Introductory Particle Cosmology

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Lecture 5

Introduction

Mathematical Tools
Generic Vector Bases
Basis Change
Vectors in Curved Geometries
Tensors, Metric Tensor
(Riemann) Manifolds
Connection
Geodesics and Curvature

General Relativity
Equivalence Principle
Einstein Equations

Cosmology
Cosmological Principle
FLRW Metric
Friedmann Equations
Cosmic Distances
Cosmological Models

Observation
CMB
Structure Formation
Red-shift/Distance

Particle Cosmology
Dark Matter (Models + Exp.)
Dark Energy (Models + Exp)
Inflationary Models
Gravitational Waves
Density Perturbations

Standard Model
Brief History
Particle Content
Gauge Principle, CPV, Strong CP
EW Symmetry Breaking
Beyond the SM

Plan

	Datum	Von	Bis	Raum
1	**Di, 17. Apr. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
2	**Do, 19. Apr. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
3	**Di, 24. Apr. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
4	**Do, 26. Apr. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
5	**Do, 3. Mai 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
6	**Di, 8. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
7	**Di, 15. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
8	**Do, 17. Mai 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
9	**Di, 22. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
10	**Do, 24. Mai 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
11	**Di, 29. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
12	**Di, 5. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
13	**Do, 7. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
14	**Di, 12. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
15	**Do, 14. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
16	**Di, 19. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
17	**Do, 21. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
18	**Di, 26. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
19	**Do, 28. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
20	**Di, 3. Jul. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
21	**Do, 5. Jul. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**

H.Minkowski
(1864-1909)

Physics Dept. Building, 5th Floor

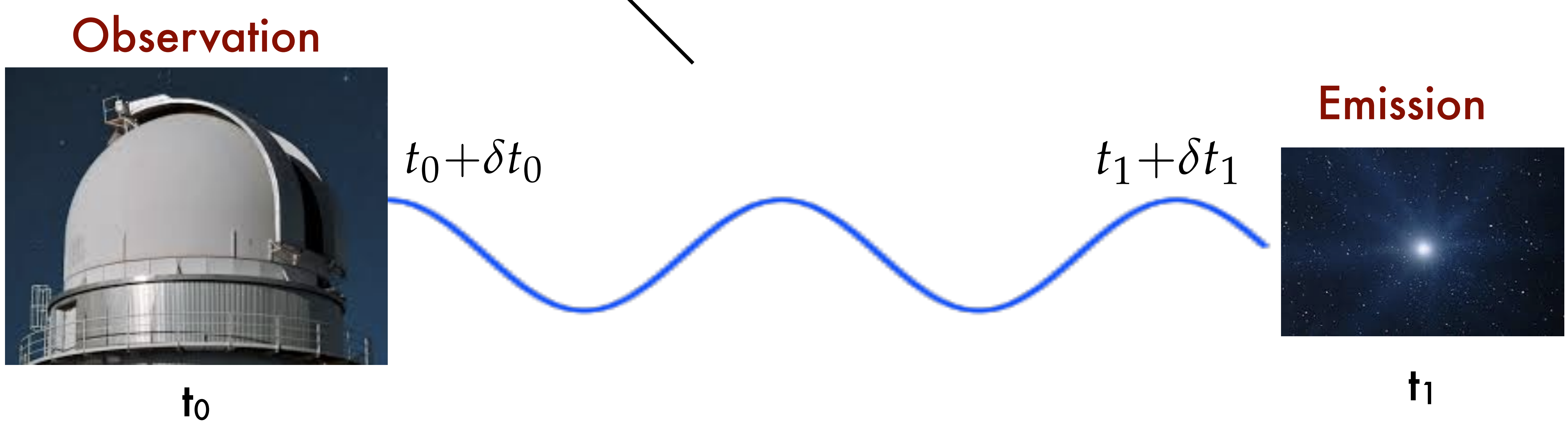
Observational Cosmology

Cosmological Red-shift
Estimation of the Age of the Universe
Cosmological Distances
Cosmography
Standard LambdaCDM Model

