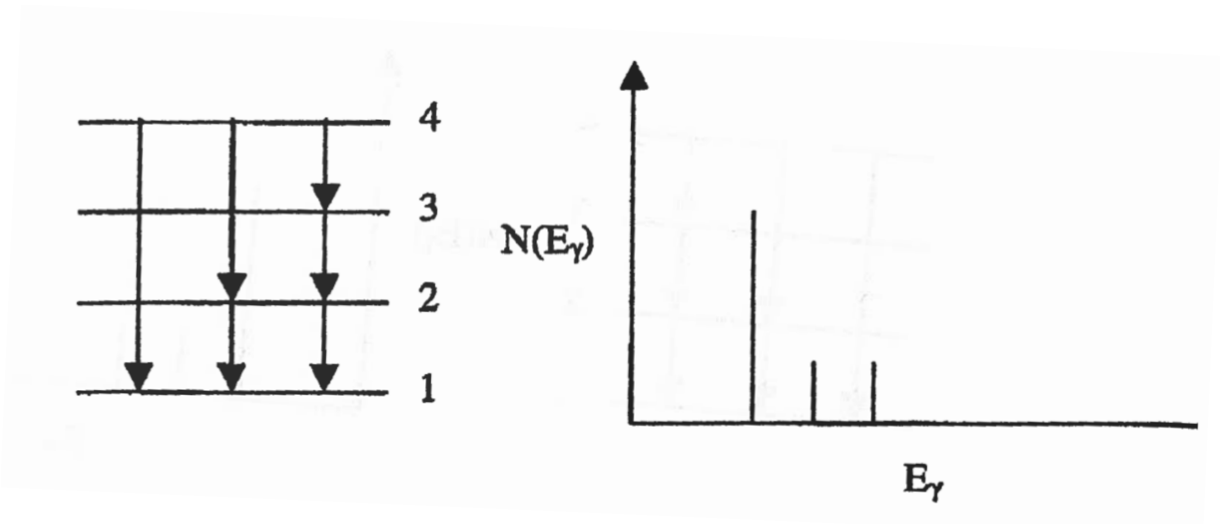
An electron from a higher shell drops down to fill the vacancy and the atomic de-excitation kicks off another orbital electron (Auger electrons)

Gamma Decay

- The gamma-ray decay occurs when a nucleus in an excited state releases its excess energy by emission of an electromagnetic radiation, i.e. a photon.

$$^{A}\mathrm{El}^{*} \rightarrow {}^{A}\mathrm{El} + \gamma$$

- There is no change in Z or A, but just a release of energy
- You can also have gamma de-excitation from an excited state to the other



Summary

- Alpha Decay —> Tunnel Effect

- Beta(+/-) Decay

- Electron Capture

- Gamma Decay