Übungsblatt 4

Exercise 1

In the notes, we estimated the speed of a shock-wave from a supernova with 10 solar masses. One ingredient for this estimation is the calculation of the gravitational energy.
Considering a spherical star of mass M, radius R, and density ρ, the potential energy U for a spherical body (angles are integrated out) is

$$U = -G \int_0^R \frac{M(r)}{r} \rho(r) 4\pi r^2 dr \quad . \tag{1}$$

$M(r) = \rho \cdot V(r)$ is the mass within a "shell" of radius r. If the density is constant, show that

$$U = -\frac{3}{5} \frac{GM^2}{R} \quad . \tag{2}$$

Exercise 2

Consider the mass of the Sun (in grams) and calculate the gravitational energy of a supernova with 10 times the solar mass. For estimating the radius, find out the solar radius and calculate the radius of a 10-times more massive star with the same density of the Sun.
If 1% of the supernova gravitational energy goes in the shock-wave after the core collapse, estimate the shock-wave velocity.

Exercise 3

The rate of primary cosmic rays arriving on earth is approximately $\Phi \approx 0.2\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$. You would like to build a cosmic ray detector for your laboratory and you have 3 m^2 of scintillators for building a flat-plane detector. How many counts per day do you expect from such a detector? Assume that cosmic rays arrive on earth isotropically.

Exercise 4

Pulsars (fast-rotating neutron stars) can be suitable accelerators for cosmic rays. The pulsar's magnetic field axis is generally not aligned with the rotation axis: this varying magnetic field B gives rise to an electric field E. We can estimate

$$\frac{E}{L} = \frac{1}{c} \frac{dB}{dt} \quad , \tag{3}$$

where L is the distance over which the particle is accelerated.
a) Which physical law justifies Eq. 3 ?
b) If we assume $L \sim R_{NS}$ (R_{NS} is the star's radius) and ω_{NS} is the angular velocity of the pulsar, show that the maximum kinetic energy that a particle with charge Ze can achieve is

$$T_{max} = \int ZeEdx = \frac{ZeR_{NS}^2 B\omega_{NS}}{c} \tag{4}$$

c) Taking the case of the Crab nebula pulsar with $\omega_{NS} R_{NS}/c \sim 0.1$, $B \sim 10^{11}$ gauss, and $R_{NS} \sim 10$ km, what is the maximum energy in TeV ? Is it beyond the "knee" or not?