

Neutrino Astrophysics



Introduction

- General considerations
- Neutrino interactions
- The solar neutrino problem
- Various experiments before the solution
- What if neutrino oscillate?
- SNO experiment and discovery of neutrino oscillation

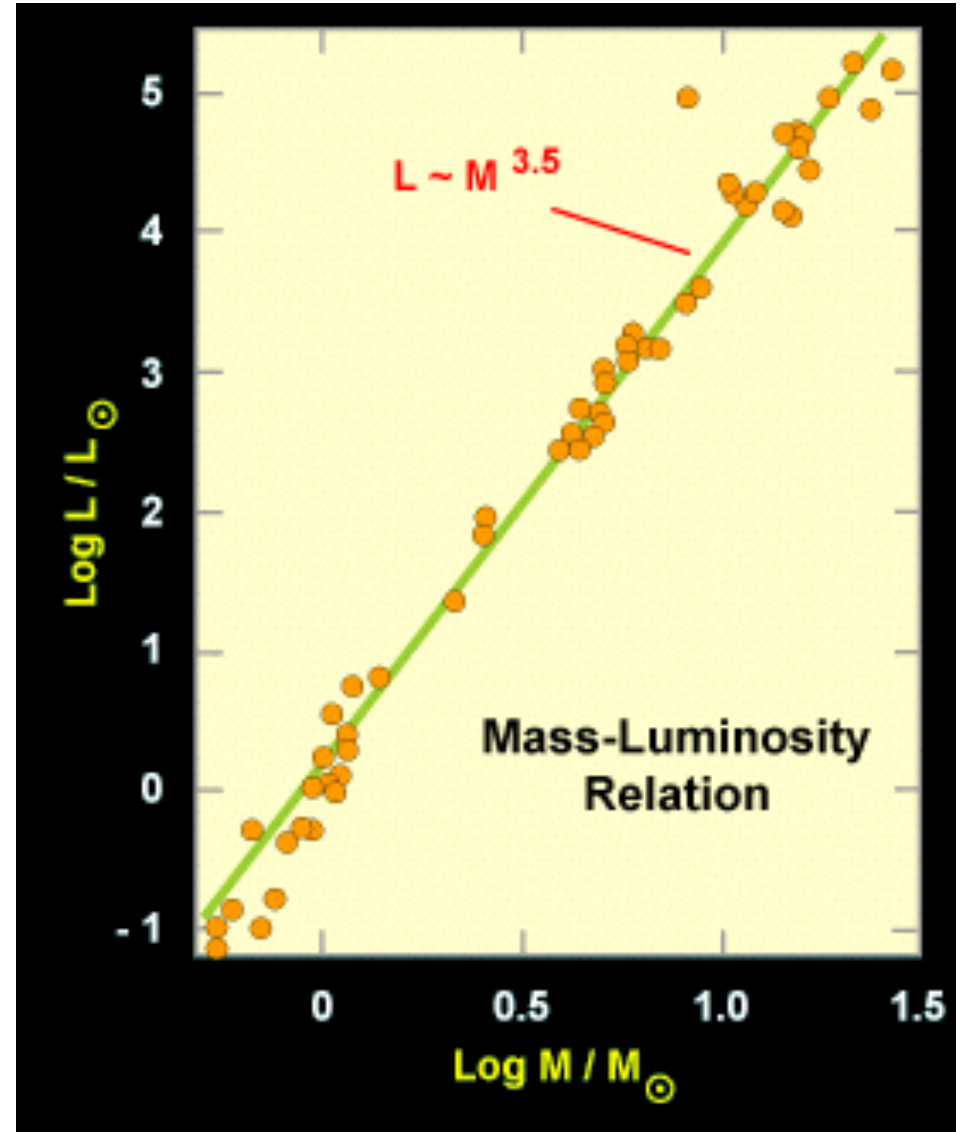
Optical Astronomy Considerations

Observations from optical astronomy give only indirect evidence for the nuclear processes occurring inside a star:

Observation 1: there is strong dependence of the luminosity on mass. This points towards a tunnelling process through the Coulomb barrier for nuclear fusion to occur.

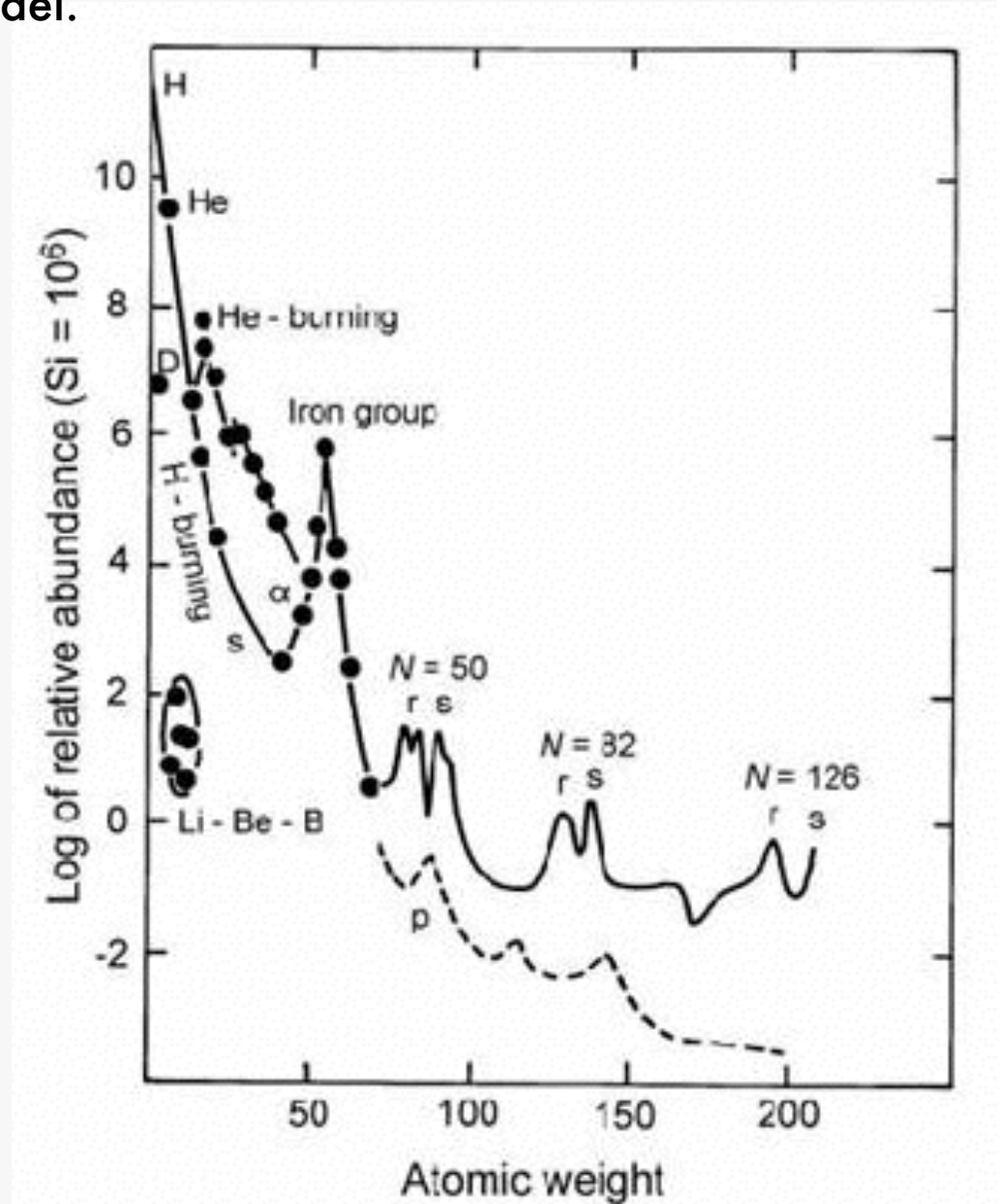
Tunnel Effect $L = \frac{dE}{dt} \sim e^{-1/kT}$

A small change in temperature results in a large change in luminosity.



Optical Astronomy Considerations

Observation 2: peaks in elemental abundances corresponding to magic numbers in the nuclear shell model.



Further Considerations

Gamma rays, X-rays, optical, infrared, UV, radio waves all have mean free path in stellar matter much smaller than the diameter of the star, so only give direct information about what's happening near the surface of the star.

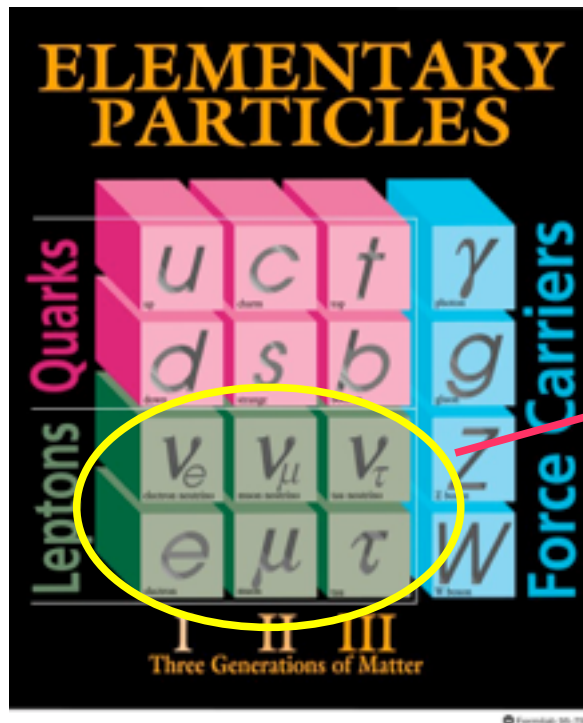
1 MeV neutrinos have mean free path of ~ 400 light years of lead, which is much larger than the diameter of a star.

The star and the earth are virtually transparent to neutrinos.

Neutrinos allow us to “see” what's actually happening in the core of a star, and their detection would verify the nuclear processes that are believed to be occurring there.



Neutrino Flavours and Interactions



FERMIONS			matter constituents spin = 1/2, 3/2, 5/2, ...		
Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c ²	Electric charge	Flavor	Approx. Mass GeV/c ²	Electric charge
ν_e electron neutrino	$<1 \times 10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	-1	d down	0.006	-1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	-1	s strange	0.1	-1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	-1	b bottom	4.3	-1/3

- * Leptons are point-like particles that do not feel the strong nuclear interaction, only the EM and weak interaction.
- * Leptons come in 3 families (flavours)“ e, μ , τ
- * You can think of a neutrino as a “lepton that has lost its electric charge”, and thus feels only the weak interaction.

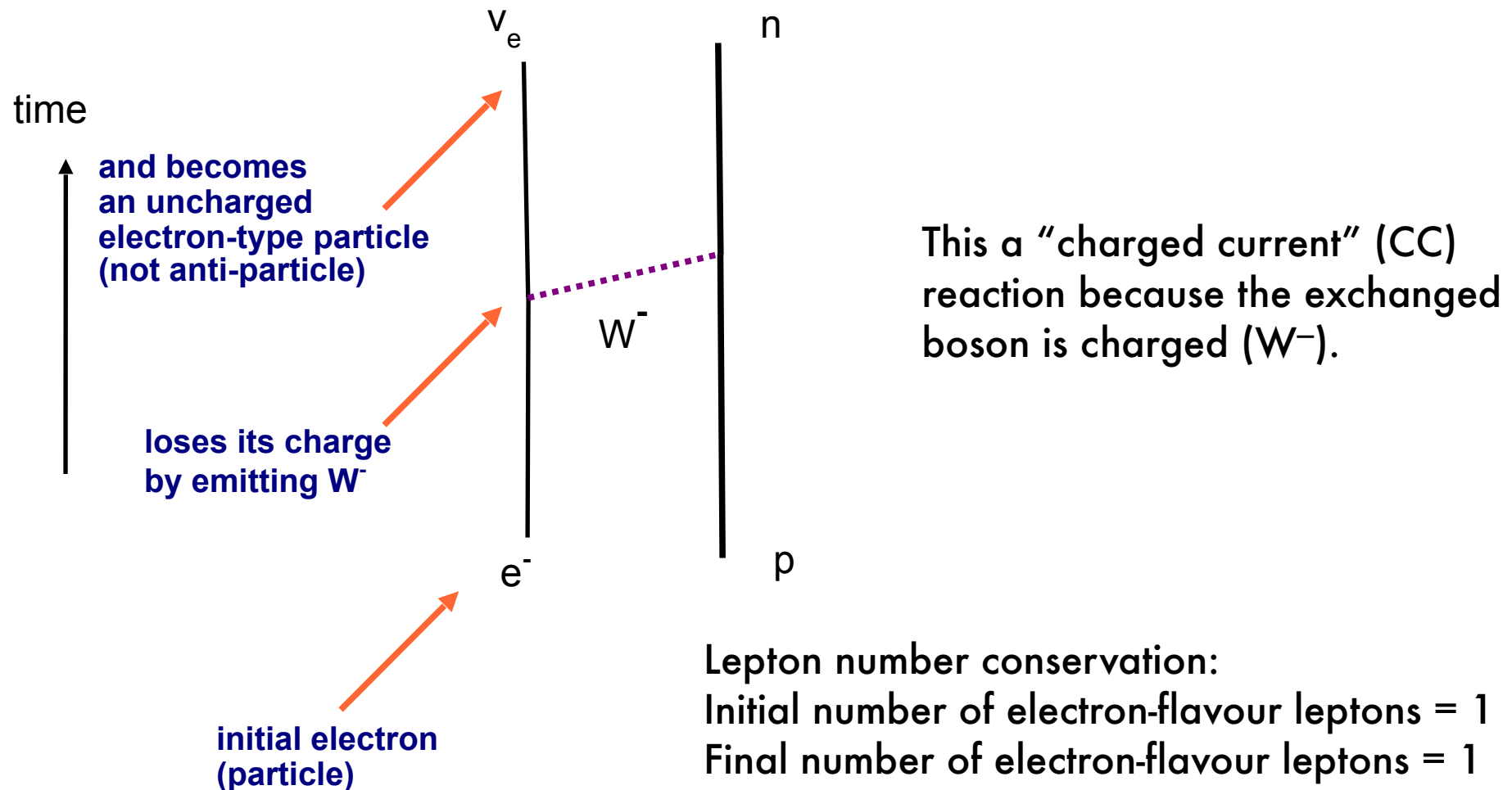
Weak interactions preserve the generation of a lepton. (“lepton flavour conservation”). *

* **except for neutrino oscillations**

Neutrino Interactions

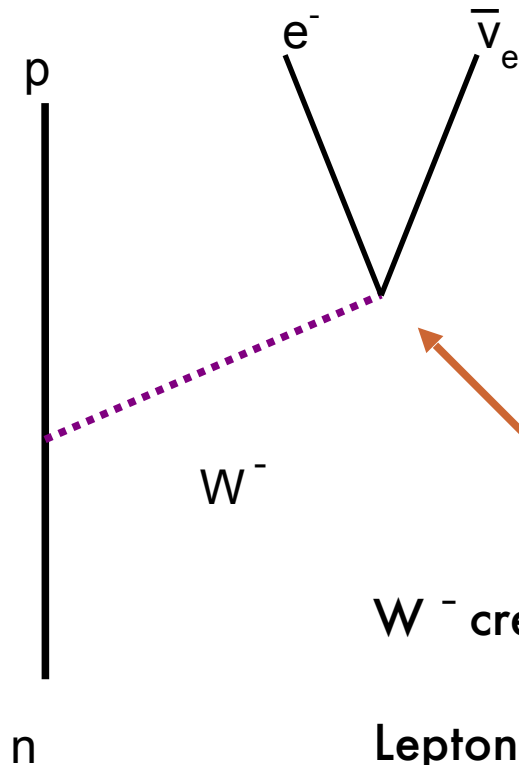
The neutrino as a “lepton that lost its charge” becomes more obvious if we draw the Feynman diagrams

Example 1: $e^- + p \rightarrow \nu_e + n$



Neutrino Interactions

Example 2: neutron beta-decay $n \rightarrow p + e^- + \bar{\nu}_e$



W^- creates an electron-type lepton-anti-lepton pair.

