Übungsblatt 3

Exercise 1

In the Notes, we obtained an evolution equation for the abundance $Y = n/s$, which can be (also) written as

$$\frac{dY}{dx} = -\frac{xs\langle\sigma v\rangle}{H}(Y^2 - Y_e^2) \quad . \tag{1}$$

The equation can be solved only numerically, but we can try to obtain an approximate solution for a *cold relic* (cold = non-relativistic). In such a case, the density n after freeze-out is much larger than the equilibrium density n_e (since the latter is exponentially suppressed). Use this approximation in Eq. 1.
We know that cross sections are proportional to some power of the temperature (energy), thus $\langle\sigma v\rangle \sim \sigma_0 x^{-n}$. The entropy density for a cold relic scales as $s = s_0 x^{-3}$. Defining $\lambda = \sigma_0 s_0/H$, show that you can rewrite approximately Eq. 1 as

$$\frac{dY}{dx} = -\frac{\lambda}{x^{n+2}}Y^2 \quad . \tag{2}$$

Exercise 2

Eq. 2 can be solved analitically by separation of variables. Show that the solution reads

$$Y_{now} = \frac{n+1}{\lambda}x_{f.o.}^{n+1} \quad , \tag{3}$$

where we approximated $1/Y_{f.o.} \ll 1/Y_{now}$ and the integration limits are from freeze-out (f.o.) and today (now).

Exercise 3

The Universe is not continuously in thermal equilibrium, and the abundances of particle spacies can change with time (density, temperature, scale factor). The change of the amount of neutrons can be written as

$$\dot{X}_n = -\lambda_{np}X_n + \lambda_{pn}(1 - X_n) \quad , \tag{4}$$

where we used the fact that by definition (in absence of nuclei) $X_n + X_p \approx 1$. λ_{np} is the $n \leftrightarrow p$ and $p \leftrightarrow n$ conversion rates. The first term on the right-hand side of the equation describes the disappearence of neutrons due to conversion to protons, the second term describes the appearance of neutrons due to proton to neutron conversion (which is proportional to the proton fraction $X_p = 1 - X_n$). At equilibrium, $\dot{X}_n = 0$ and $X_n/X_p = e^{-Q/T}$.
Find a relationship between λ_{np} and λ_{pn} in terms of $Q = (m_n - m_p)$ and T.