The Cosmological Red-shift

Light ray in the FLRW space-time: $$d\tau^2 = dt^2 - a^2(t)\frac{dr^2}{1-kr^2} = 0$$

Separating the variables: $$\int_{t_1}^{t_0}\frac{dt}{a(t)} = \int_o^{r_1}\frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \sin^{-1} r_1 & k=1 \\ r_1 & k=0 \\ \sinh^{-1} r_1 & k=-1 \end{cases}$$



$$\int_{t_1+\delta t_1}^{t_0+\delta t_0}\frac{dt}{a(t)} - \int_{t_1}^{t_0}\frac{dt}{a(t)} = \int_{\delta t_0}^{\delta t_1}\frac{dt}{a(t)} = 0 \quad \Rightarrow \quad \boxed{\frac{\delta t_1}{\delta t_0} = \frac{a(t_1)}{a(t_0)} = \frac{\nu_0}{\nu_1} = \frac{\lambda_1}{\lambda_0}}$$

a(t) does not vary much between two crests

The Red-shift

Red-shift Parameter

$$\boxed{z = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1}$$

$z > 0 \Rightarrow \lambda_0 > \lambda_1$ Red-shift The Universe is expanding

$z < 0 \Rightarrow \lambda_0 < \lambda_1$ Blue-shift The Universe is contracting

Hubble Parameter and Densities

Second Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho_c \left[\sum_i \frac{\rho_i}{\rho_c} + \frac{\rho_k}{\rho_c}\right]$$

Many components

$$\rho_k = k/(a^2)$$

ATTENTION: just a formal analogy!

Introducing the present-day critical density (and the present-dat Hubble parameter H_0)

$$\boxed{H^2 = H_0^2\left[\frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \frac{\Omega_k^0}{a^2} + \Omega_\Lambda^0\right]}$$

Matter Radiation Curvature Cosmological Constant

Hubble Parameter and Densities

The previous formula can be rewritten using the red-shift parameter

$$\boxed{H^2 = H_0^2 \left[\Omega_m^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \Omega_k^0 (1+z)^2 + \Omega_\Lambda^0\right]}$$

since $H = -\frac{1}{1+z}\frac{dz}{dt}$ and $\Omega_r^0 \approx 0$ so $\Omega_k^0 = 1 - \Omega_m^0 - \Omega_\Lambda^0$

we obtain the approximate formula

$$\Delta t = \frac{1}{H_0} \int_0^z \frac{dz'}{1+z'} \frac{1}{\sqrt{(1+\Omega_m^0 z')(1+z')^2 - z'(2+z')\Omega_\Lambda^0}}$$

where the integral extends from today (z=0) to some epoch characterized by z.

Age of the Universe

extend to ∞

$$\Delta t = \frac{1}{H_0} \int_0^{z} \frac{dz'}{1+z'} \frac{1}{\sqrt{(1+\Omega_m^0 z')(1+z')^2 - z'(2+z')\Omega_\Lambda^0}}$$

