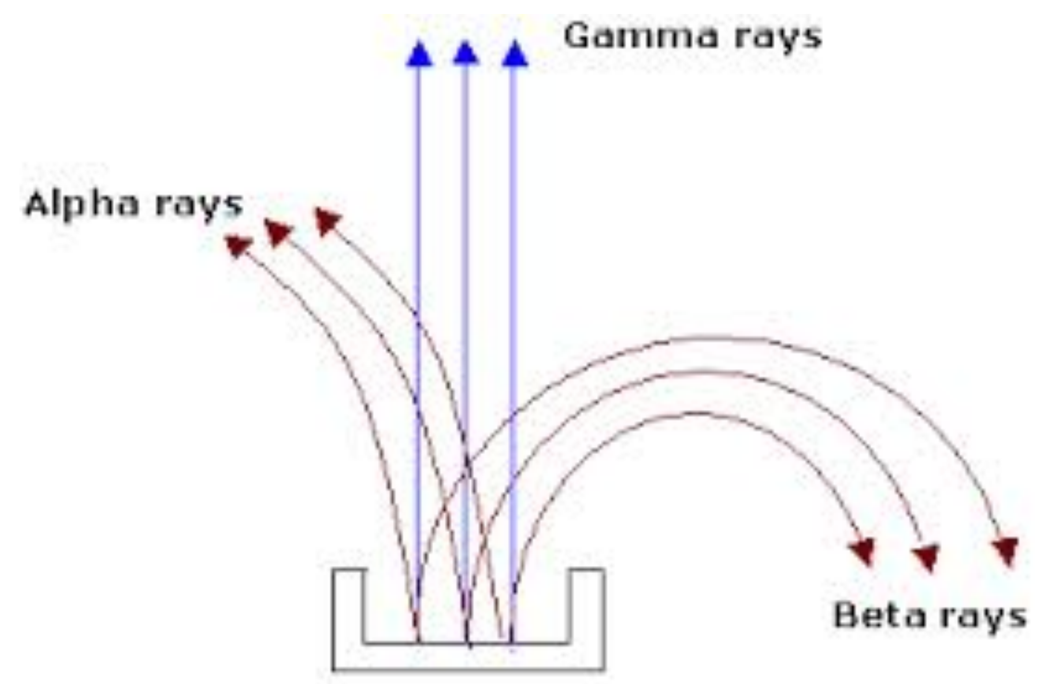
NUSC 341 Summary
Topics Relevant for the FINAL EXAM

α, β, and γ rays

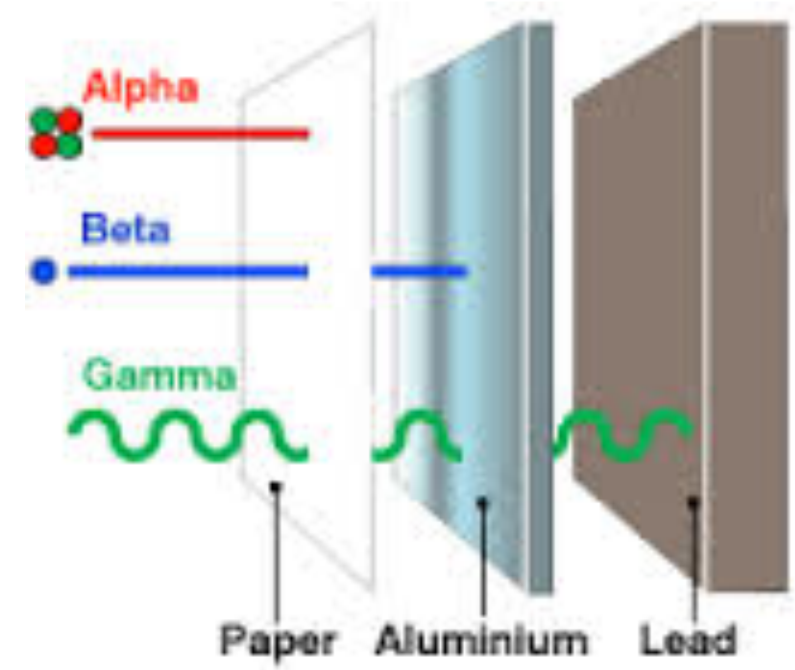
α rays: deflected by electric and magnetic fields in a direction opposite to that of cathode rays: positively charged

β rays: behaved in the same manner as the cathode rays, so they thought they were negatively charged

γ rays: extremely penetrating and unaffected by electrical or magnetic fields



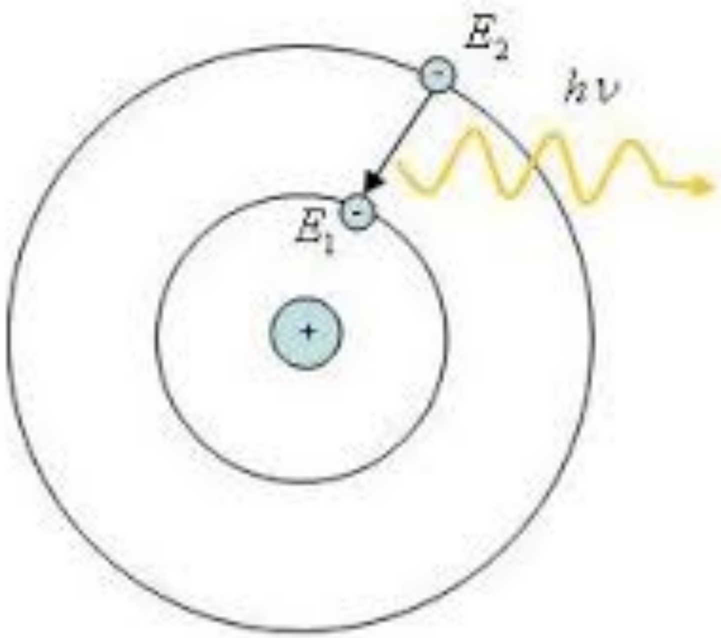
REVIEW LECTURE 8 about the details of the radioactive decays!