Lepton flavour conservation:

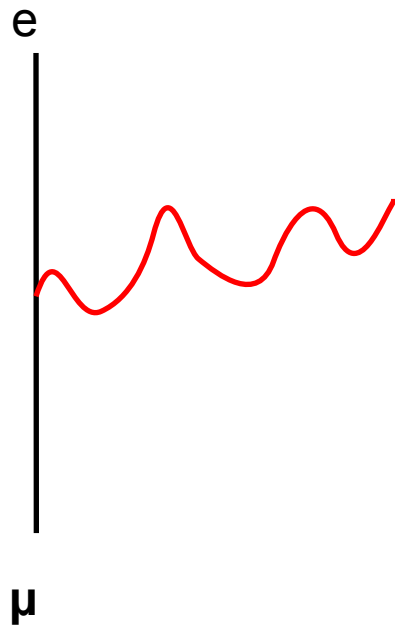
Initial number of electron-like leptons = 0

Final number of electron-like leptons

= (1 for the e^-) + (-1 for the anti-neutrino) = 0

Neutrino Interactions

Example 3: $\mu^- \rightarrow e^- + \gamma$: does not happen in the SM



This decay is forbidden because it violates lepton flavour conservation i.e. it changes a 1st flavour lepton into a 2nd flavour lepton.

Lepton flavour not conserved in this decay!

Initial number of electron-flavour leptons=0

Final number of electron-flavour leptons=+1

Initial number of muon-flavour leptons=+1

Final number of muon-flavour leptons=0

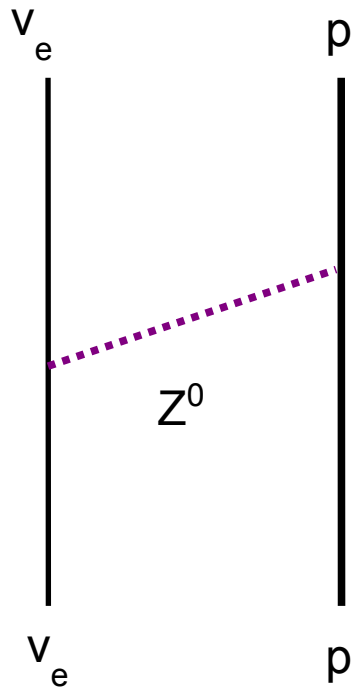
This has been experimentally searched for (and it still is!), and the current experimental limit on the branching ratio

$$R = \sigma(\mu \rightarrow e\gamma) / \sigma(\text{all decays})$$

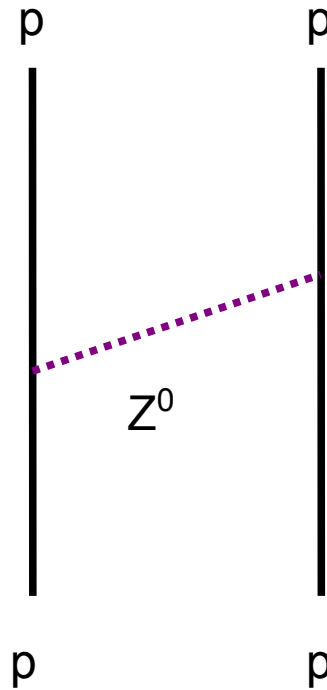
is less than 2.4×10^{-12} (MEG experiment at PSI).

Neutrino Interactions

There are also neutral current reactions where the exchanged boson is a Z^0 ,
Examples:



elastic neutrino-proton
scattering



weak interaction between two protons (this is in
addition to the much stronger strong and EM
interactions). This is responsible for the small
parity mixing in nuclei.

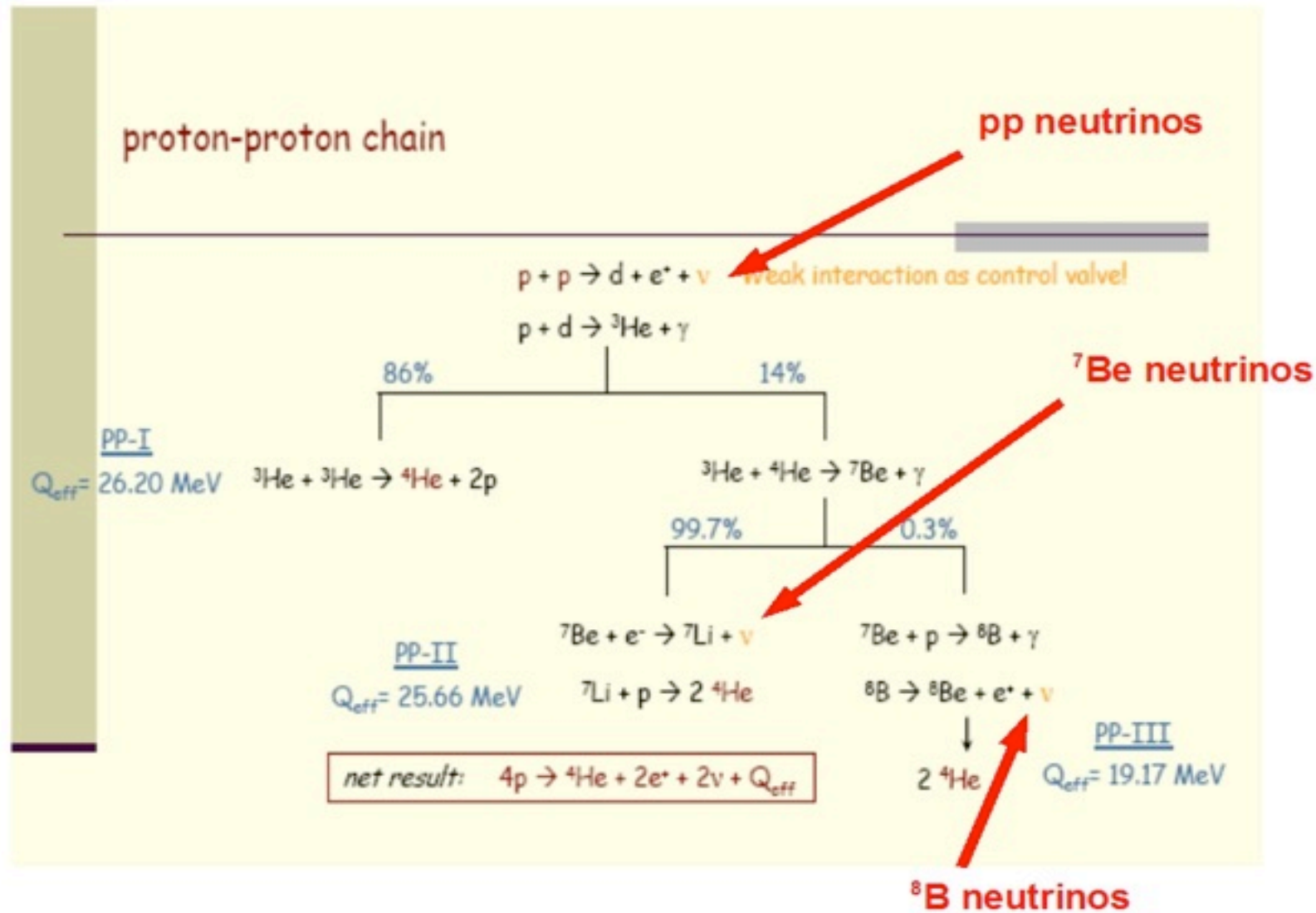
The solar neutrino problem

If the p-p chain of fusion reactions is the main mode of energy production in the Sun, we should measure the expected flux of electron neutrinos coming from the Sun.

But in a variety of neutrino detectors, the measured flux of neutrinos is much smaller than predicted by the Standard Solar Model

Why is that?

Neutrino production in the Sun



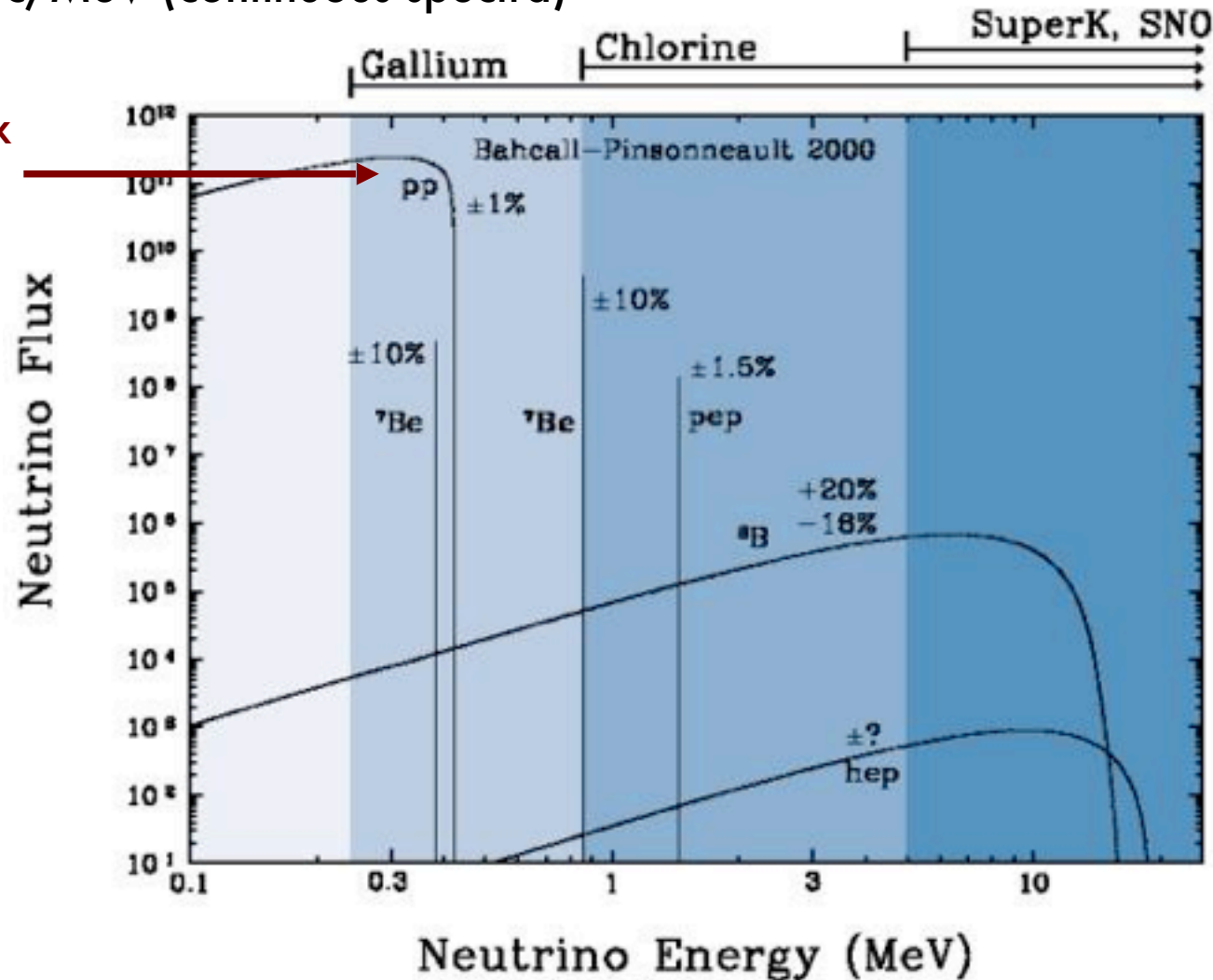
1 time in 400, instead of $p+p \rightarrow d+e^+\nu$, have $p + e^- + p \rightarrow d + \nu$ **← pep neutrinos**

${}^3\text{He} + p \rightarrow {}^4\text{He} + e^+ + \nu$ **← hep neutrinos**

Neutrino production in the Sun

Solar neutrino flux (Standard Solar Model) in number/cm²/sec (discrete lines) or number/cm²/sec/MeV (continuous spectra)

pp neutrino flux
 6×10^{10} /cm²/s



Note that the reactions with 2-body final states have discrete neutrino energies, while the reactions with 3-body final states have continuous spectra.

The Homestake experiment



R. Davis (1914–2001)

The first solar neutrino detector used the reaction



Natural Chlorine is 24% ${}^{37}\text{Cl}$, and 76% ${}^{35}\text{Cl}$, so in one molecule of perchlorethylene (C_2Cl_4 , dry cleaning fluid), there is on average about 1 atom of the desired ${}^{37}\text{Cl}$ isotope.

The chemist **Ray Davis** built a solar neutrino detector consisting of 100,000 gallons (615 tons) of cleaning fluid located 1480 metres underground in the **Homestake** Mine in South Dakota.