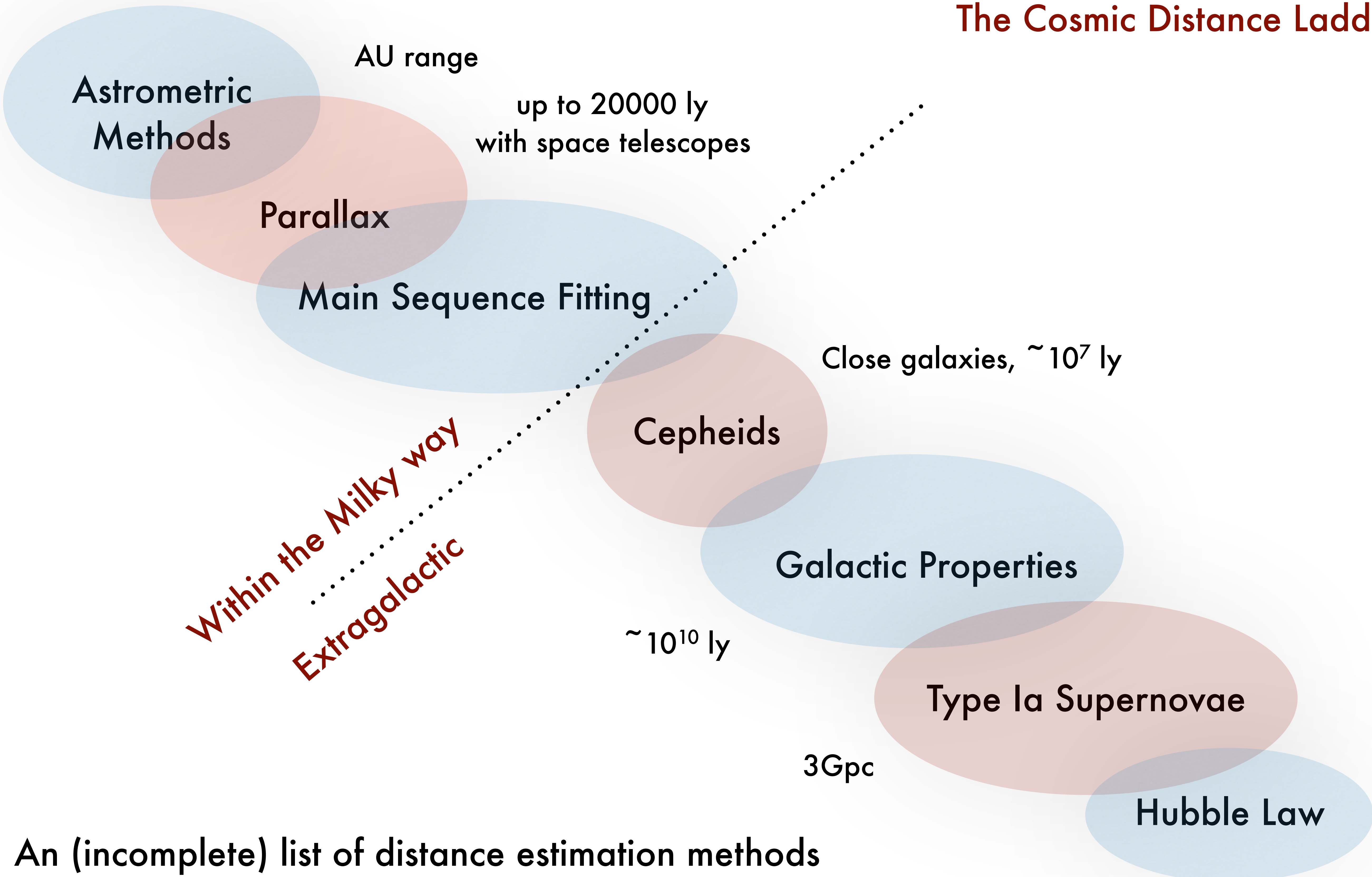
This integral is O(1), so Age of the Universe $\approx$ 1/H$_0$ $\approx$ 14 Gyr

Simple analytical case: flat Universe with no cosmological constant:

$$A = \frac{1}{H_0} \int_0^{\infty} \frac{dz'}{(1+z')^{5/2}} = \frac{2}{3H_0} \sim 10 \text{ Gyr}$$

Cosmological Distances

The Cosmic Distance Ladder



An (incomplete) list of distance estimation methods

Luminosity Distance

Detector Area

$$R_i = R(t_i)$$

Power received $$P = L \cdot \left[\frac{R_1^2}{R_0^2}\right] \cdot \left[\frac{A}{4\pi R_0^2 r_1^2}\right]$$

True luminosity
(i.e. power)

Photon Energy redshift

Distance taking
into account expansion

Apparent luminosity $$l = \frac{P}{A}$$

r_1

In Euclidean space $$l = \frac{L}{4\pi d_L^2}$$

We can thus define the **luminosity distance** $$d_L = \frac{R_0^2}{R_1^2} r_1 = (z+1) r_1 R_0$$

r_0

Cosmological Distances

Expand the scale factor $a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \frac{1}{6} j_0 H_0^3 (t - t_0)^3 + \ldots \right]$