DeBroglie Wavelenght

What is the wavelength associated to a level transition?

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar c}{pc} \longrightarrow \frac{2\pi\hbar c}{E}$$



- The limit is ultra-relativistic (v~c) or (equivalently) we assumed m=0.
- $\hbar = h/2\pi$ (read: h "bar")

Electric Potential

Electrostatic Potential between two particles with a and b elementary charges separated by a distance R:

$$V = \frac{1}{4\pi\epsilon_0}\frac{(ae)(be)}{R}$$

Introducing the fine structure constant:

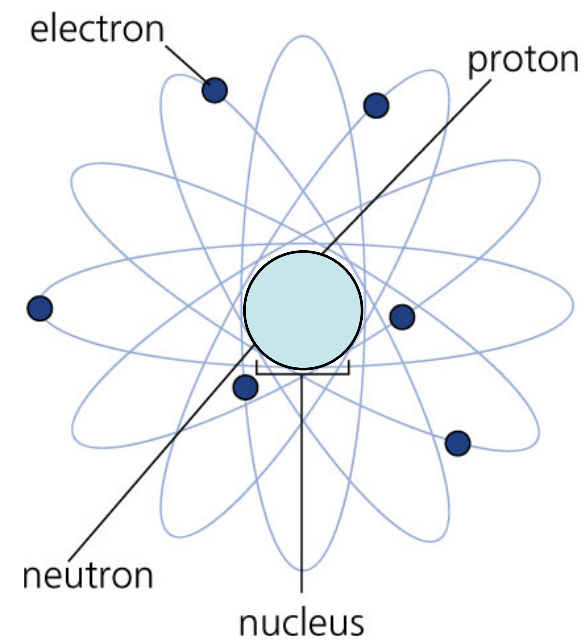
$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

We can rewrite the potential (in MeV) as:

$$V = (\alpha\hbar c)\frac{ab}{R} = 1.44[\mathrm{MeV}\cdot\mathrm{fm}]\frac{ab}{R[\mathrm{fm}]}$$

Binding Energy

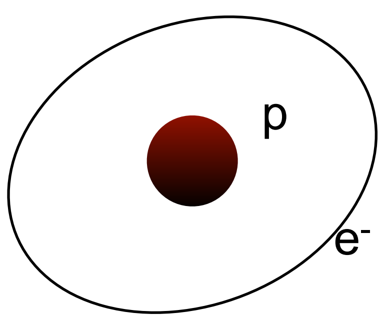
- The binding energy is a quantity that can be defined also for an atom



The binding energy of Z electrons in an atom is

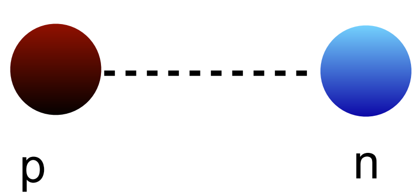
$$BE_{elec}(Z) = Zm_e c^2 + m(Z,N)c^2 - m_{atom}c^2$$

Consider the simplest atom (Hydrogen)



The atomic binding energy is 13.6 eV

Consider the simplest nucleus (deuteron)



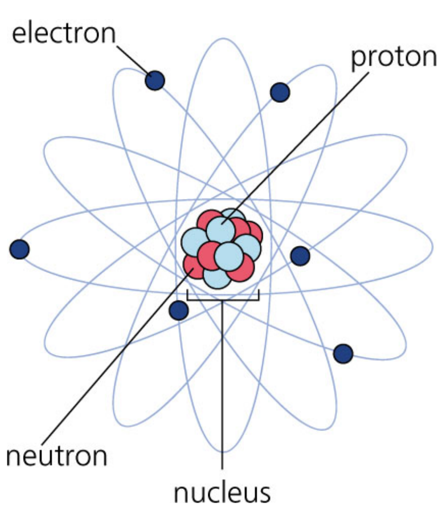
The nuclear binding energy is 2.22 MeV

- How do we know the binding energy of a nucleus? We need an operative way to define the mass of the nucleus. What exactly is $m(Z,N)$

Binding Energy

- BE are measured from **masses of the atoms**, since they are much better determined than nuclear masses

$$BE(Z,N) = Zm_pc^2 + Nm_nc^2 - BE_{elec}(Z) + Zm_ec^2 - m_{atom}c^2$$



- Atomic masses are referred to the Hydrogen atom $m_{atom}c^2 = Zm_Hc^2$

$$m_Hc^2 = m_ec^2 + m_pc^2 - BE_{elec} \quad \times Z$$

$$\Rightarrow \quad Zm_Hc^2 = Zm_ec^2 + Zm_pc^2 - ZBE_{elec}$$

$$ZBE_{elec} \simeq BE_{elec}(Z)$$

$$BE(Z,N) = Zm_Hc^2 + Nm_nc^2 - m_{atom}c^2$$

Summarizing on the Chart of Nuclides:

- For the light elements: N=Z
- With increasing Z for achieving nuclear stability, the N/Z ratio increases from 1 to ~1.5 (at Bi).
- Pairing of nucleons is not a sufficient criterion, but a certain N/Z must also exist.
- At high-Z, a new mode of decay appears (α-decay) in addition to β-decay.
- Nuclei far from the valley of stability (see later):
    - high N/Z (neutron-rich): β-decay for lowering N
    - low N/Z (proton-rich): β^+-decay for lowering Z

For better understanding all the collected facts, we need a more quantitative description of the nucleus, ie we need:

Nuclear Models

Collective Models

- Try to describe the nucleus as a whole
- Identify collective variables.