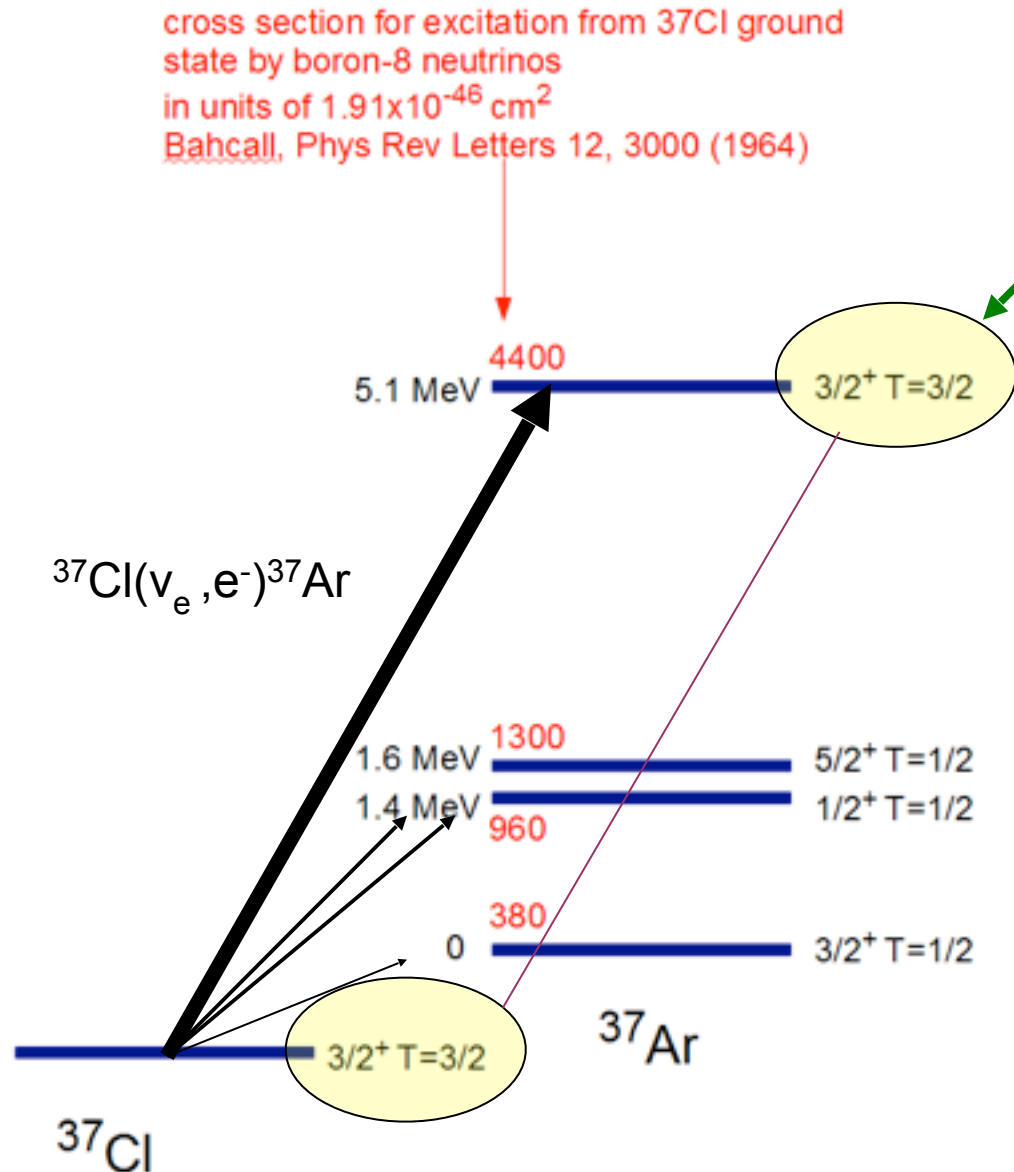
Unit: 1 SNU (solar neutrino unit) = 10^{-36} capture per second per target atom

If the Sun's energy came mainly from the pp chain as expected from the Standard Solar Model, the neutrino flux on Earth and the capture rate should be 7.8 SNU.

In the 100,000 gallons of C_2Cl_4 , 1 SNU = 1 capture every 6 days.

so according to the Standard Solar Model, there should be 39 captures per month, i.e. 39 atoms of ${}^{37}\text{Ar}$ produced in 100,000 gallons of cleaning fluid per month!

The Homestake experiment



This is the isobaric analog state of the ^{37}Cl ground state – same space and spin wf, same spin, parity and isospin.

Only difference is that a neutron has replaced a proton.

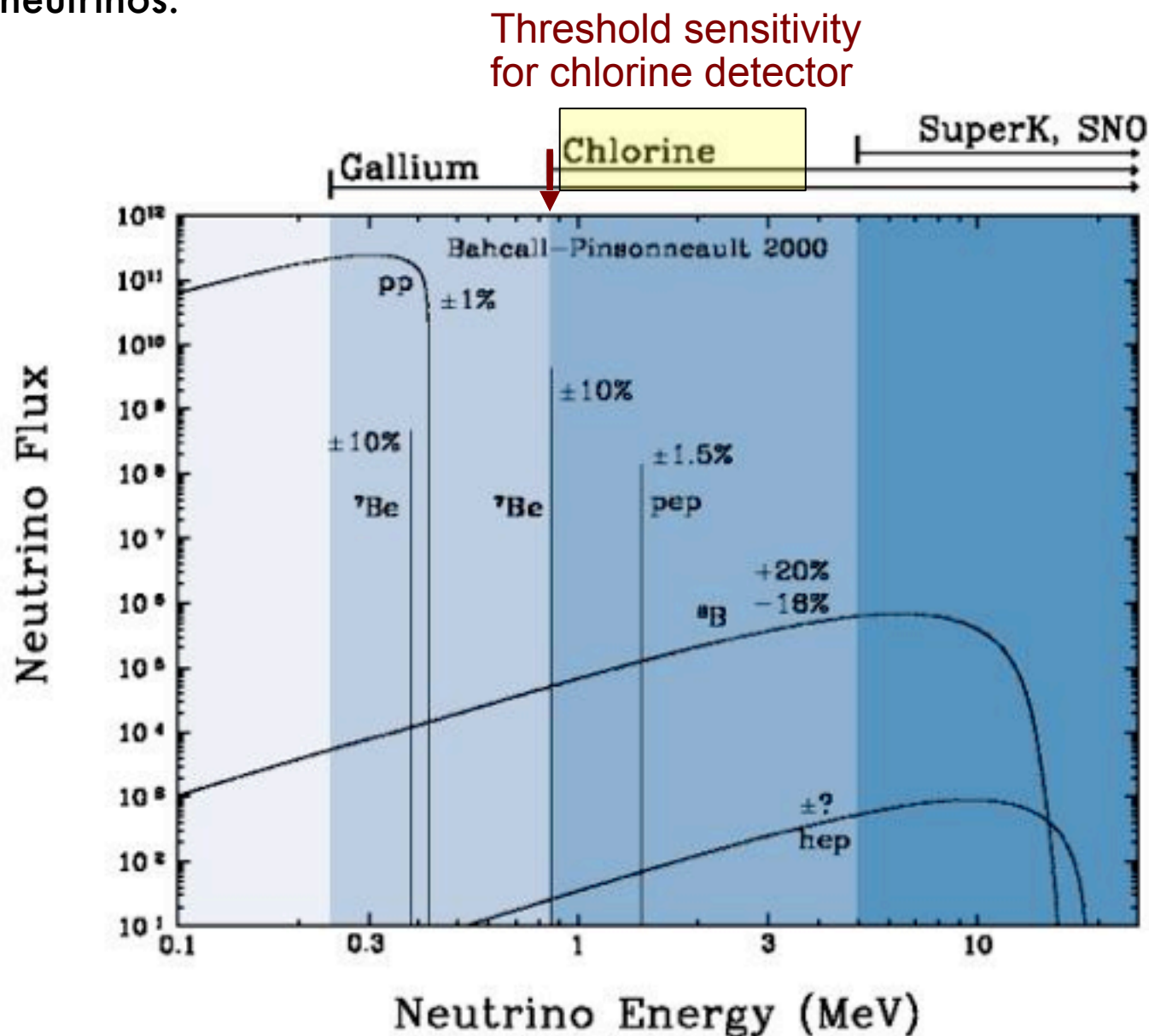
This means there is maximum overlap between the initial and final wf – a so-called “super-allowed” transition with extra-large cross section.

All the neutrino has to do is gently nudge one of the neutrons and change it into a proton, with no drastic re-arrangements of the nucleus.

It was John Bahcall's realization of this in 1964 that made Davis' chlorine detector feasible – transitions to the ground state of ^{37}Ar alone would be too slow to be practical.

Neutrino production in the Sun

Note that the chlorine detector is not sensitive to the pp neutrinos, only to the less numerous ${}^7\text{Be}$ and ${}^8\text{B}$ neutrinos.



The Homestake experiment

Detector in Homestake Mine



Ray Davis (L) and John Bahcall (R)



Result

Result:

Only ~33% of anticipated number of ^{37}Ar atoms seen (2.56 instead of 7.6 SNU)

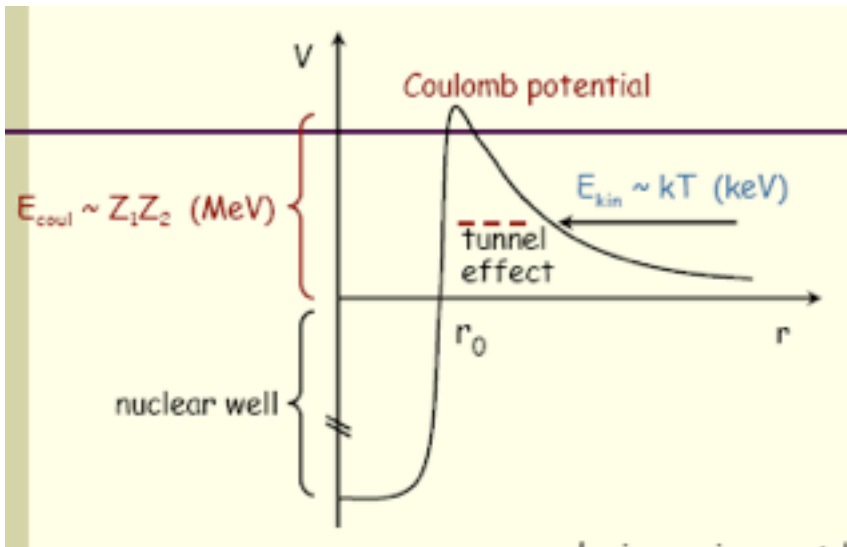
A deficiency of solar neutrinos observed!

What could be wrong?

- 1) Standard Solar Model calculation is wrong?
- 2) something is happening to the neutrinos between the Sun's core and the detector here on Earth (neutrino oscillations)

Uncertainties in the Standard Model?

Flux (of ^8B neutrino) $\sim T^{25}$: extremely sensitive to temperature, because all nuclear fusion reactions in the Sun are inhibited by the Coulomb barriers of the reacting nuclei.



Recall from alpha decay:

$P \sim e^{(-2\alpha d)}$ where d =barrier thickness

small change in energy \rightarrow large change in tunnelling probability P

The $^7\text{Be}(p,\gamma)$ reaction responsible for the ^8B neutrinos has a higher Coulomb barrier than the pp reaction, so is much more sensitive to small changes in temperature. A small change in temperature that has negligible effect on the pp reaction could drastically change the flux of ^8B neutrinos.

Perhaps the core of the Sun is a bit cooler than expected?
Unexpected convection mixing of cooler material further away from the core reducing the core temperature ("solar spoon")?

- Bahcall claimed that such effects were ruled out by other constraints, e.g. total solar irradiance, helioseismology

More Results

The ^8B neutrinos are only a small side-branch of the pp chain, and they could even disappear without much affecting the power output of the Sun.

A detector made of Gallium (liquid metal) is sensitive to the pp neutrinos via



SAGE and GALLEX detectors: chemically separate Ge from 60 and 30 tons of liquid Gallium metal

Table 1
Standard Model Predictions (BP98): solar neutrino fluxes and neutrino capture rates, with 1σ uncertainties from all sources (combined quadratically).

Source	Flux ($10^{10} \text{ cm}^{-2}\text{s}^{-1}$)	Cl (SNU)	Ga (SNU)
pp	$5.94 \left(1.00^{+0.01}_{-0.01}\right)$	0.0	69.6
pep	$1.39 \times 10^{-2} \left(1.00^{+0.01}_{-0.01}\right)$	0.2	2.8
hep	2.10×10^{-7}	0.0	0.0
${}^7\text{Be}$	$4.80 \times 10^{-1} \left(1.00^{+0.09}_{-0.09}\right)$	1.15	34.4
${}^8\text{B}$	$5.15 \times 10^{-4} \left(1.00^{+0.19}_{-0.14}\right)$	5.9	12.4
${}^{13}\text{N}$	$6.05 \times 10^{-2} \left(1.00^{+0.19}_{-0.13}\right)$	0.1	3.7
${}^{15}\text{O}$	$5.32 \times 10^{-2} \left(1.00^{+0.22}_{-0.15}\right)$	0.4	6.0
${}^{17}\text{F}$	$6.33 \times 10^{-4} \left(1.00^{+0.12}_{-0.11}\right)$	0.0	0.1
Total		$7.7^{+1.2}_{-1.0}$	129^{+8}_{-6}

Bahcall and Basu, arXiv:astro-ph 9805135

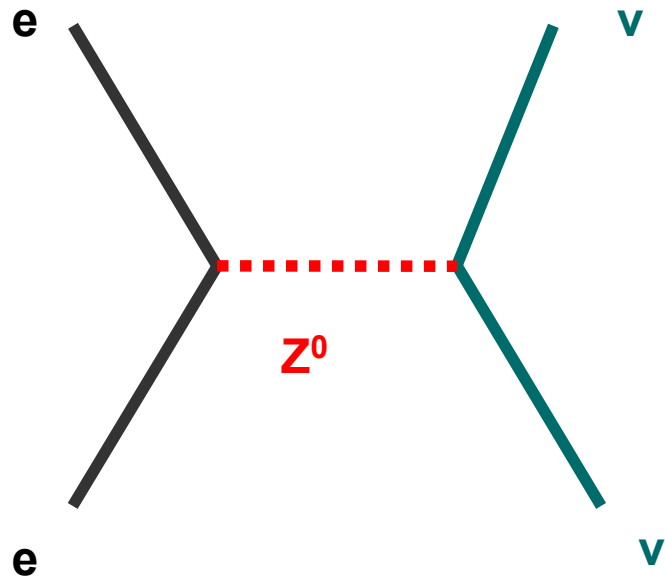
A deficit observed again!

Considerations

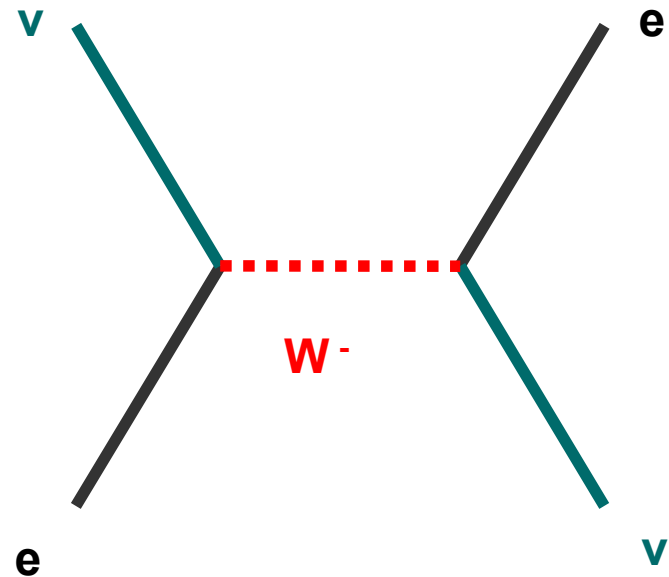
So far these are radiochemical experiments – you only measure the amount of the chemical products of neutrino interactions after months of exposure to the alleged flux of solar neutrinos.

You can't actually see where the neutrinos are coming from, in real time. And since you can't turn off the Sun to do a “beam off” measurement, you can't be absolutely sure that what you're measuring is the effect of solar neutrinos at all.