Relation to redshift $1 + z = \frac{a(t_0)}{a(t_0 - D/c)}$

Luminosity distance $d_L = \frac{a_0^2 r_0}{a(t_0 - D/c)}$

$$\left.\begin{array}{l} 1 + z = \frac{a(t_0)}{a(t_0 - D/c)} \\ d_L = \frac{a_0^2 r_0}{a(t_0 - D/c)} \end{array}\right\} \text{Measurable}$$

Fit to the d_L / z data:

$$d_L(z) = \frac{cz}{H_0} \left[1 + \frac{1}{2}(1 - q_0) z - \frac{1}{6} \left(1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2 a_0^2} \right) z^2 \right.$$

$$\left. + \frac{1}{24} \left(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0 + \frac{2kc^2(1 + 3q_0)}{H_0^2 a_0^2} \right) + \ldots \right.$$

Discovery of the Accelerated Expansion

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

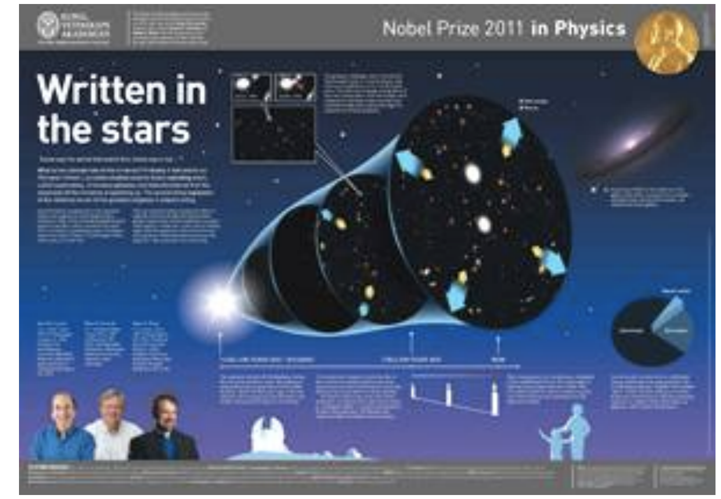
Saul Perlmutter

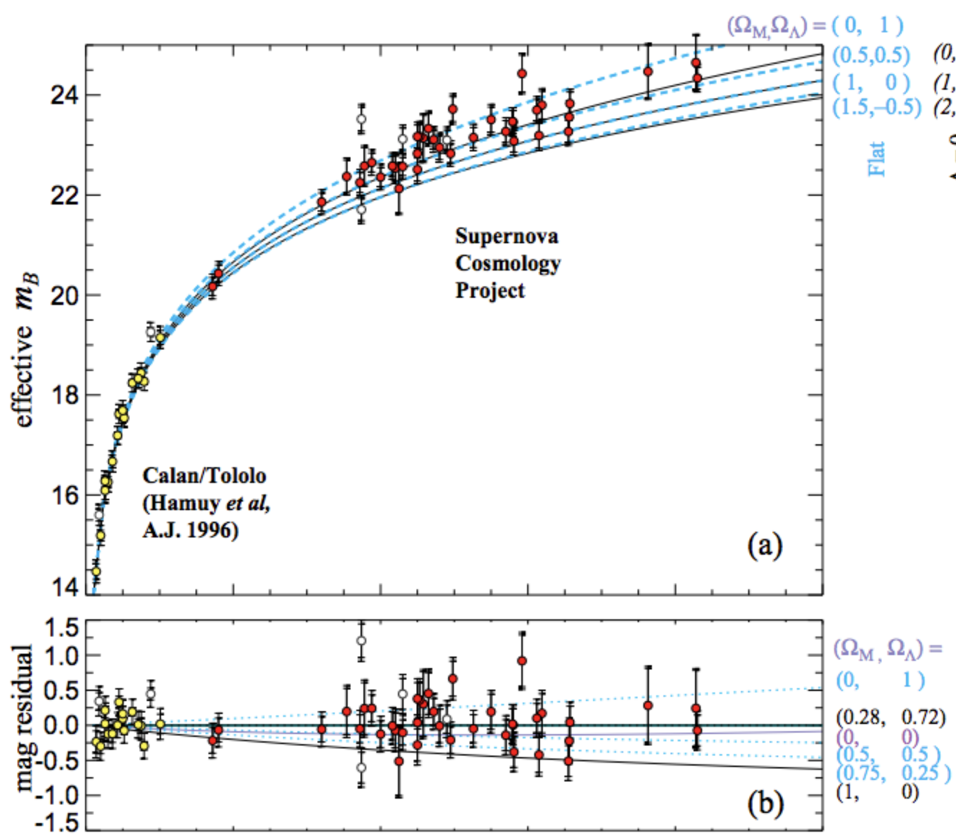
Photo: Belinda Pratten, Australian National University

