Exercise Sheet 6

Exercise 1

Consider the following system of non-linear differential equations:

$$\begin{cases} \frac{dx}{dt} &= \sigma(y-x) \\ \frac{dy}{dt} &= x(\rho - z) - y \\ \frac{dz}{dt} &= xy - \beta z \end{cases} \tag{1}$$

This is the famous Lorenz system displaying chaos. With "Chaos", we define a dynamical system which is extremely sensitive to the initial conditions. This means that no matter how close two solutions start, they will diverge from each other exponenially fast. Along the lines of the Predator-Prey code of the Notes, implement RK2 and RK4 for the Lorenz model.

Exercise 2

Choosing parameters $\sigma = 10$, $\beta = 8/3$, $\rho = 28$, show the accuracy of the two algorithms calculating the relative error (RK4-RK2)/RK2 in percentage for one solution (choose a suitable starting point (x_0, y_0, z_0)).

Exercise 3

Consider the same parameter values as before and use RK4 calculating two solutions S1 and S2: one starting at (x_0, y_0, z_0) and one starting at $(x_0 + h, y_0 + h, z_0 + h)$, where h=0.001. Plot the distance $|S1 - S2|$ of the two solutions as function of time.

Exercise 4

Consider the case $\rho < 0$ and compare it with the previous case. What do you observe?