Microscopic Models

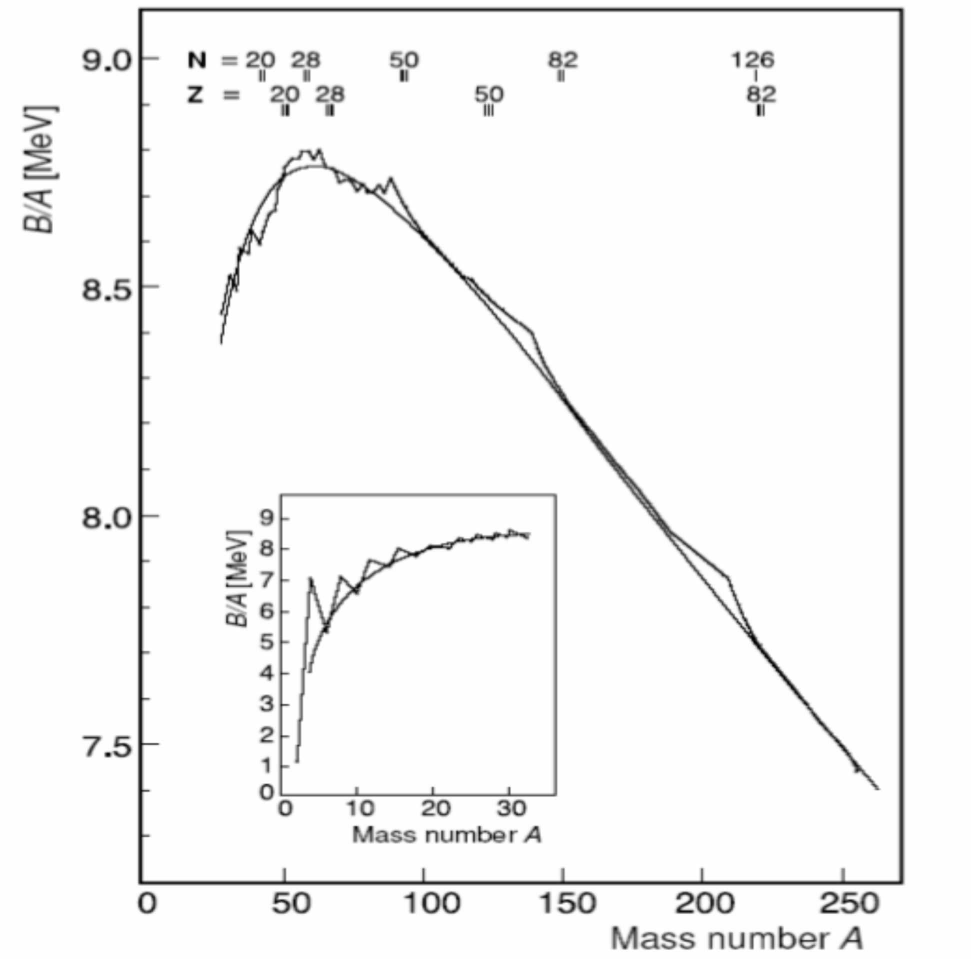
- Try to describe the nucleus using variables relative to the single nucleons.

The Liquid Drop Model

$$BE(Z,N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A} - a_5 \Delta$$

A fit to a set of nuclides data gives:

a1 = 15.67 MeV
a2 = 17.23 MeV
a3 = 0.714 MeV
a4 = 23.29 MeV
δ = 25/A MeV



Fits might yield slightly different results depending on the dataset. The pairing parameter is the most difficult to determine.

