Introductory Particle Cosmology

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Lecture 6

Introduction

Mathematical Tools

Generic Vector Bases
Basis Change
Vectors in Curved Geometries
Tensors, Metric Tensor
(Riemann) Manifolds
Connection
Geodesics and Curvature

General Relativity

Equivalence Principle
Einstein Equations

Cosmology

Cosmological Principle
FLRW Metric
Friedmann Equations
Cosmic Distances
Cosmological Models

Observation

CMB
Structure Formation
Red-shift/Distance

Standard Model

Brief History
Particle Content
Gauge Principle, CPV, Strong CP
EW Symmetry Breaking
Beyond the SM

Particle Cosmology

Dark Matter (Models + Exp.)
Dark Energy (Models + Exp)
Inflationary Models
Gravitational Waves
Density Perturbations

Plan

	Datum	Von	Bis	Raum
1	**Di, 17. Apr. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
2	**Do, 19. Apr. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
3	**Di, 24. Apr. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
4	**Do, 26. Apr. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
5	**Do, 3. Mai 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
6	**Di, 8. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
7	**Di, 15. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
8	**Do, 17. Mai 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
9	**Di, 22. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
10	**Do, 24. Mai 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
11	**Di, 29. Mai 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
12	**Di, 5. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
13	**Do, 7. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
14	**Di, 12. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
15	**Do, 14. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
16	**Di, 19. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
17	**Do, 21. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
18	**Di, 26. Jun. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
19	**Do, 28. Jun. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**
20	**Di, 3. Jul. 2018**	**10:00**	**12:00**	**05 119 Minkowski-Raum**
21	**Do, 5. Jul. 2018**	**08:00**	**10:00**	**05 119 Minkowski-Raum**


H.Minkowski
(1864-1909)