The Super-Kamiokande water Cherenkov detector can detect solar neutrinos in real time by scattering of the neutrinos from atomic electrons. There is both a neutral (15%) and a charged current (85%) contribution:

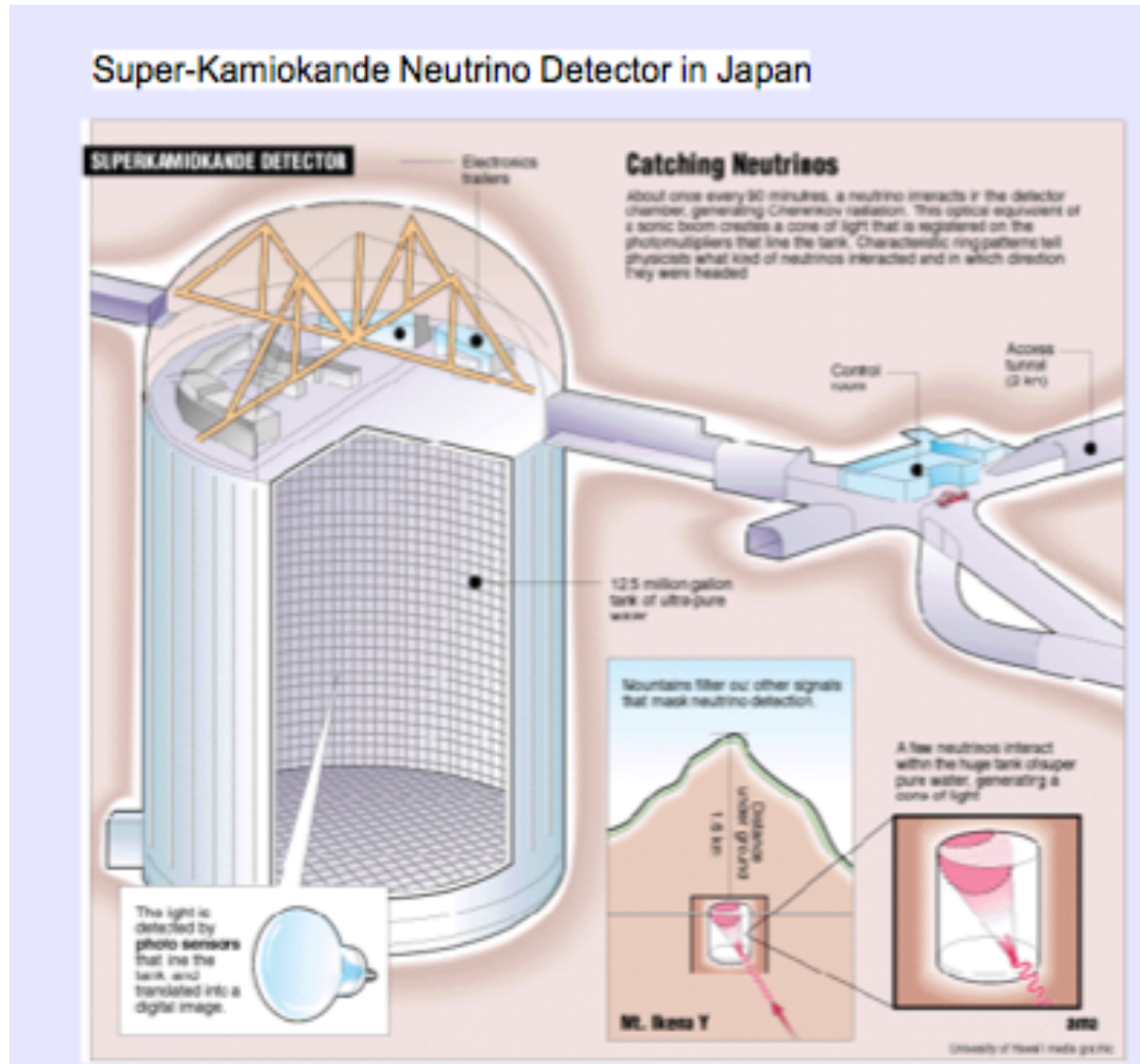


Neutral current: any flavor ν



Charged current: only ν_e contributes

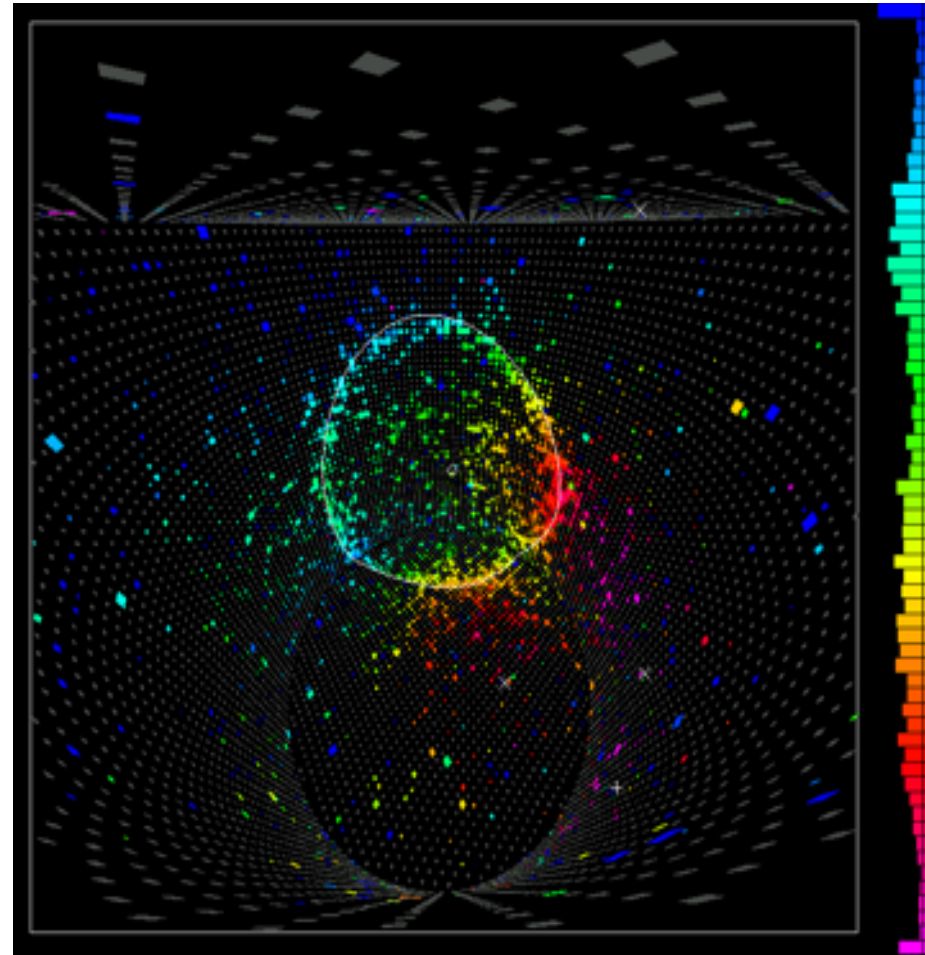
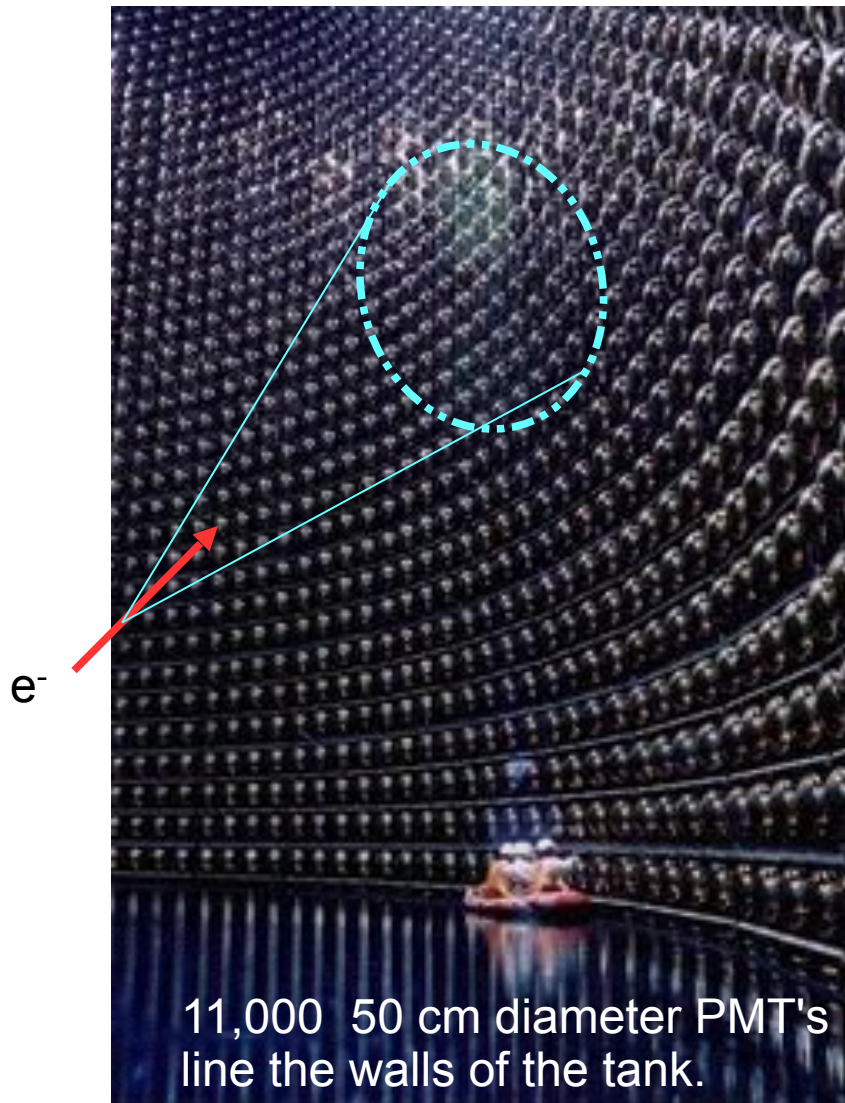
Super Kamiokande



39 m diameter x 41 m tall, 50,000 kilo-tonnes of ultra-pure water, under 1.6 km of rock to filter out the charged cosmic ray background.

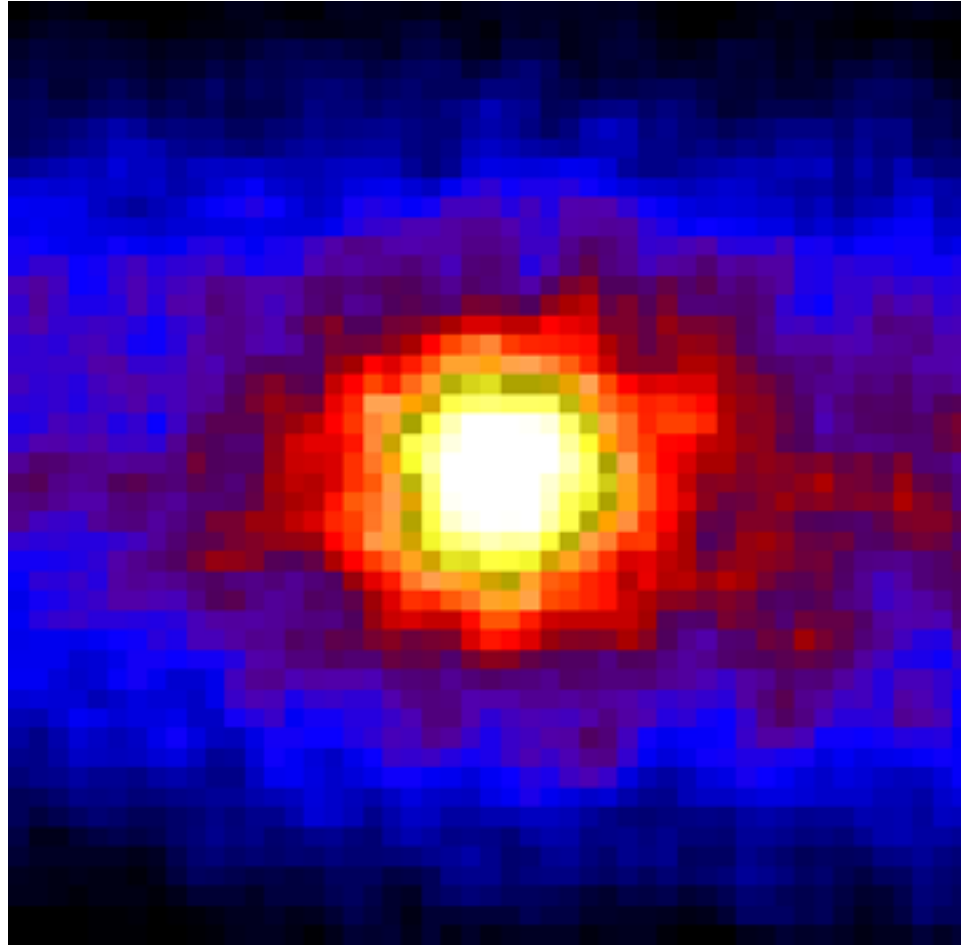
Super Kamiokande

Electron scattering by neutrinos is forward-peaked, so the direction of the emerging electron tells us (roughly) the direction of the incident neutrino. And the direction of the electron can be inferred by the direction of the cone of Cherenkov light!



Event display of a Cherenkov cone in Super-K. Each pixel is one photomultiplier tube. The colours encode the relative time of the light reaching that photomultiplier.

Super Kamiokande

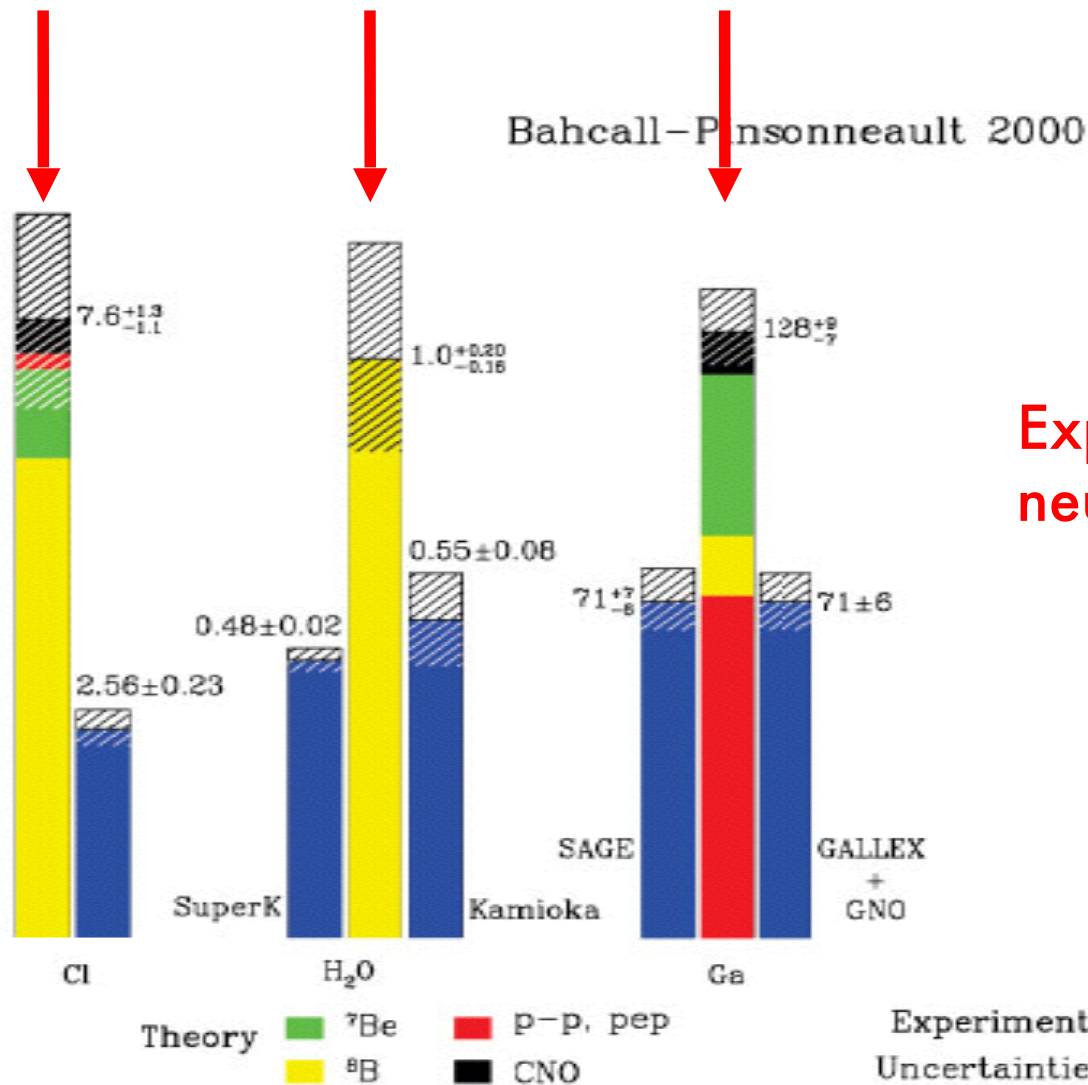


The Sun, imaged by observing the direction of neutrinos in SuperK. The angular resolution (pixel size) is about 2 degrees. The colours denote intensity.