Residuals of the Liquid Drop Model

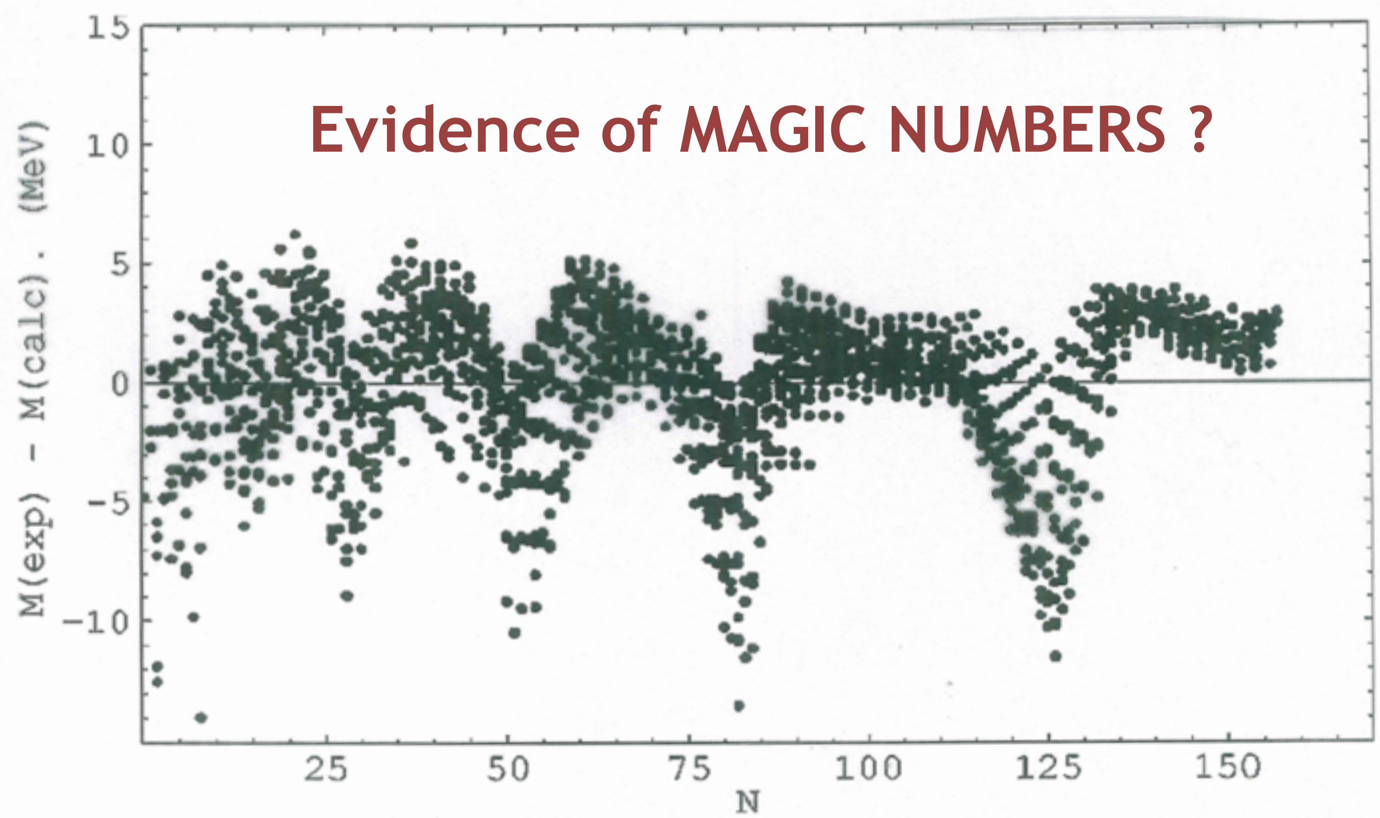
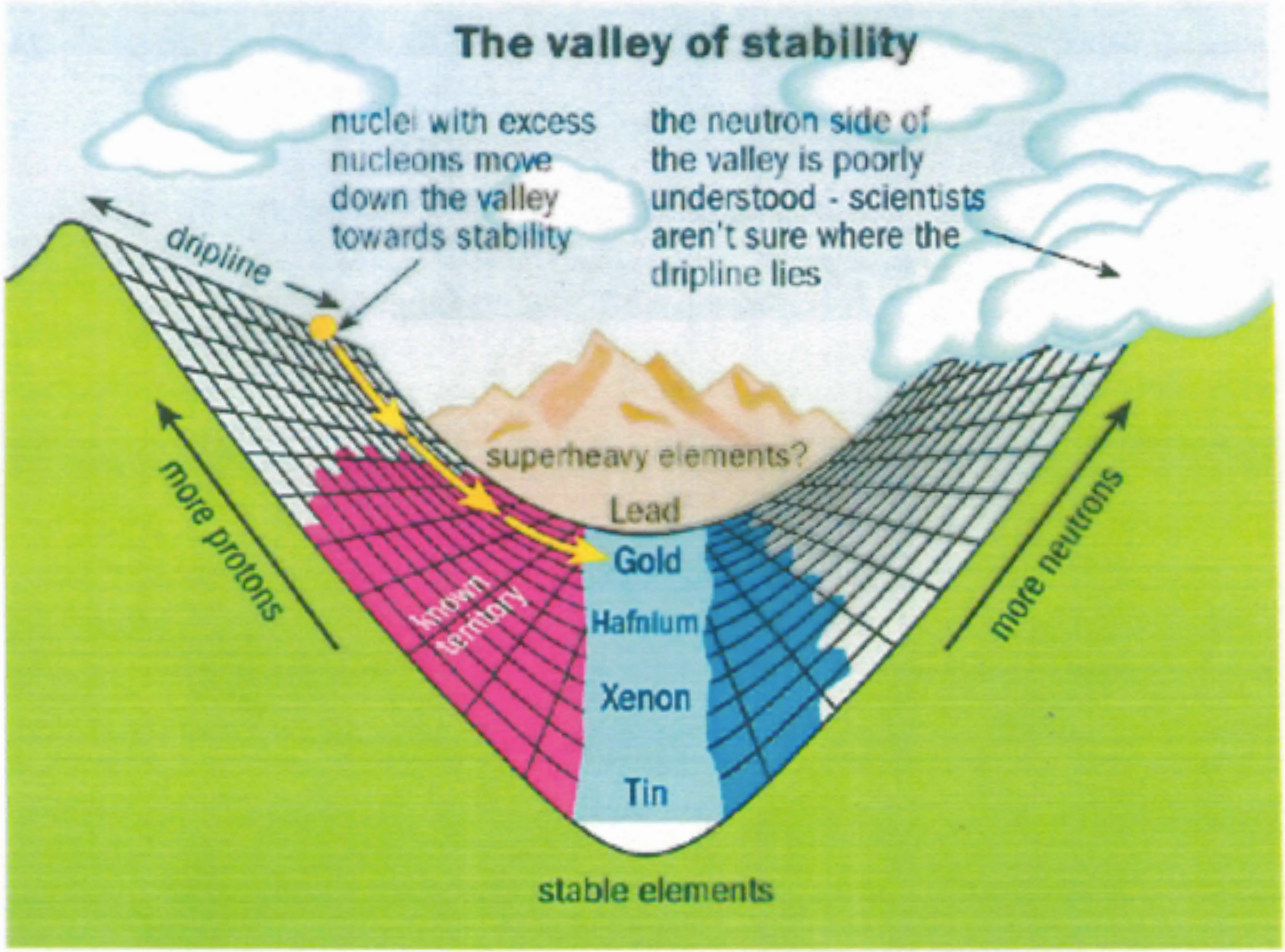


FIG. 8. Deviations from experiment of the von Weizsäcker mass formula (9), shown as a function of neutron number N.

Review EVERYTHING about the liquid drop model!

