TEST for the Final Exam (Klausur) 6

Exercise 1

1) Briefly discuss the main errors present in a numerical calculation.
2) If two quantities x_1 and x_2 with absolute errors Δx_1 and Δx_2 are numerically calculated, what is the error of their sum? And the relative error?

Exercise 2

1) Name at least one numerical algorithm for solving a linear system $Ax = b$.
2) What is a necessary condition for solving the system?
3) How can you assess if it is "easy" to solve the system numerically?
4) Given the following linear system:

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1}$$

show in datails the steps performed by the Gauss elimination algorithm. 5) Is the Choleski decomposition possible for solbinf the previous system?

Exercise 3

Explain what are the main features of the splies cubic interpolation with respect to lower-order interpolation algorithms.

Exercise 4

1) Given the equation

$$x^3 + x^2 = 0 \quad , \tag{2}$$

find the analytic solutions and then calculate 3 steps of the bisection algorithm explicitly. Show that you are converging to one solution of the equation. 2) Which other algorithms for root finding do you know? 3) Describe the working principle of one of them.

Exercise 5

What is the difference between trapezoid integration and Simpson's algorithm?

Exercise 6

Consider the interval $[0, 1]$ and the function $f(x) = x^2 + x + 1$.
Dividing the interval in 10 mesh points (N=10), calculate the *forward* derivative of f

at the point x=0.5.

Exercise 7

Given the differential equation

$$\frac{dy(x)}{dx} = y(x) + x^2 + 1 \quad , \tag{3}$$

Given the initial condition $y(0) = 0$ and a mesh size $h = 0.01$, perform 3 integration steps of Newton's algorithm for finding its solution.

Exercise 8

Consider the following partial differential equation for the function $\psi = \psi(x, y)$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y} = C \quad , \tag{4}$$

where C is a real constant.
1) Derive a numerical approximation scheme for solving the equation, assuming an equal grid spacing h for both variables x and y.
2) Draw the stencil diagram corresponding to the derived method.