The neutrinos are clearly coming from the Sun!

Experimental evidence

Chlorine Water Cherenkov Gallium

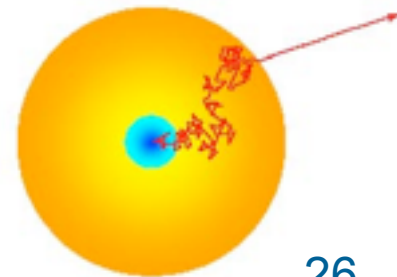


Experiments all show deficits of neutrinos compared to theory!

Considerations

Hard to believe at this point that the Standard Solar Model could be wrong. The pp neutrinos come from the pp chain that is believed to be the main mode of energy production in the Sun. Only way out is if:

- the pp chain is NOT the main mode of energy production (but calculations show the CNO cycle would produce 6 times MORE neutrinos in chlorine detector, not fewer)
- the Sun is not shining by nuclear fusion energy at all, and is therefore not billions of years old
- the Sun has temporarily stopped producing nuclear energy (photons produced in the core of the Sun take about 10,000 to 170,000 years to diffuse their way to the surface, but neutrinos come out immediately)
- or **something has happened to the neutrinos on their way from the Sun** (neutrino oscillations)

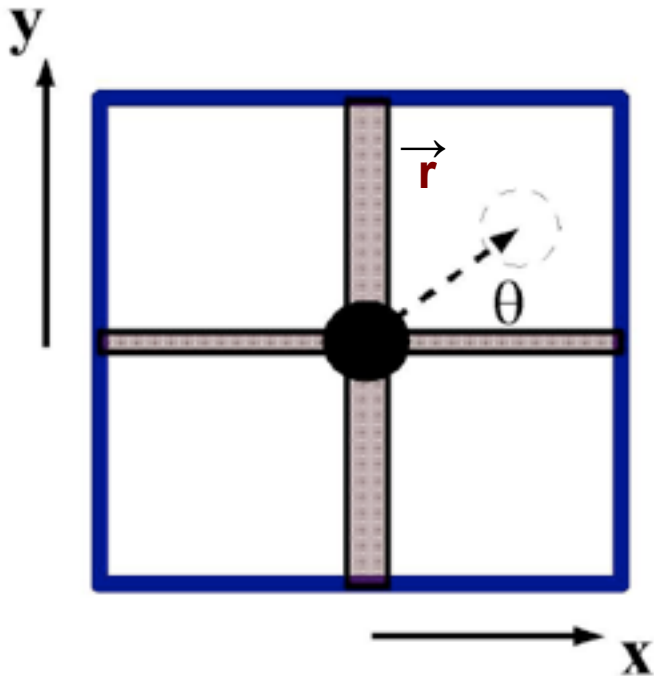


Neutrino Oscillations

Normal modes of vibrating systems

Consider the following mechanical system.

A mass M lies inside a square frame, constrained by elastic bands (or springs) in the x and y directions.



Suppose the spring constant of the elastics in the x and y directions are k_x and k_y .

The angular frequencies for vibrations along the x and y directions are given by those of a harmonic oscillator, i.e.

$$\omega_x = \sqrt{\frac{k_x}{M}} \quad \omega_y = \sqrt{\frac{k_y}{M}}$$

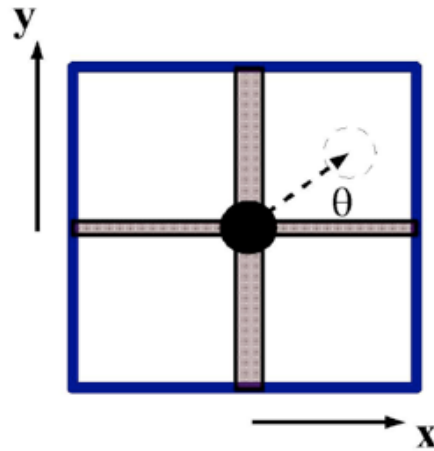
If we displace the mass in the x direction from equilibrium and let go, it will vibrate forever along the x direction with frequency ω_x .

Similarly, if we displace it from equilibrium in the y direction and let go, it will vibrate forever along the y direction with frequency ω_y .

Normal modes of vibrating systems

Modes that vibrate independently of each other are called the normal modes of the system. Once started in a normal mode, the system will continue vibrating in that mode forever. Vibrations in the x and y directions are hence normal modes of the system.

But suppose we initially displace the mass at some angle θ relative to the x axis.

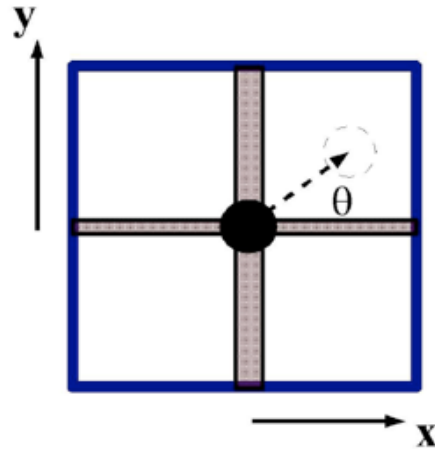


If the vector $r = (x, y)$ describes the displacement of the mass from the origin as a function of time, and $r_0 = (x_0, y_0)$ is the initial displacement, then at time t ,

$$x(t) = x_0 \cos(\omega_x t) \qquad y(t) = y_0 \cos(\omega_y t)$$

If the elastic bands in the x and y directions were of equal strength, then $\omega_x = \omega_y$, the vibrations in the x and y directions would remain in synchronization, and the mass will continue vibrating at the same angle θ forever.

Normal modes of vibrating systems



If the elastic bands in the x and y directions are of **different strengths**, then $\omega_x \neq \omega_y$ and the vibrations in the x and y directions will not be synchronized, i.e. a vibration in the x direction will not reach maximum amplitude at the same time as a vibration in the y direction.

and hence the mass will NOT continue vibrating in the initial direction specified by angle θ .

The x- and y-vibrations will go in and out of phase with each other with the beat frequency

$$\omega_{\text{beat}} = \omega_1 - \omega_2 = \Delta\omega$$

Derivation of the beat frequency

After one full cycle in the x direction, the y-direction will have completed $(\omega_y/\omega_x) = (\omega_x + \omega_y - \omega_x)/\omega_x = 1 + (\omega_y - \omega_x)/\omega_x \equiv 1 + \Delta\omega/\omega_x \equiv 1 + \delta$ cycles

Let $T_x = 2\pi/\omega_x$ be the period of one cycle in the x direction. We make a table of how many cycles have been completed in the x and y directions

Elapsed time	No. of x cycles	No. of y cycles
T_x	1	$1 + \delta$
$2T_x$	2	$2 + 2\delta$
\dots	\dots	\dots
NT_x	N	$N + N\delta$

How long does it take for the x and y vibrations to get back into phase?

From the table above, when $N\delta = 1$, there will be exactly N x-cycles completed and $(N+1)$ y-cycles completed. Thus

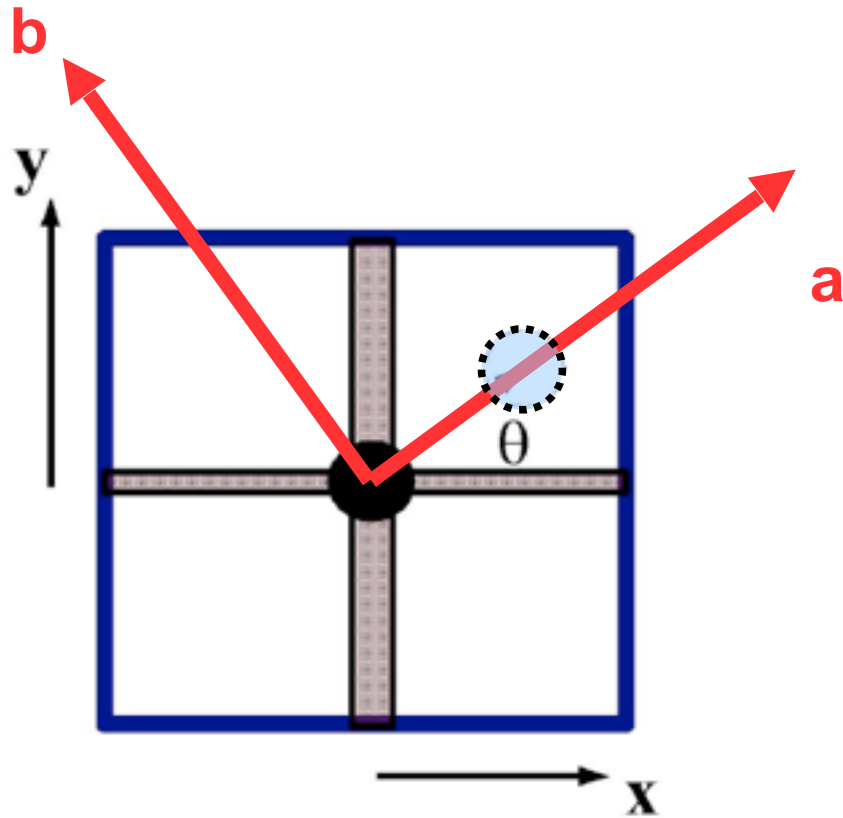
$$1 = N\delta = N \Delta\omega/\omega_x \quad \text{or} \quad N = \omega_x/\Delta\omega$$

1 complete x-cycle takes time $1/f = 2\pi/\omega_x$

N complete x-cycles takes time $T_x = 2\pi N/\omega_x$ which corresponds to an angular beat frequency of $\omega_{\text{beat}} = 2\pi/T_x = \omega_x/N = \omega_x / (\omega_x/\Delta\omega) = \Delta\omega$

We obtain the well-known result that two vibration modes with frequencies ω_1 and ω_2 will be go in and out of phase with each other with beat frequency $\omega_{\text{beat}} = \omega_1 - \omega_2$.

Normal modes of vibrating systems



We define x and y to be the normal directions, i.e. vibrations along x and y will be normal modes of vibration.

We define the skew directions a and b by setting a to be the initial direction of displacement of the mass, and b to be orthogonal to a

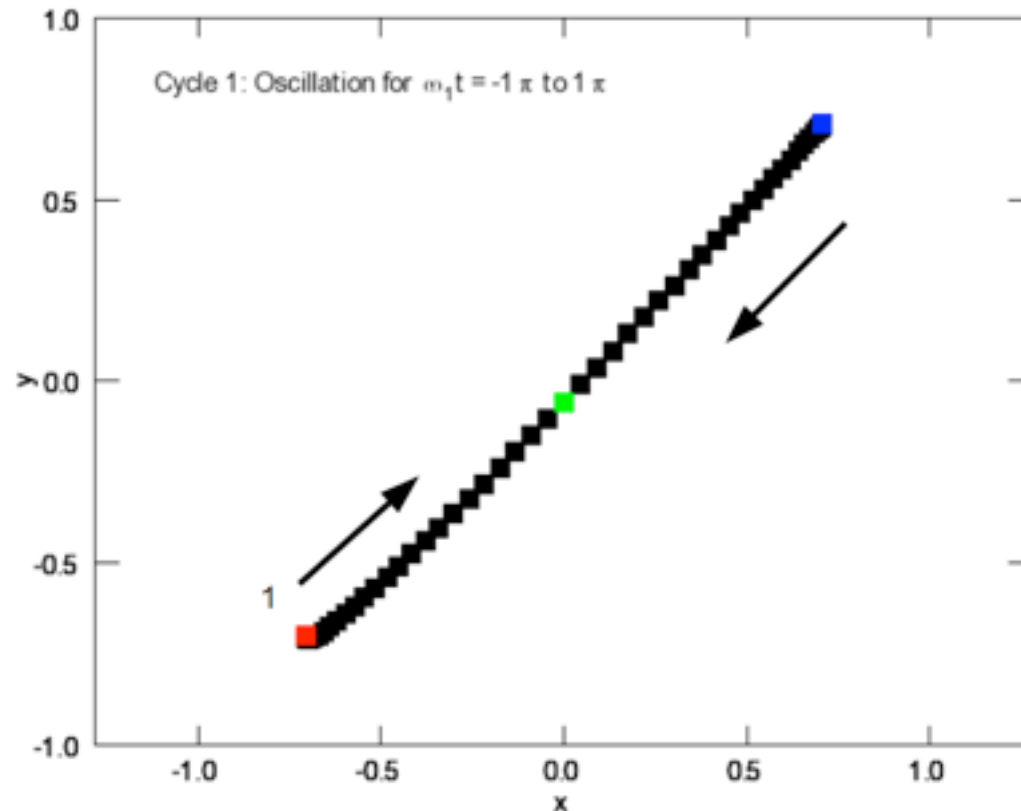
We call θ , the angle between the normal and skew axes, the mixing angle.

Graphical example

Suppose we take the initial angle $\theta = 45^\circ$, $\omega_x = 1.00$, $\omega_y = 1.05$.

