# Übungsblatt 5

## Exercise 1

The main hadronic mechanism for creating high-energy gamma rays is the $\pi^0$ production and decay.
Show explicitly that the energy spectrum for this decay is

$$\frac{dN}{dE_\gamma} = \frac{1}{2}\frac{1}{\beta\Gamma p^*_\gamma} \quad . \tag{1}$$

Pay attention to the 1/2 factor, which comes from the fact that the pion has spin zero.

## Exercise 2

In the propagation of cosmic rays in the atmosphere an important quantity is the *atmospheric depth*

$$X = \int_h^\infty \rho(h')dh' \quad , \tag{2}$$

since the air density $\rho$ changes with the height h.

1. Show that $\frac{mg}{S} = gX$, where S is a surface. Remember that pressure $P = F/S$ (F is the force) and $\rho = m/V$ (V is a volume).

2. From Eq. 2, $\rho = -dX/dh$ (we choose the "-" sign because the density decreases as altitude increases). Using the law of perfect gases $PV = kT$, show that the variation of temperature with altitude is

$$T(h) = -\frac{M}{k}\frac{gX}{dX/dh} \quad , \tag{3}$$

   where M is the molecular mass of air ($\approx 10^{-23}$g) and k the Boltzmann constant.

3. Under the *isothermal approximation* where we consider T approximately constant with the altitude ($T(h) \sim T$), find a solution to the differential equation Eq. 3.

4. In your solution, identify the *atmospheric scale height* $h_0 = kT/Mg$ and calculate its magnitude in $km$ assuming $T = 290K$.

The above formulas allow you to make calculations about the interaction of cosmic rays considering the atmosphere as a calorimeter with a varying density. $h_0$ sets the scale over which the atmospheric depth varies significantly.