Übungsblatt 2

Exercise 1

Consider the Weiszäcker formula for the binding energy of a nucleus with only the volume, surface, and Coulomb terms:

$$E(Z, A) = \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 \frac{Z^2}{A^{1/3}} \quad . \tag{1}$$

This approximation is good for symmetric nuclei (with $Z = N$) and thus $Z = A/2$. Build the new function $f(A) = E(Z, A)/A$. We would like to calculate what is the mass number A of the nucleus with the highest binding energy per nucleon.
To this end, we have to find the mass number A_{max} which *maximixes* $f(A)$. Use the numerical values of the constants in the Notes of the course.
We know that iron has the highest binding energy per nucleon: does this fact approximately correspond to your calculations? Check on the periodic table!

Exercise 2

Given the general expression for the energy density,

$$\rho = \frac{g}{(2\pi)^3} \int E(\bar{k}) f(\bar{k}) d^3k \quad , \tag{2}$$

show that in the case of relativistic bosons

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}}{e^{(E-\mu)/T} - 1} E^2 dE \quad . \tag{3}$$

Exercise 3

The Boltzmann equation in an expanding Friedman Universe is

$$\dot{n} + 3Hn = \langle \sigma v \rangle \left(n_{eq}^2 - n^2 \right) \quad . \tag{4}$$

Following the Notes, show explicitly that the previous equation can be written as

$$\frac{x}{Y_{eq}} \frac{dY}{dx} = -\frac{\Gamma}{H(m)} \left(\frac{Y^2}{Y_{eq}^2} - 1 \right) \quad . \tag{5}$$