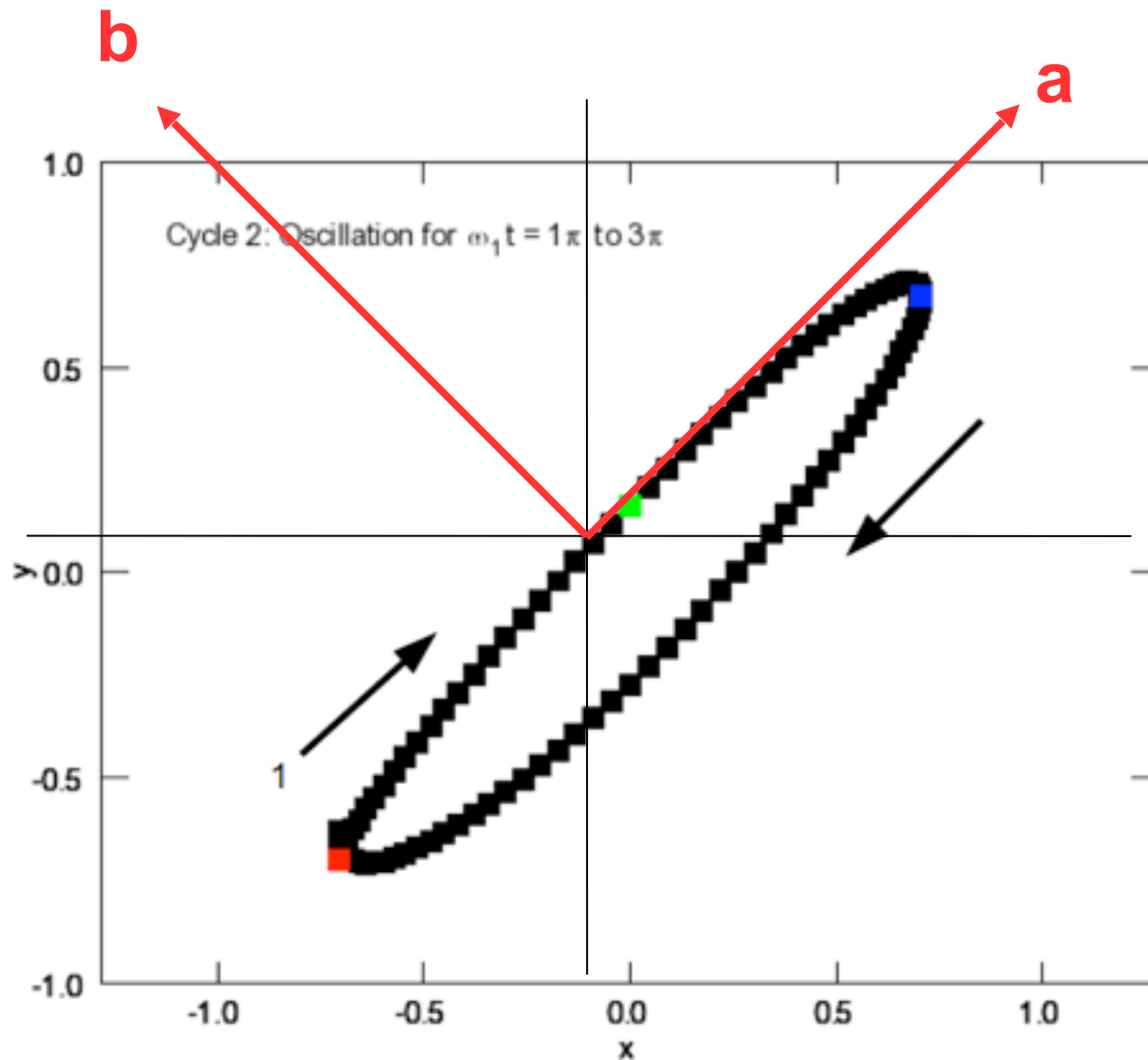
We expect the beat frequency to be 0.05, i.e. it will take 20 full cycles for the x and y vibration modes to fall back into phase.

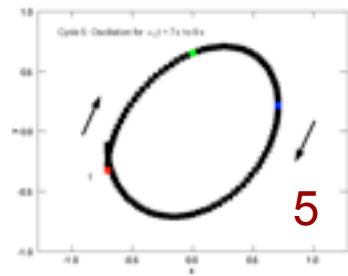
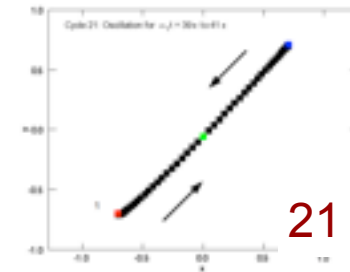
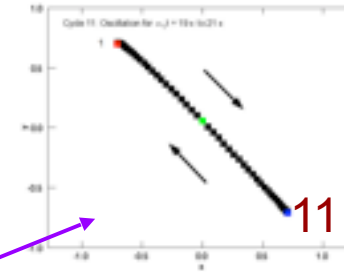
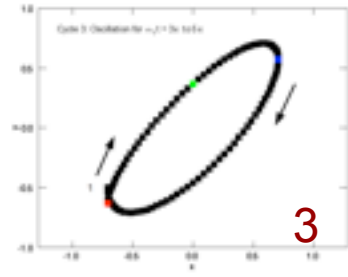
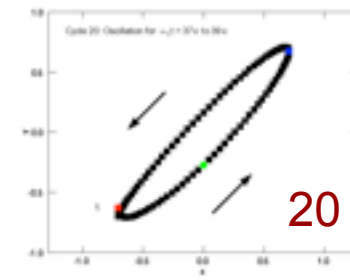
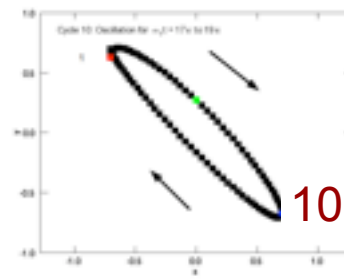
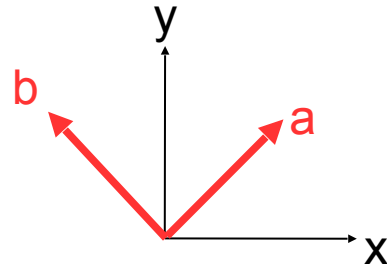
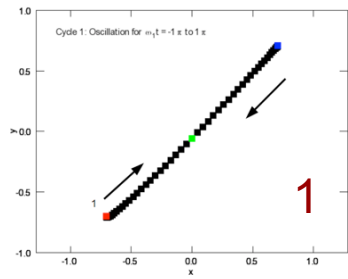
The position of the mass for the first cycle of $\omega_x t$ from $-\pi$ to $+\pi$ looks like this: almost a perfect line along the α axis (45° line).



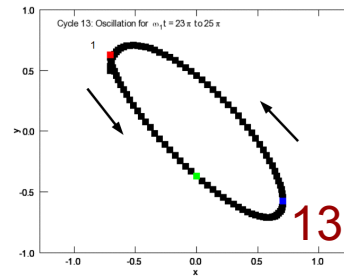
Graphical example

But cycle #2, for $\omega_x t = \pi$ to 3π looks like this: no longer purely a vibration along the **a** axis, but now with a component of vibration along the **b** axis.

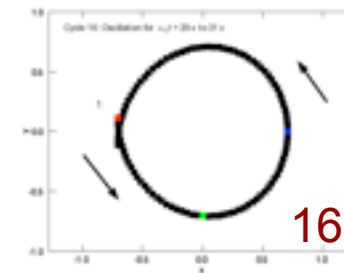
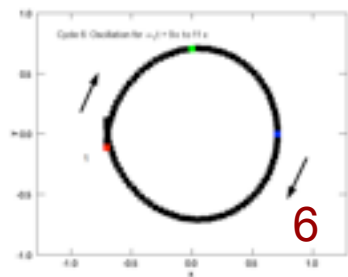




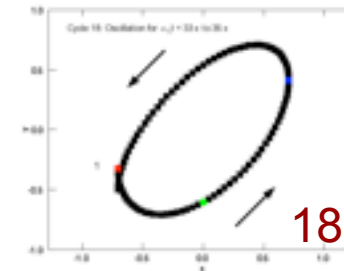
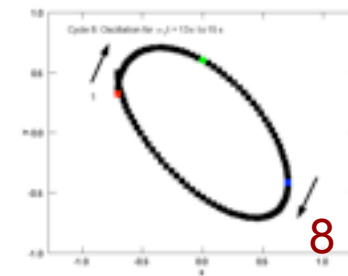
complete conversion
of a-vibration to
b-vibration
for mixing angle
 $\theta=45^\circ$



Graphs of motion for mixing
angle $\theta = 45^\circ$ through 21 cycles
of $\omega_1 = 1$ beat cycle.



Note that halfway through the
beat cycle, the vibration at
cycle 11 is exactly
orthogonal to
the original skew direction **a**
(no vibration along **a**, all along **b**),

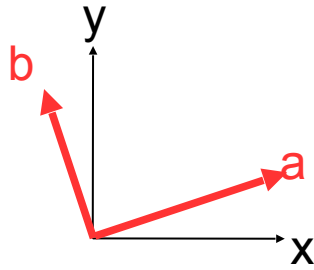
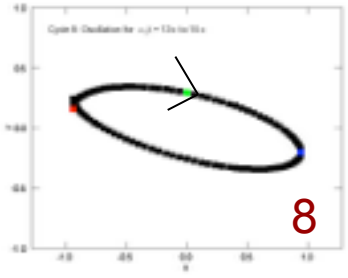
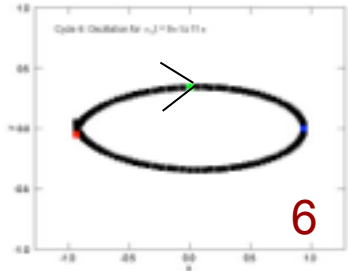
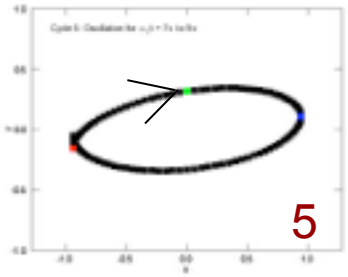
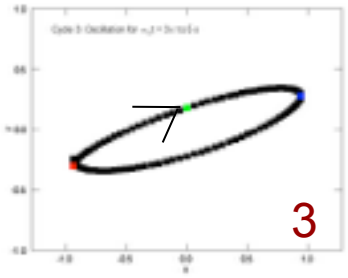
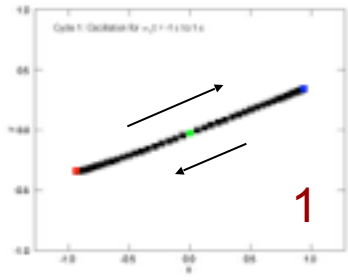


and that at the 1/4 and 3/4
points at cycles 6 and 16,
the motion is an equal mix
of vibrations along
skew directions **a** and **b**.

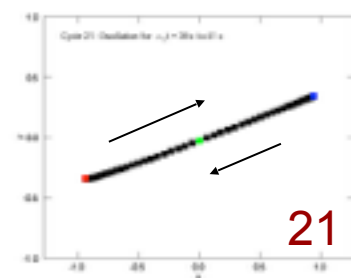
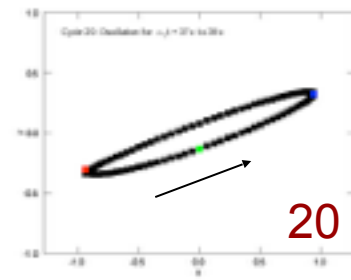
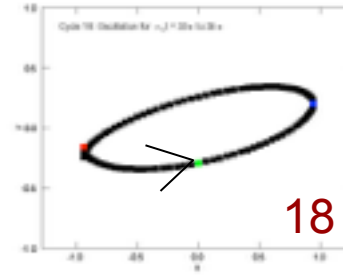
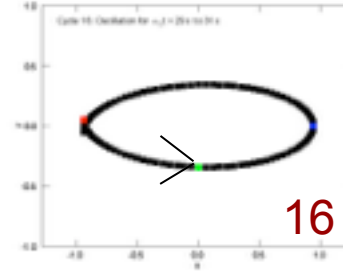
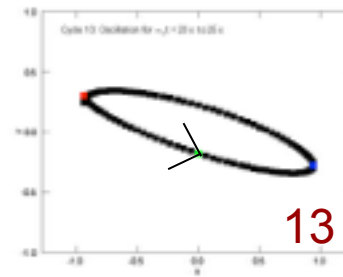
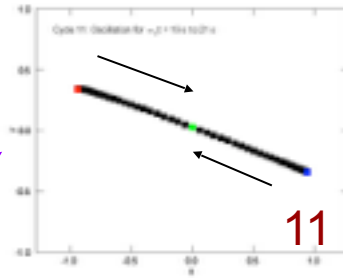
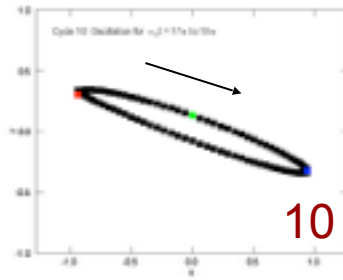
Graphical example

A mixing angle of $\theta = 45^\circ$ is as severe a mixing between the normal and skew axes as is possible, and as we have seen, half-way through the beat cycle, the initial vibration along the **a** direction has been transformed into a vibration purely along the orthogonal **b** direction.

What happens if we choose a less severe mixing angle, say $\theta = 20^\circ$?



partial conversion of a-vibration to b-vibration if mixing angle $\theta \neq 45^\circ$



Graphs of motion for mixing angle $\theta = 20^\circ$, through 21 cycles.

Note that halfway through the beat of 21 cycles, the vibration at cycle 11 has as little of initial skew direction a and as much of skew direction b as it ever gets, but there is still a component of vibration along direction a.

At the 1/4 and 3/4 marks at cycles 6 and 16, the motion is an unequal mix of motions along skew directions a and b, but weighted towards a.

Considerations

Thus, a maximal mixing angle of 45° (maximum angle between the normal and skew axes) results in the initial vibration along the **a** axis transforming completely at one point in the beat cycle to a vibration along the orthogonal **b** axis.

A less than maximal mixing angle results in the initial vibration along the **a** axis transforming partially to an vibration along the orthogonal **b** axis, but some component of vibration along the original **a** axis always remains.

We will now apply what we have learned about vibrating systems with two different frequencies, to an analogous quantum mechanical system, namely, a system of two neutrinos with different masses.

Neutrino Flavour Oscillations in Vacuum

A neutrino that is created as one flavour eigenstate (e.g. ν_e) can turn into a different flavour eigenstate (e.g. ν_μ or ν_τ).

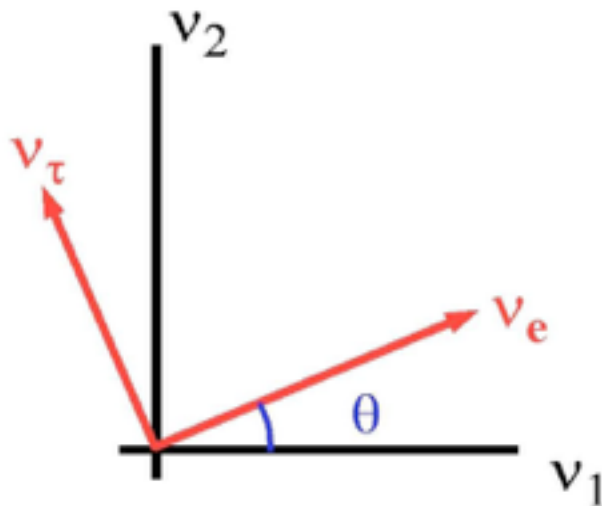
This happens because the **flavour eigenstates are not the same as the mass eigenstates**, i.e. neutrinos are created as a flavour eigenstates, but they propagate as mass eigenstates.

By analogy with our treatment of a vibrating mass, the flavour eigenstates are NOT the normal modes of the system, so a vibration along one flavour axis can partially or totally transform into a vibration along the orthogonal flavour axis.

Neutrino Flavour Oscillations in Vacuum

Imagine that the vibrations are not vibrations of a mass on a spring, but the phase vibrations of the wave-function of a neutrino.

To simplify things, imagine that there are only two neutrino flavours, say ν_e and ν_τ . Suppose that these are not the same as the normal modes of the system, but the normal modes are some linear combination of (ν_e, ν_τ) . Call these normal modes ν_1 and ν_2 . The picture now looks like this:



Just like the mass constrained by elastic bands, if we start vibrating in one of the normal modes ν_1 or ν_2 , the system will stay in that mode.