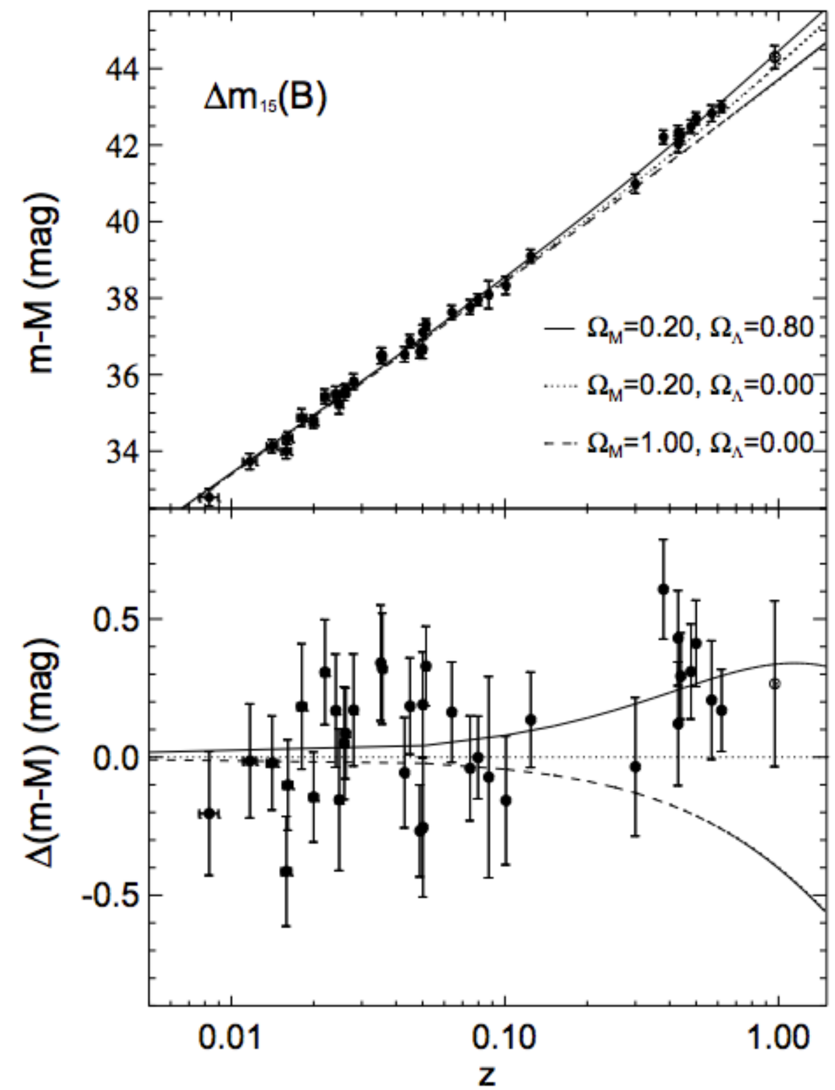
Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess







Type 1a Supernovae

Type 1a supernovae are used as standard candles.

SNe Ia occur in binary systems where one of the two stars is a carbon-oxygen white dwarf.

The efficiency of the explosion after accretion is determined by the core temperature and ultimately by the mass of the star

The peak luminosity can be used as standard candle, since this type of stars have similar masses.