The valley of stability

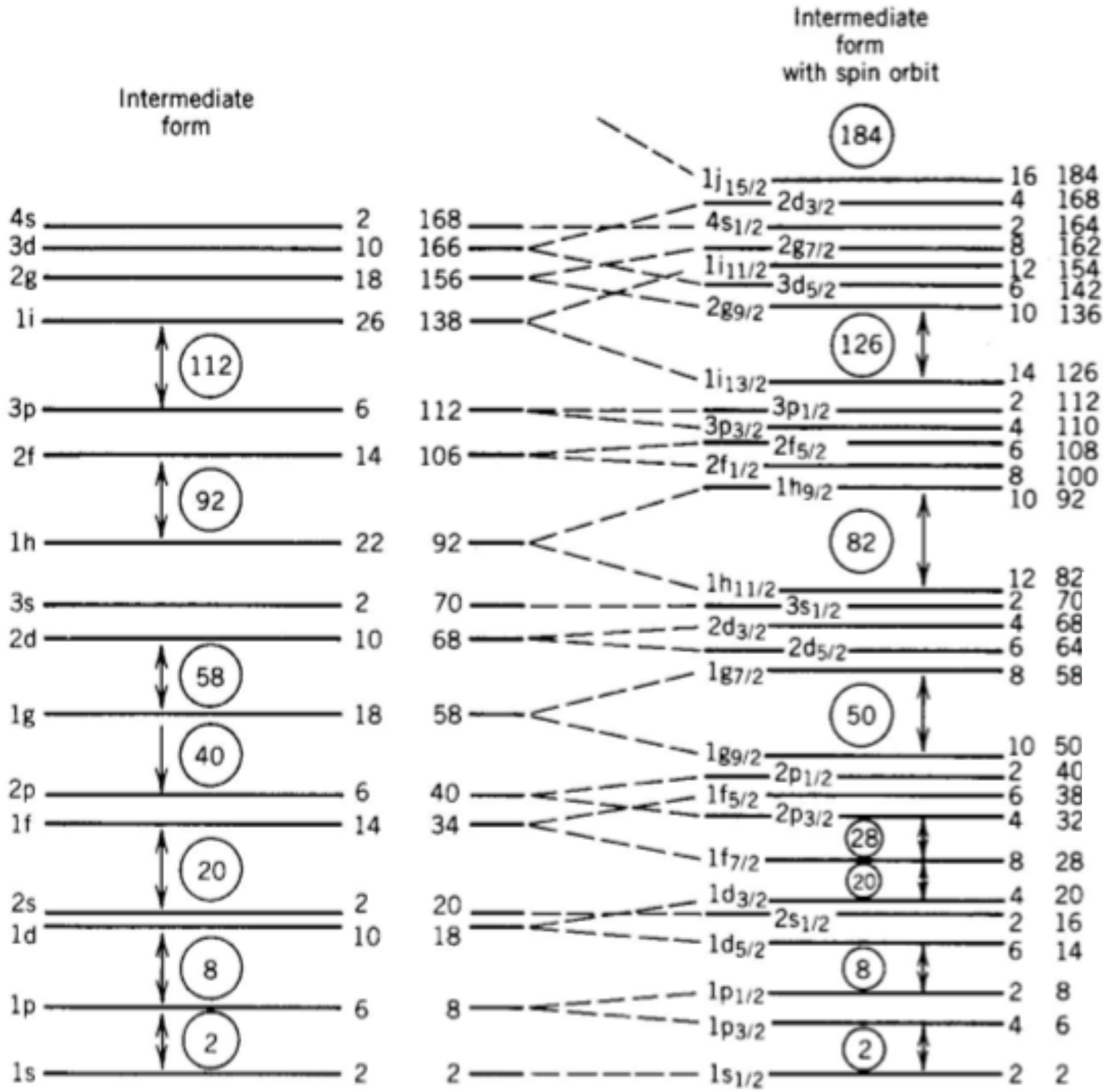


The "valley of stability" - new nuclear machines such as the Rare Isotope Accelerator will open up studies of nuclear phenomena using beams of short-lived isotopes, which form the high "walls" of the valley.

Nuclear Shell Model

With the addition of the spin-orbit, the magic numbers are reproduced

2, 8, 20,28, 50, 82, 126



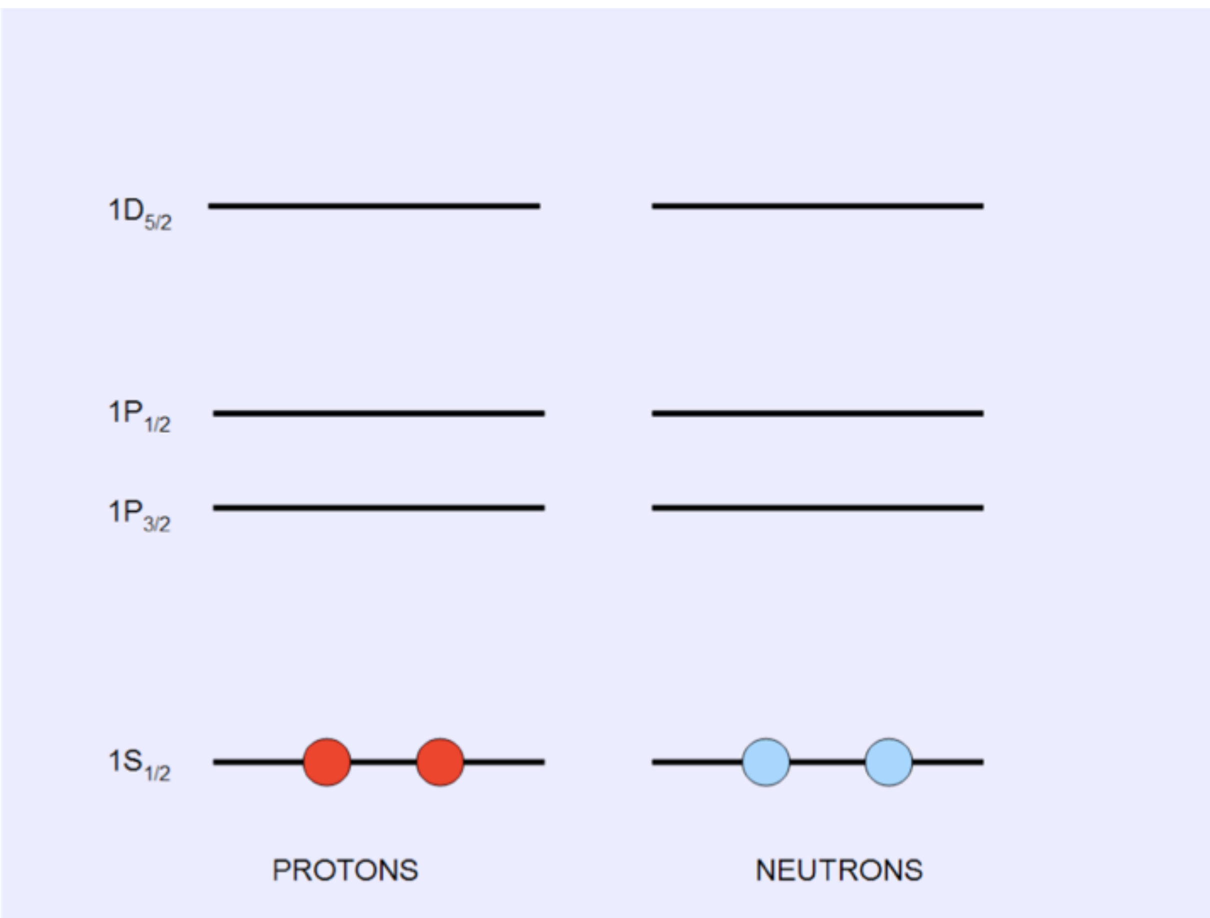
Maria Goppert-Mayer and Hans Jensen
Nobel prize in 1963