But if we start off in state ν_e , the system will NOT stay as pure ν_e , but will evolve with time into a linear combination of ν_e and ν_τ .

$$|\psi(t)\rangle = a(t) |\nu_e\rangle + b(t) |\nu_\tau\rangle \quad \text{where } a^2 + b^2 = 1 .$$

Neutrino Flavour Oscillations in Vacuum

What defines the “normal modes” ν_1 and ν_2 for neutrinos?

And why do they vibrate with different frequencies?

This happens if neutrinos are not massless but have small differences in mass.

In quantum mechanics, the normal modes are the eigenstates of the Hamiltonian operator H .

If wave-function Ψ is an eigenfunction of the Hamiltonian operator H , with energy eigenvalue E , then

$$H\Psi = E\Psi$$

and the wave-function Ψ can be written as a product of space and time components

$$\Psi(x,t) = \psi(x, t=0) e^{-i\omega t} \quad \text{where } \omega = E / \hbar$$

and hence, if ν_1 and ν_2 are energy eigenstates

$$\nu_1(t) = \nu_1(0) \exp(-iE_1 t)$$

$$\nu_2(t) = \nu_2(0) \exp(-iE_2 t) \quad \text{where we set } \hbar=1 \text{ for simplicity}$$

Neutrino Flavour Oscillations in Vacuum

Since momentum is conserved, as the neutrino oscillates between various linear combinations of ν_1 and ν_2 , the momentum must remain the same. Call this momentum p . Using $E^2 = p^2 + m^2$, in the limit that $m \ll p$, we get that

$$E_i = p + m_i^2 / 2p$$

and the vibration frequency $\omega_i = E_i / \hbar$ $i=1, 2$

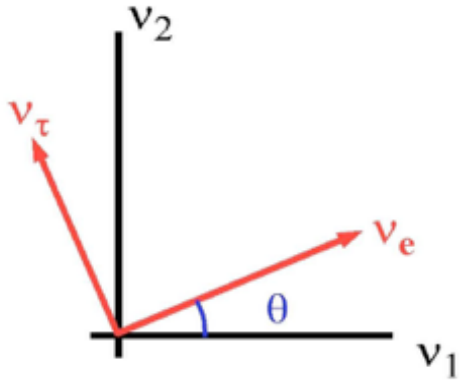
If the masses of the two neutrino eigenstates are different, $m_1 \neq m_2$, the two eigenstates ν_1 and ν_2 will vibrate with different frequencies $\omega_1 = E_1$ and $\omega_2 = E_2$ and exhibit a beat frequency

$$\omega_{beat} = (m_1^2 - m_2^2) / 2p = \Delta(m^2) / 2p$$

This is exactly analogous to the skew oscillators we examined earlier, except in place of directions "a" and "b" we write " ν_e " and " ν_τ ".

If the initial vibration direction ν_e is at some angle θ to the normal mode ν_1 then the subsequent vibrations will be a changing mixture of ν_e and ν_τ – that's what neutrino flavour oscillations are!

Neutrino Flavour Oscillations in Vacuum



From this figure, the flavour eigenstates (ν_e and ν_τ) are related to the mass eigenstates (ν_1 and ν_2) by a rotation matrix

$$\begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

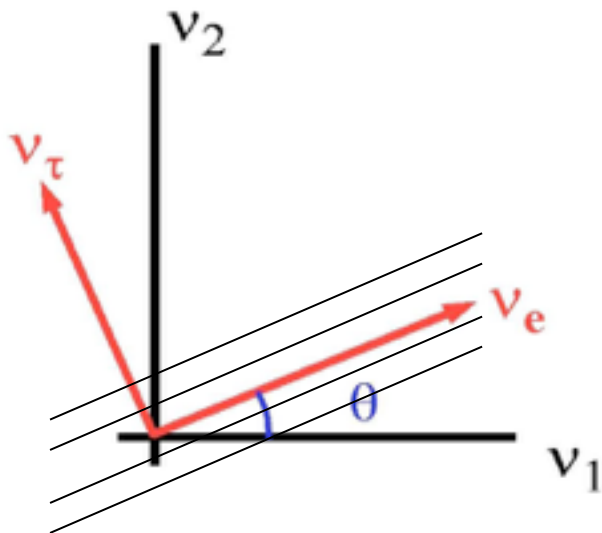
The angle θ is the “mixing angle”; it tells, e.g. how much mass eigenstate ν_1 there is in the flavour eigenstate ν_e .

Neutrino Flavour Oscillations in Vacuum

So, if we start off with a beam of pure ν_e (which is what the Sun produces), and let that beam travel for a while, what arrives at the earth is not pure ν_e , but a mixture of ν_e and ν_τ !

This change of flavor is what we call **neutrino flavour oscillations** (“neutrino oscillations” for short).

If the detector is sensitive **ONLY** to ν_e (which chlorine and gallium both are, and SuperK mostly), then a deficit of neutrinos will be observed.



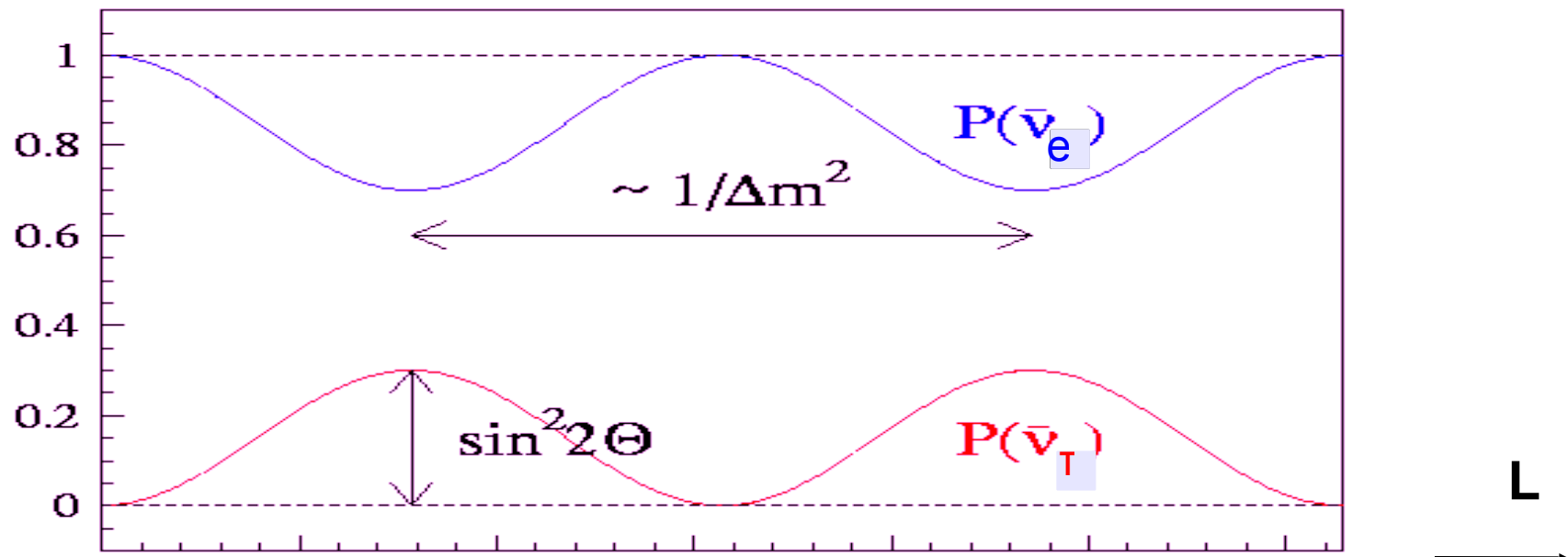
A chlorine or gallium neutrino detector is like a polaroid filter which admits only vibrations along the ν_e direction.

Neutrino Flavour Oscillations in Vacuum

$$P = 1 - \sin^2 2\theta \cdot \sin^2 \left(1.27 \Delta m^2 \frac{L}{E_\nu} \right)$$

θ = mixing angle
 Δm^2 in eV^2
 L in km
 E in GeV

so in our picture with two types of neutrinos, ν_e and ν_τ and starting with pure ν_e the relative numbers of the two types is shown below. Note that the depth of the flavour oscillation is governed by $\sin^2 2\theta$, whereas the wavelength of the flavour oscillation is governed by $1/\Delta(m^2)$.



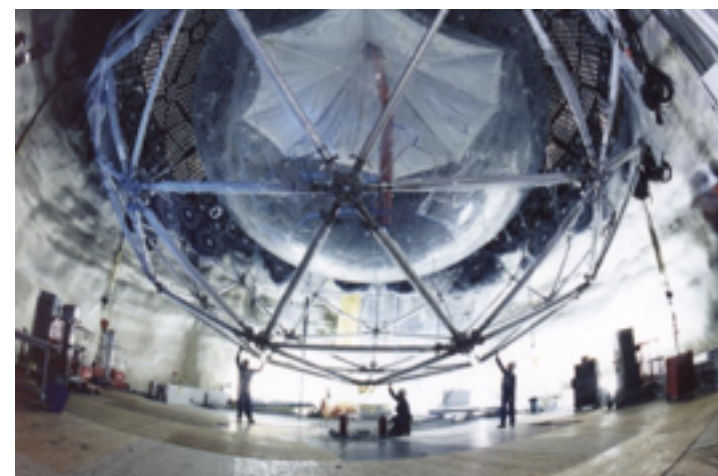
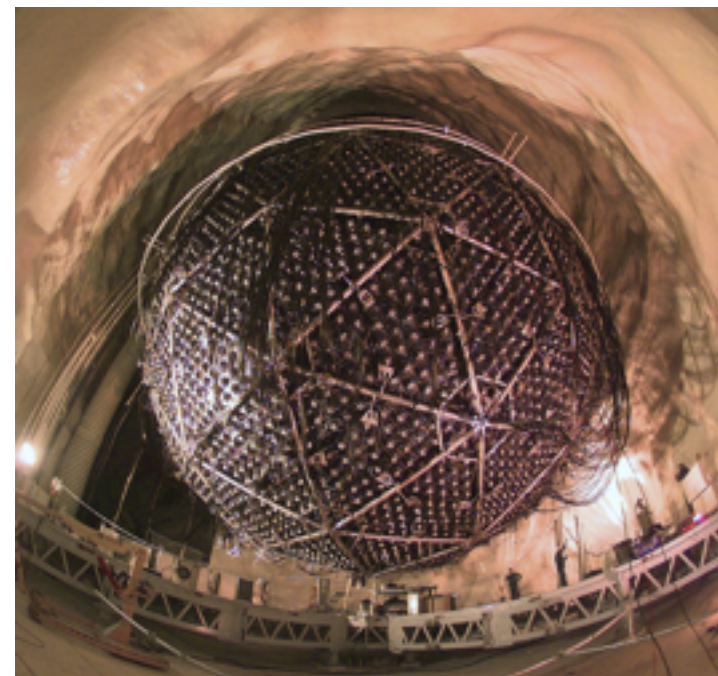
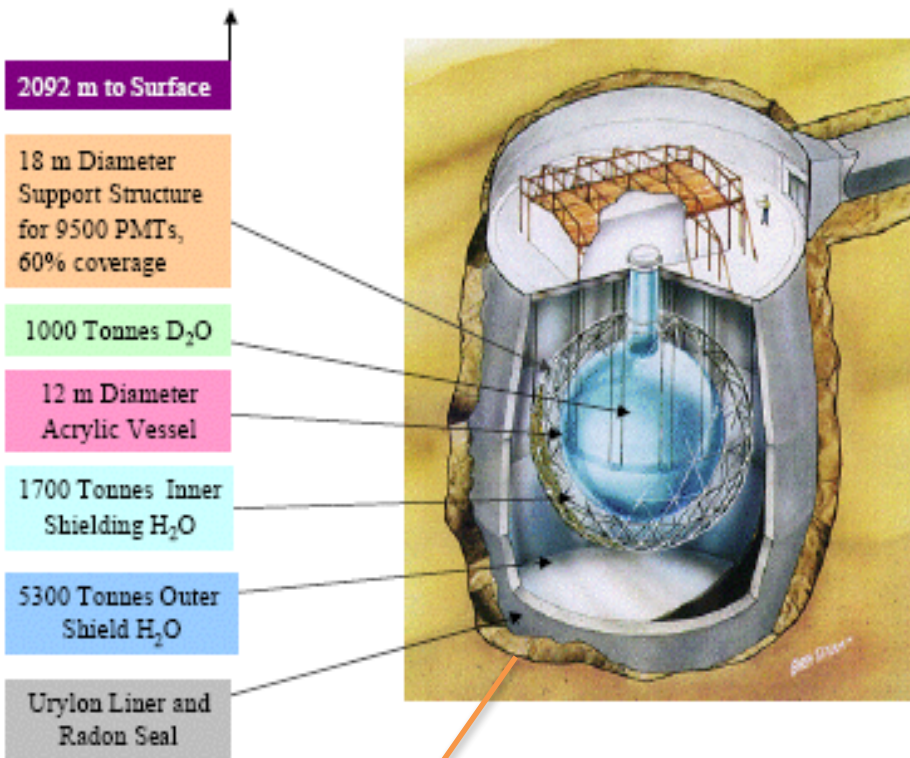
Neutrino Flavour Oscillations in Vacuum

We note that if the mixing angle θ is the maximal mixing angle of 45° then an initial ν_e beam can transform completely into the orthogonal flavour state ν_τ , exactly as we found with our vibrating mass. Any other mixing angle would result in partial transformation into the orthogonal flavour.

It turns out that for solar neutrinos, $\theta \approx 33^\circ$ and $\Delta(m^2) = 7 \times 10^{-5} \text{ eV}^2$.

