Übungsblatt 1

Exercise 1

In the Natural Units system (NU), $c = \hbar = \epsilon_0 = k_B = 1$. In NU, every unit is converted to some power of the energy, which is conventionally choosen to be measured in GeV. Units in the International System (SI) are generally conbinations of kilograms (kg), meters (m) and seconds (s). For converting SI units in NU units and vice-versa, we should have

$$kg^{\alpha} m^{\beta} s^{\gamma} = E^{a} \hbar^{b} c^{d} \quad . \tag{1}$$

Show explicitly that the following conversion formula is correct, deriving

$$kg^{\alpha} m^{\beta} s^{\gamma} = E^{\alpha-\beta-\gamma} \hbar^{\beta+\gamma} c^{\beta-2\alpha} \quad , \tag{2}$$

which means determining the exponents a, b, d as functions of α, β, γ.
Hint: remember the dimensions of $\hbar$ and c.

Exercise 2

Using the "Planck" quantities l_P, m_P, t_P defined in class (see Lecture 1 slides), derive the "Planck Force" and give a numeric estimate of it.

Exercise 3

Consider the following curve in three dimensional space expressed in parametric form

$$f(\theta) = \begin{cases} x = R\cos\theta \\ y = R\sin\theta \\ z = R\theta \end{cases} \tag{3}$$

which represent an helix.

a) Calculate the curvilinear abscissa $S(\theta)$.

b) Calculate the tangent vector $\bar{t} = \frac{df}{dS} = \frac{df}{d\theta} / \frac{dS}{d\theta}$ and verify that it is normalized to 1.

c) Calculate $\frac{d\bar{t}}{dS} / \left\| \frac{d\bar{t}}{dS} \right\|$. At this point you should have the curvature $C = \left\| \frac{d\bar{t}}{dS} \right\|$.

d) The *radius of curvature* is defined as $\rho = 1/C$. Does it make sense to you the obtained result? Explain briefly why.

e) Do you expect a non-vanishing torsion τ? Why?