Phys. Rev. **75**, 1969 (1949)

Nuclear Shell Model

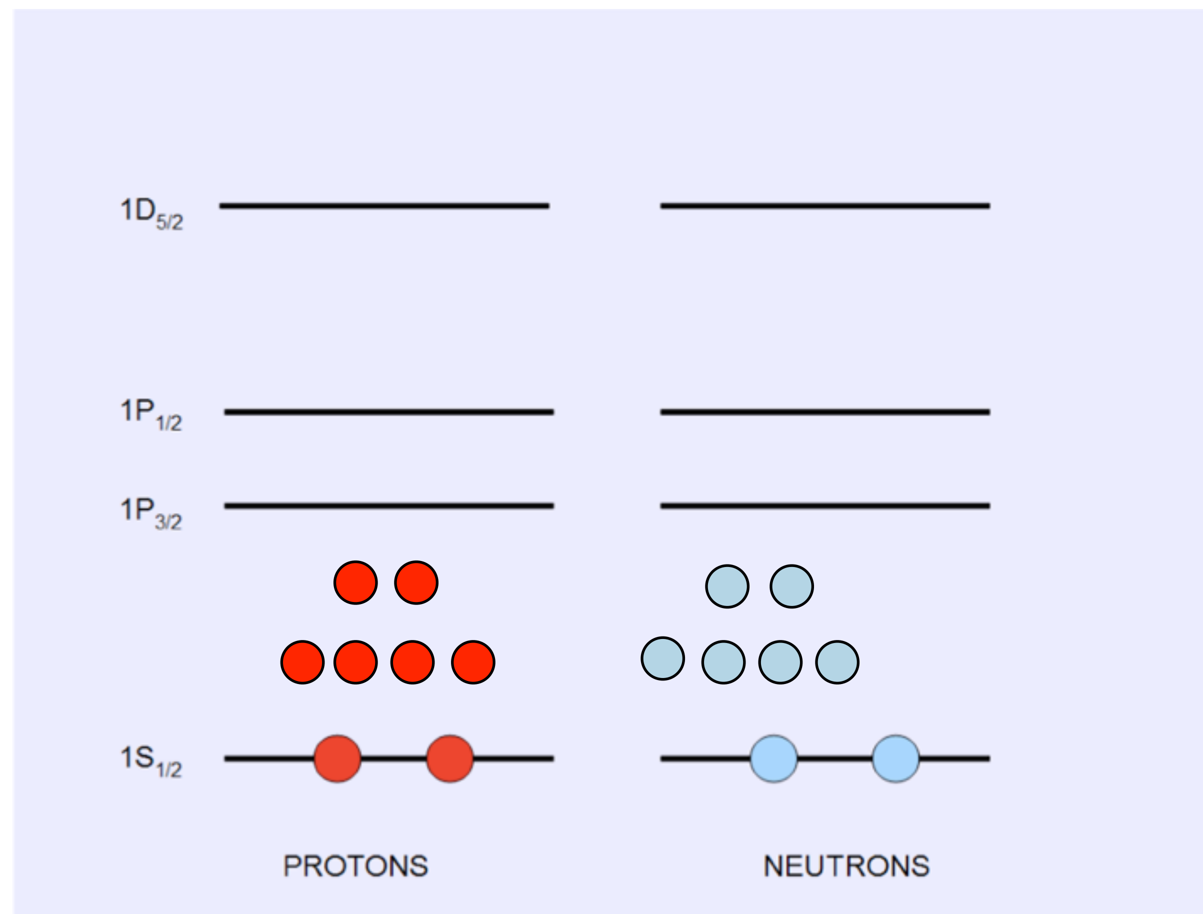
We can build the shell structure of nuclei, just like we fill up the electron shells in atoms. Only now there are separate proton and neutron shells.

^{4}He



Double closed s-shell

^{16}O



Double closed p-shell

Double magic nuclei:
extra binding energy, extra small radius, extra low reaction probability

Radioactive Decay of a Nuclide

- The activity is directly proportional to the number of nuclei we have:

$$A = -\frac{dN}{dt} = \lambda N$$

Solving the above first order differential equation considering:

- A positive proportionality constant λ (the **decay constant**)
- The boundary condition $N(t=0) = N_0$

we obtain:

$$N(t) = N_0 e^{-\lambda t}$$

The constant must have the dimension of [1/time] and therefore we can set $\lambda = 1/\tau$ with τ the **average lifetime**.

Review EVERYTHING about the A->B decay (Lecture 10)

Cross Section

$$R = I_0 \sigma \rho \Delta x$$

We can consider the last equation as the definition for the cross section. σ is not the geometrical cross section of the nucleus πR^2 but expresses the probability for a specific reaction to occur.