Solar neutrinos at Sudbury Neutrino Observatory (SNO)

Sudbury Neutrino Observatory



Solar neutrino interactions in SNO

Solar ν Interactions in SNO

Elastic Scattering (ES)



- Rate $\propto \phi(\nu_e) + 0.15\phi(\nu_{\mu\tau})$

mainly electron-type, with a little sensitivity to non-electron types

Charged Current (CC)



- sensitive only to electron-type

Neutral Current (NC)

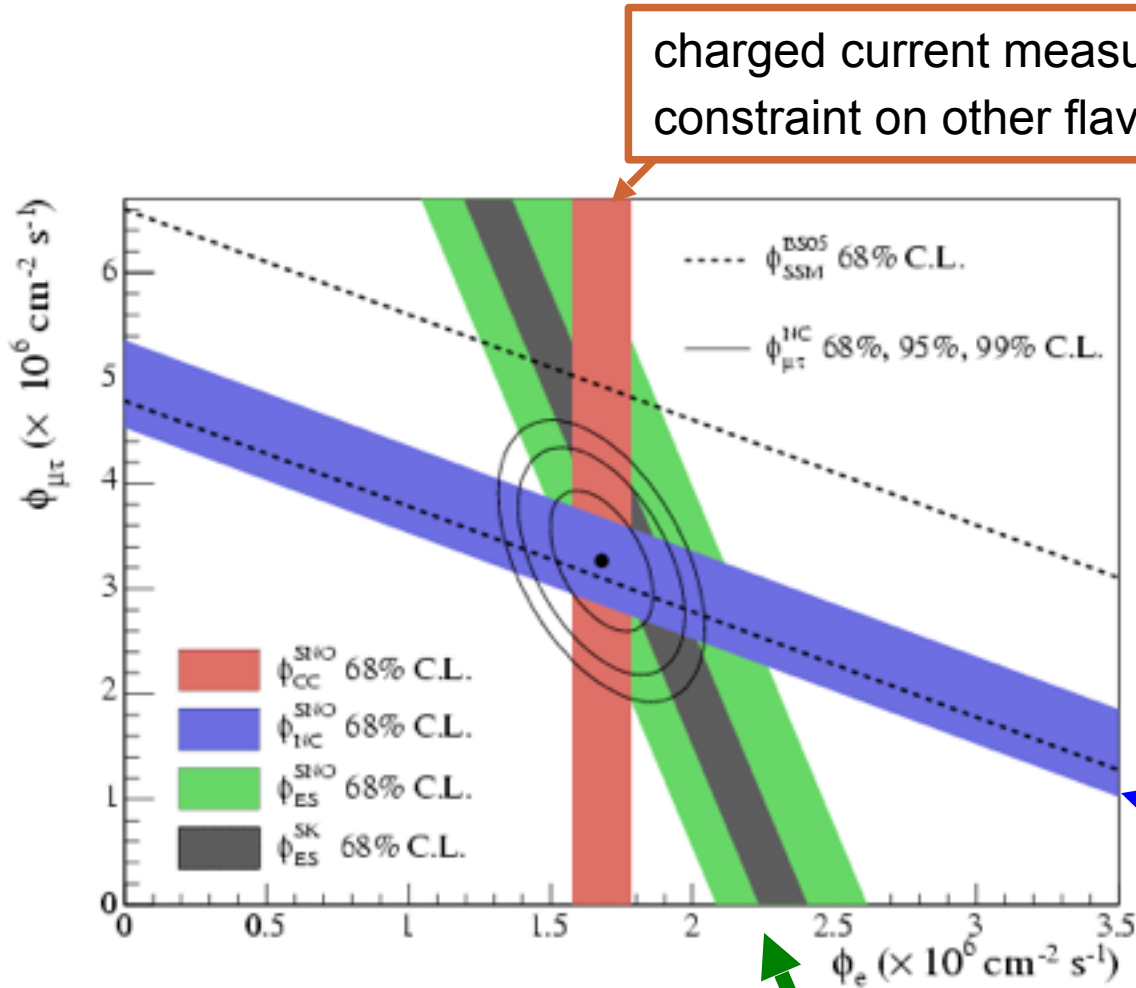


- sensitive to all neutrino flavours equally
- Detect neutrons by capture on d or ^{35}Cl or using $^3\text{He}(n,p)t$ neutron counters

Each reaction has different sensitivity to ν flavour content

Solar neutrino interactions in SNO

The cross sections for each of these 3 reactions corresponds to a band in a plot of flux of ν_μ, ν_τ on the vertical axis, versus flux of ν_e on the horizontal axis.



charged current measures flux of ν_e only, no constraint on other flavours, so vertical band

neutral current measures all flavours; more ν_e means less ν_μ and ν_τ , so diagonal band with negative slope

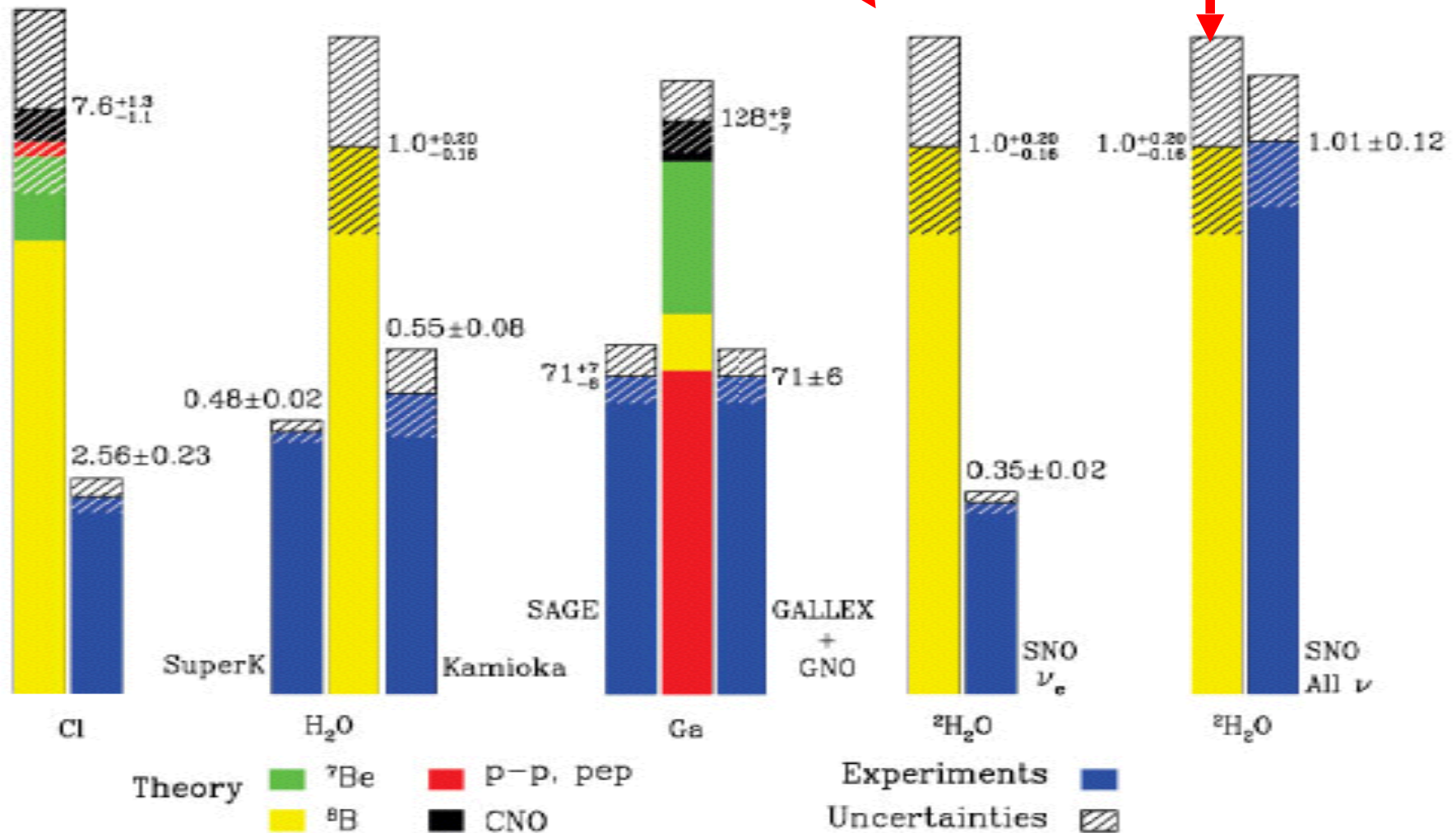
Elastic scattering measures mostly ν_e and a little bit of other flavours, so almost vertical band with a bit of slope

Solar neutrino interactions in SNO

SNO ν_e flux shows deficit

SNO neutral current measures all ν flavours, agrees with SSM !

Bahcall-Pinsonneault 2000



Solar neutrino interactions in SNO

Neutrino oscillations discovered!

What we call a " ν_e " is actually a superposition of two mass eigenstates ν_1 and ν_2 .

A neutrino which starts out as a pure ν_e can change as it propagates into a linear superposition of ν_e and ν_τ , and then back to pure ν_e again, because the two frequencies of the two mass eigenstates beat against each other.

Detectors which are only sensitive to ν_e will observe a deficit, but a detector like SNO which is sensitive to all neutrino flavours sees the full number predicted by the Standard Solar Model.

Noble Prize



The Nobel Prize in Physics 2015

Takaaki Kajita, Arthur B. McDonald

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The Nobel Prize in Physics 2015



Photo: A. Mahmoud
Takaaki Kajita
Prize share: 1/2

neutrino oscillations observed
in atmospheric neutrinos,
with Super-Kamiokande detector



Photo: A. Mahmoud
Arthur B. McDonald
Prize share: 1/2

neutrino oscillations
observed
in solar neutrinos with
Sudbury Neutrino
Observatory

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

Neutrino Oscillations

Neutrino oscillations have been explained thus far in two-dimensional vector space

$$(\nu_e, \nu_\tau) \leftrightarrow (\nu_1, \nu_2)$$

In fact, there are 3 neutrino flavours known $(\nu_e, \nu_\mu, \nu_\tau)$ so there must be 3 mass eigenstates (ν_1, ν_2, ν_3) , 2 mixing angles and 2 mass differences

Relation of these states expressed by a unitary matrix

$$\begin{array}{c} \text{Flavor eigenstates} \\ \text{(interaction)} \end{array} \begin{array}{c} \left(\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \right) \end{array} = \begin{array}{c} \left(\begin{array}{ccc} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{array} \right) \end{array} \begin{array}{c} \left(\begin{array}{c} \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \right) \end{array} \begin{array}{c} \text{Mass eigenstates} \\ \text{(propagation)} \end{array}$$



Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

Neutrino Oscillations

- “ ν_e appearance” : sensitive to θ_{13} , δ_{CP} , mass hierarchy

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_\nu} \right) \left(1 + \frac{4\sqrt{2}G_F n_e E}{\Delta m_{31}^2} (1 - 2 \sin^2 \theta_{13}) \right) - \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_\nu} \right) \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E_\nu} \right)$$

- “ ν_μ disappearance” : sensitive to θ_{23} , Δm_{32}^2

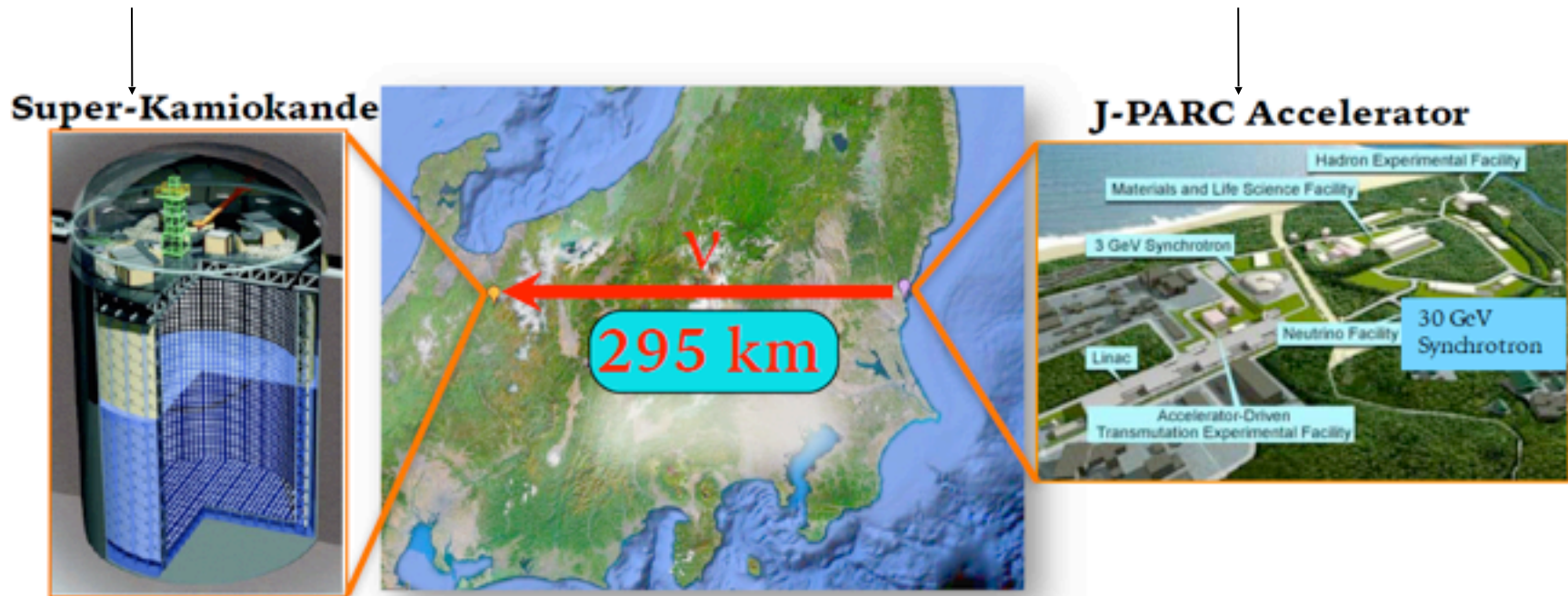
$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - (\cos^4 \theta_{13} \sin^2 2\theta_{23} + \sin^2 \theta_{23} \sin^2 2\theta_{13}) \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_\nu} \right)$$

If we start with a beam of pure ν_μ and measure the probability of a ν_μ changing into ν_e , then that gives a measure of the unknown parameter θ_{13}

T2K

look for ν_e via the reaction $\nu_e + n \rightarrow e^- + p$ in the Super-K detector

produce beam of (almost) pure ν_μ from pion decay
 $\pi^+ \rightarrow \mu^+ + \nu_\mu$



- **Tokai to Kamioka (T2K)** long-baseline neutrino oscillation experiment
- **Muon neutrinos from J-PARC** \Rightarrow **Super-Kamiokande @ 295 km**
- **T2K accomplishment**
 - **Discovery of ν_e appearance in 2013**

Neutrino Oscillations

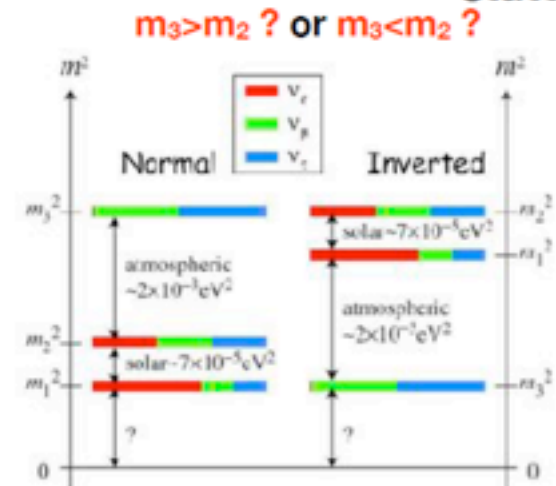
PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix}}_{\text{Atmospheric term}} \underbrace{\begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{i\delta} & 0 & \cos \theta_{13} \end{pmatrix}}_{\text{Reactor term}} \underbrace{\begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar term}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Flavor eigenstate Mass eigenstate

- **Current measured values (PDG2016)**

- $\sin^2 \theta_{12} = 0.307 \pm 0.013$
- $\sin^2 \theta_{13} = 0.0021 \pm 0.00011$
- $\sin^2 \theta_{23} = 0.51 \pm 0.04$ ($m_3 > m_2$) or 0.50 ± 0.04 ($m_3 < m_2$)
- $\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
- $\Delta m_{32}^2 = (2.45 \pm 0.05) \times 10^{-3} \text{ eV}^2$ ($m_3 > m_2$) or $(-2.52 \pm 0.05) \times 10^{-3} \text{ eV}^2$ ($m_3 < m_2$)
- Sign is not determined yet \Rightarrow **Mass hierarchy problem**
- δ_{CP} : **not determined yet**



but preliminary data from T2K indicates $\delta_{CP} \neq 0$, i.e. neutrinos oscillate differently from anti-neutrinos; possible way to create matter-antimatter asymmetry of the universe (Google 'leptogenesis' for more info.)