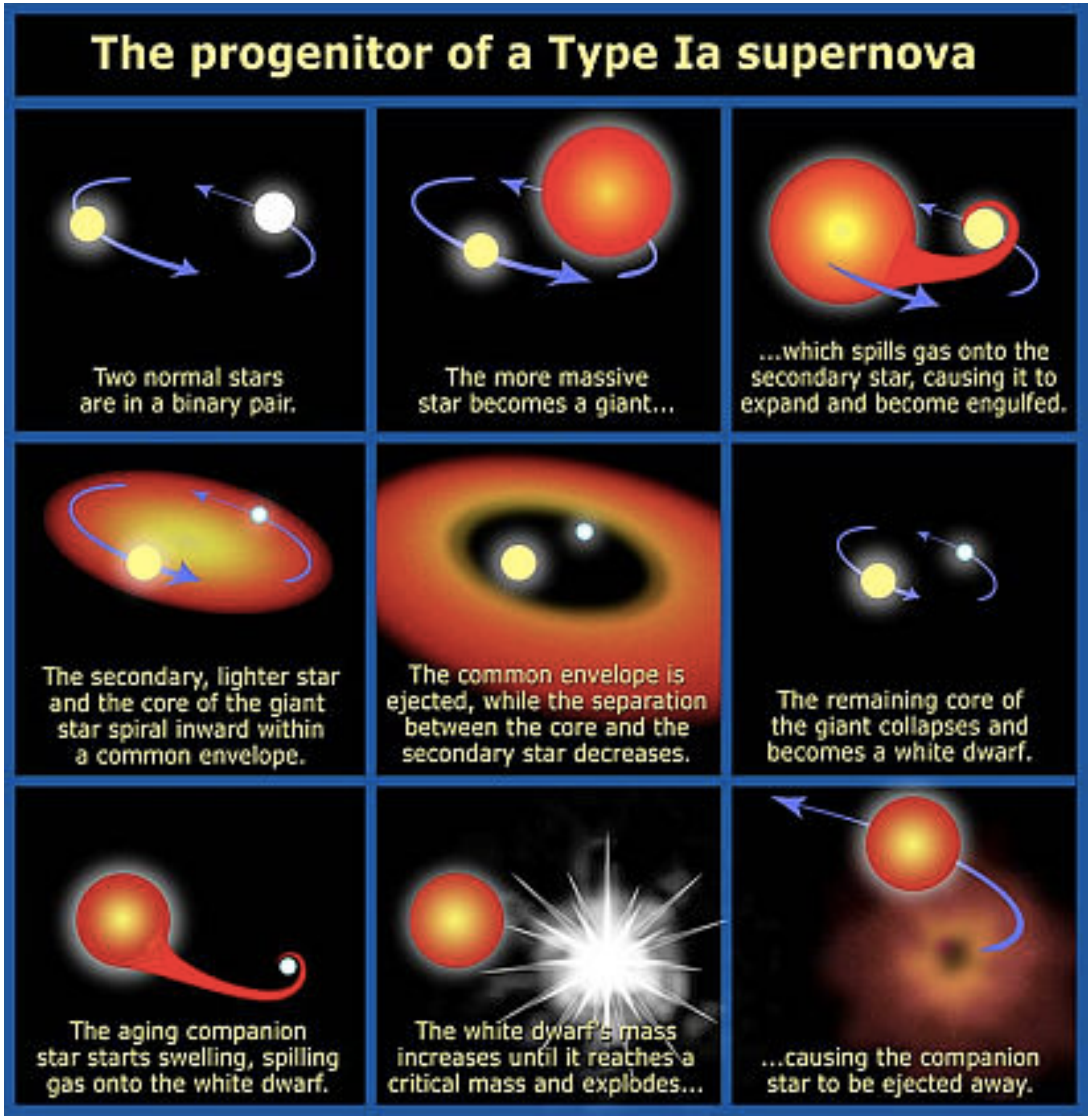


Image from : NASA, ESA and A. Feild (STScI)

Recent Data

Rebinned version of the data from arxiv-1401.4064