# Übungsblatt 2

**Exercise 1**

On the 2-sphere, the geodesics equation

$$\frac{d^2x^\mu}{dt^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = 0 \quad , \tag{1}$$

becomes

$$\frac{d^2\theta}{dt^2} + 2\frac{\cos\phi}{\sin\phi}\frac{d\theta}{dt}\frac{d\phi}{dt} = 0$$
$$\frac{d^2\phi}{dt^2} - \sin\phi\cos\phi\left(\frac{d\theta}{dt}\right)^2 = 0 \quad .$$

Write the parametric equation of a maximum circle on the sphere (a "meridian", geographically speaking) and verify that it is indeed a geodesic: this means verifying that it is indeed a solution of Eq. 2.

**Exercise 2**

The trace of the Einstein equations is

$$-R + 4\Lambda = 8\pi T \quad . \tag{2}$$

Explicitly prove that its substitution back into the Einstein equations leads to the trace-inverted form

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}) \quad . \tag{3}$$

**Exercise 3**

Show that the term $\ddot{a}/a$ contained in the first Friedmann equation can be written as $\dot{H} + H^2$.
*hint:* Try to calculate $dH/dt$ remembering that the Hubble parameter is $H = \dot{a}/a$.