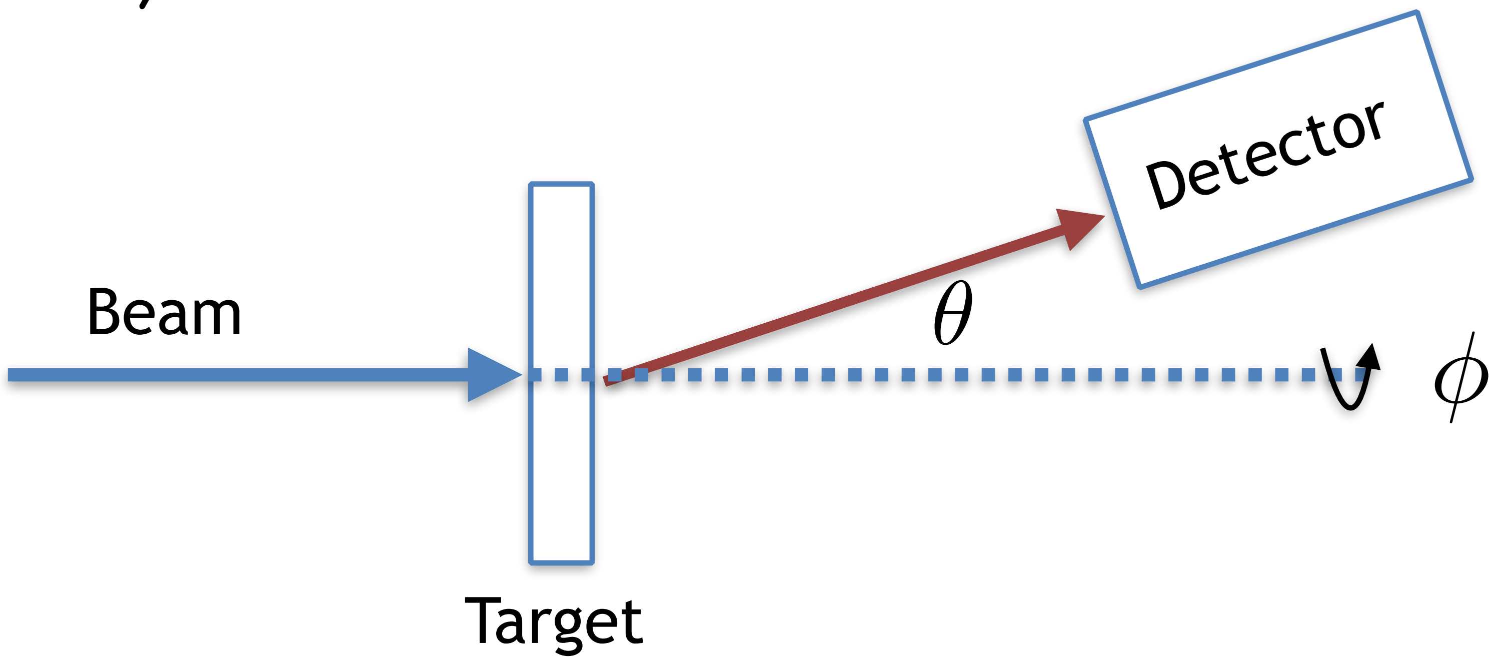
Cross section unit: the **barn (b)**.

$$1 \text{ b} = 10^{-28} \text{ m}^2$$

The name comes from the first experiments where a surprisingly large cross section was measured: "It is as big as a barn !".

Differential Cross Section

Up to now, we discussed the **total cross section** but in reality what is really measured is almost always a **differential cross section**.
This means that usually we cannot measure all the particles emerging from a reaction at all the angles and all the energies. This is mainly due to the limited area and sensitivity of our detectors:



If the detector subtends a solid angle $\Delta\Omega$ the cross section is:

$$R = I_0 \frac{d\sigma}{d\Omega}(\theta, \phi)\Delta\Omega \cdot \rho\Delta x$$

..back to Rutherford's Experiment

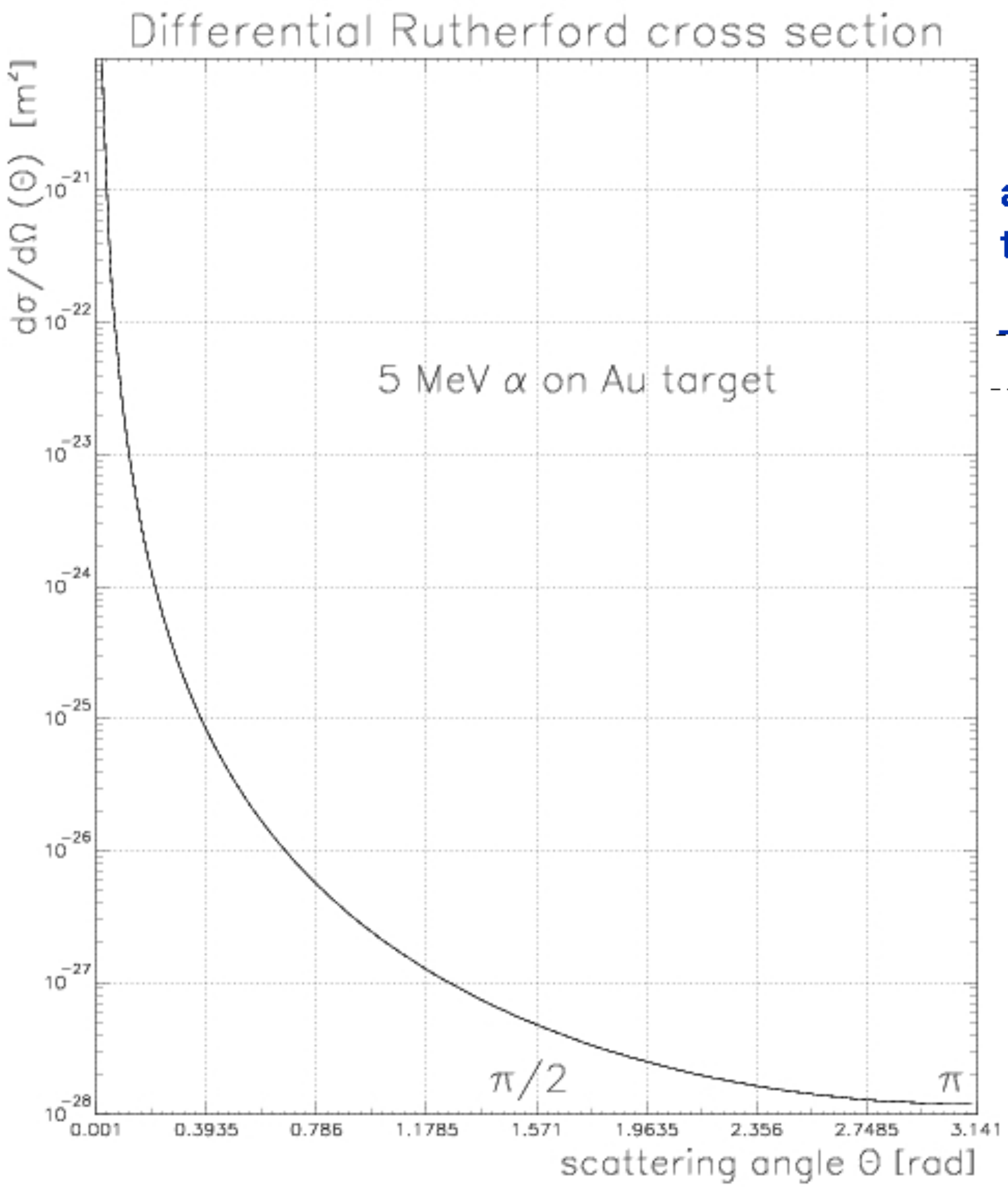
By alpha-scattering, Rutherford deduced that the atom was made by a dense object in its centre. At that time, the structure of the nucleus was not resolved. Assuming the nucleus as a point with positive charge e, its electric potential is the one of a point-like charge V~1/r .