Physics Dept. Building, 5th Floor

Quick Standard Model Summary

Problems of the Standard Big-Bang Scenario

Horizons

Inflation

Inflationary Scenarios

Standard Model Particle Content

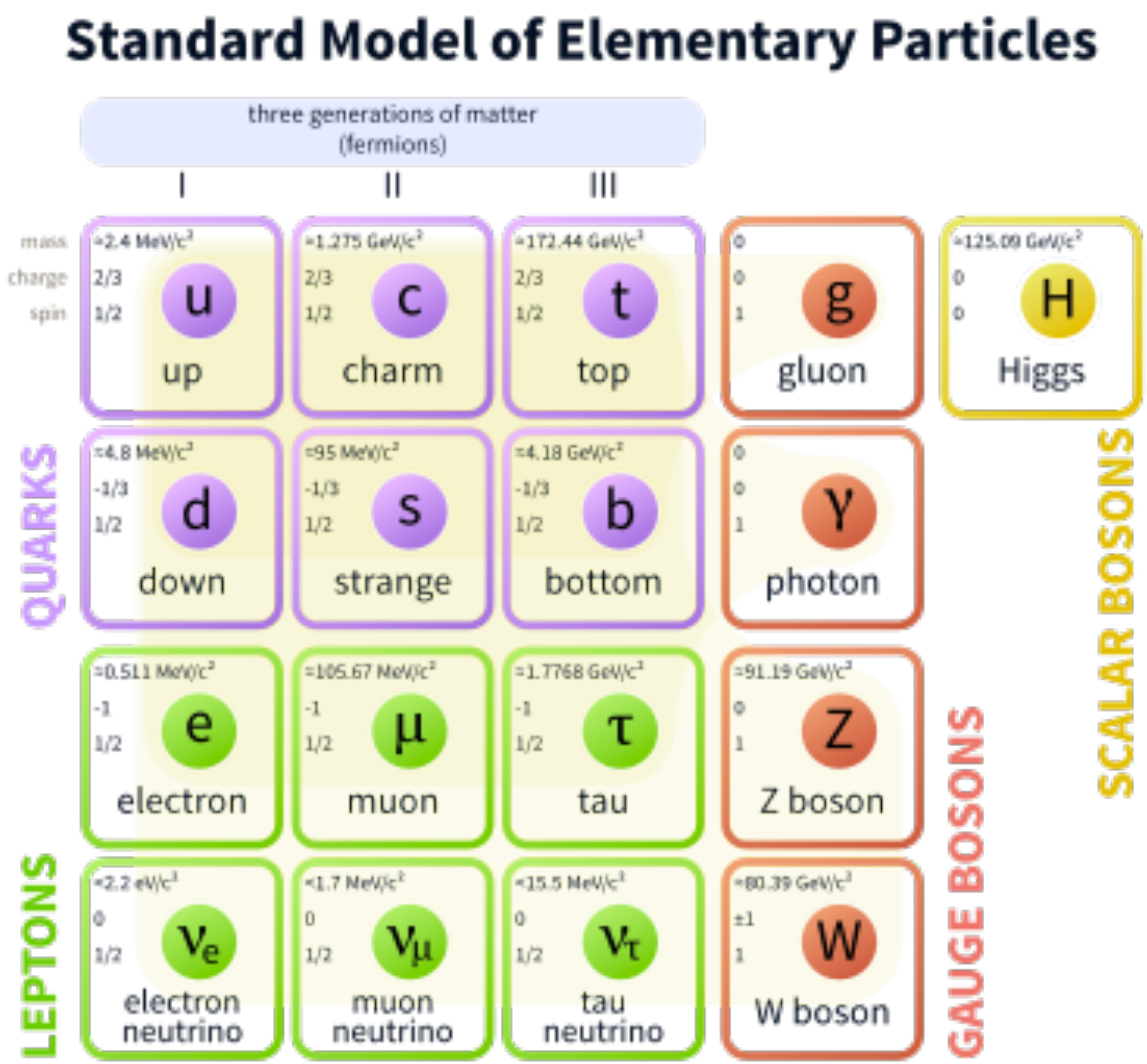
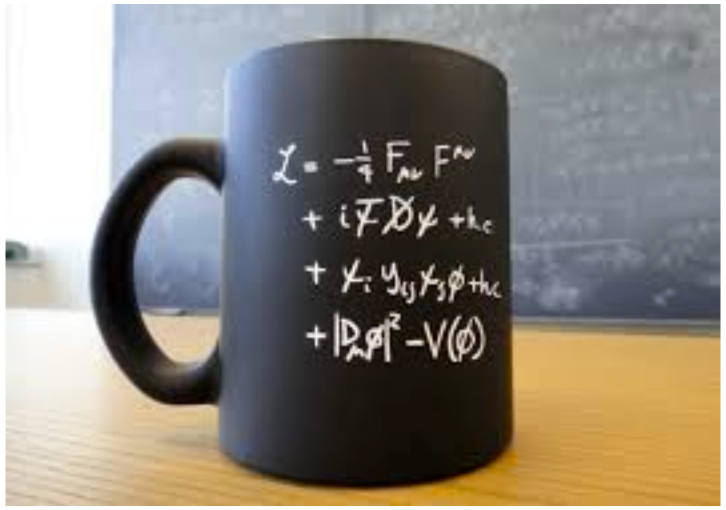


Image from wikipedia

The Standard Model



Brief History

1964 Murray Gell-Mann and George Zweig tentatively put forth the idea of quarks. Mesons and baryons are composites of three quarks or antiquarks, called up, down, or strange (u, d, s) with spin 0.5 and electric charges 2/3, -1/3, -1/3, respectively Since the fractional charges had never been observed, the introduction of quarks was treated more as a mathematical explanation of flavor patterns.

1964 Since leptons had a certain pattern, several papers suggested a fourth quark carrying another flavor to give a similar repeated pattern for the quarks, now seen as the generations of matter. Sheldon Glashow and James Bjorken coin the term "charm" for the fourth (c) quark.

1965 O.W. Greenberg, M.Y. Han, and Yoichiro Nambu introduce the quark property of color charge.

1966 The quark model is accepted rather slowly because quarks hadn't been observed.

1967 Steven Weinberg and Abdus Salam separately propose a theory that unifies electromagnetic and weak interactions into the electroweak interaction. Their theory requires the existence of a neutral, weakly interacting boson (now called the Z^0) They also predict an additional massive boson called the Higgs Boson that has not yet been observed.

1968-69 At the SLAC, in DIS on protons, the electrons appear to be scattering off small hard cores inside the proton. James Bjorken and Richard Feynman analyze this data in terms of a model of constituent particles inside the proton .