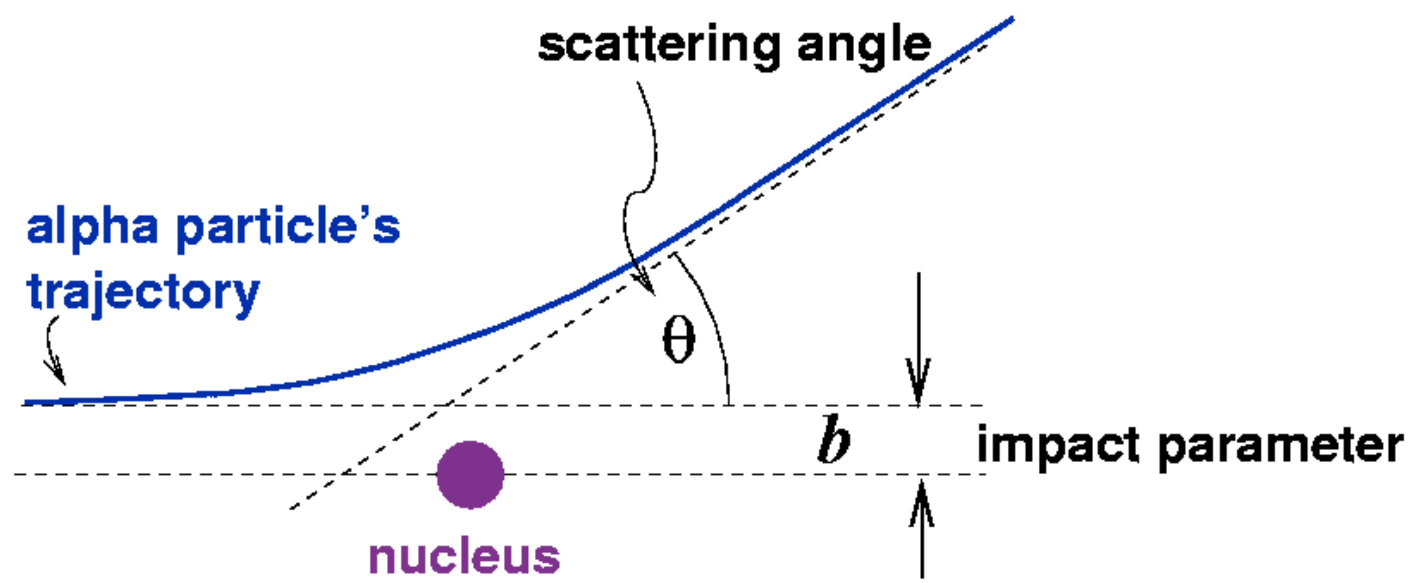
The differential cross-section for such a configuration can be calculated and the result is:

$$\frac{d\sigma}{d\Omega} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{4p^2}\frac{1}{\sin^4(\theta/2)}$$

This is the distribution Rutherford found as a function of θ. Note that the differential cross section does not depend on ϕ since the problem is symmetric wrt the beam axis.

..back to Rutherford's Experiment





The cross section can be written as a function of the impact parameter. In this way, Rutherford was able to estimate the size of the nucleus.

$$\frac{1}{2}mv^2 = \frac{e^2}{4\pi\epsilon_0}\frac{1}{b} \Rightarrow$$

$$b = \frac{2e^2}{4\pi\epsilon_0}\frac{1}{mv^2}$$

The Cyclotron (II)

Working principle of the cyclotron:

The magnetic field is orthogonal to the trajectory, therefore, neglecting here the acceleration force, the Lorenz force acting on the orbiting particle is:

$$F = qv \times B = qvB$$

The centrifugal force is: $F_C = \frac{mv^2}{R}$ and in equilibrium $F_C = F$ yielding:

$$qvB = \frac{mv^2}{R} \Rightarrow \nu = \frac{1}{2\pi}\frac{qB}{m}$$

Important observation: the revolution frequency does not depend on the radius of the orbit, therefore we can use a fixed-frequency generator.
The longer trajectory at every turn is compensated exactly by the higher speed of the particle.

Additional Topics:

Radiation-Matter interaction

Dating Techniques

REVIEW THE MID-TERM EXERCISES!

Final Exam:

Room: AQ 5018
Date: December 12, h15;30

Remember:

- Calculator
- Scrap paper


תודה
Dankie
Gracias
Спасибо
شكراً
Merci
Takk
Köszönjük
Terima kasih
Grazie
Dziękujemy
Dėkojame
Ďakujeme
Vielen Dank
Paldies
Kiitos
Täname teid
谢谢
Thank You
Tak
感謝您
Obrigado
Teşekkür Ederiz
감사합니다
Σας Ευχαριστούμ
ขอบคุณ
Bedankt
Děkujeme vám
ありがとうございます
Tack