Brief History

1970 Sheldon Glashow, John Iliopoulos, and Luciano Maiani recognize the importance of a fourth type of quark. A fourth quark allows a theory that has flavor-conserving Z^0-mediated weak interactions but no flavor-changing ones.

1973 Donald Perkins, re-analyzes some old data from CERN and finds indications of weak interactions with no charge exchange (those due to a Z^0 exchange.)

1973 A QFT of strong interaction is formulated. This theory of quarks and gluons is similar in structure to QED. Quarks carry a color charge. Gluons are massless quanta of the strong-interaction field. The theory was first suggested by Harald Fritzsch and Murray Gell-Mann.

1973 David Politzer, David Gross, and Frank Wilczek discover that QCD has "asymptotic freedom."

1974 In a summary talk for a conference, John Iliopoulos presents the Standard Model.

1974 (Nov.) Burton Richter and Samuel Ting, leading independent experiments, announce on the same the discovered of the J/Psi particle. The J/psi particle is a charm-anticharm meson.

1976 Gerson Goldhaber and Francois Pierre find the D0 meson (anti-up and charm quarks).

1976 The tau lepton is discovered by Martin Perl and collaborators at SLAC.

Brief History

1977 Leon Lederman and collaborators at Fermilab discover the “bottom” quark.

1978 Charles Prescott and Richard Taylor observe a Z^0 mediated weak interaction in the scattering of polarized electrons from deuterium which shows a violation of parity conservation, as predicted by the Standard Model.

1979 Strong evidence for a gluon radiated by the initial quark or antiquark if found at PETRA at DESY

1983 The $W^{\pm}$ and Z0 intermediate bosons are observed by two experiments using the CERN synchrotron using techniques developed by Carlo Rubbia and Simon Van der Meer.

1989 Experiments carried out in SLAC and CERN strongly suggest that there are three and only three generations of fundamental particles. This is inferred by showing that the Z0-boson lifetime is consistent only with the existence of exactly three very light (or massless) neutrinos.

1995 CDF and D0 experiments at Fermilab discover the top quark at the unexpected mass of 175 GeV.

2012 Almost half a century after Peter Higgs predicted a Higgs boson the ATLAS and CMS experiments at the CERN lab discover the Higgs boson.

Gauge Invariance

In classical electrodynamics the fields $\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t};\ , \qquad \vec{B} = \vec{\nabla} \times \vec{A}$

do not change if $\phi \to \phi' = \phi - \frac{\partial \chi}{\partial t}\ , \qquad \vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\chi$

In QED $\mathcal{L}_\psi = \bar{\psi}(i\ \partial\!\!\!/ - m)\psi$

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

the U(1) local field transformation $\psi \to \psi' = \exp\left[-i\alpha(x)\right]\psi$

changes the lagrangian as

$$\begin{aligned}
\mathcal{L}_\psi \to \mathcal{L}'_\psi &= \bar{\psi}'\left[(i\ \partial\!\!\!/ - e\ A\!\!\!/') - m\right]\psi' \\
&= \bar{\psi}\exp(+i\alpha)\left[i\ \partial\!\!\!/ - e\left(A\!\!\!/ + \frac{1}{e}\ \partial\!\!\!/\alpha\right) - m\right]\exp(-i\alpha)\psi \\
&= \mathcal{L}_\psi - e\bar{\psi}\gamma_\mu\psi A^\mu
\end{aligned}$$

Gauge Invariance

$$
\begin{aligned}
\mathcal{L}_\psi \to \mathcal{L}'_\psi &= \bar{\psi}'\left[(i\,\not{\partial} - e\,\not{A}') - m\right]\psi' \\
&= \bar{\psi}\exp(+i\alpha)\left[i\,\not{\partial} - e\left(\not{A} + \frac{1}{e}\,\not{\partial}\alpha\right) - m\right]\exp(-i\alpha)\psi \\
&= \mathcal{L}_\psi - \boxed{e\bar{\psi}\gamma_\mu\psi A^\mu}\,.
\end{aligned}
$$

The gauge invariance requirement fixes how fermion fields are couplet to photons

where we can define the covariant derivative $D_\mu \equiv \partial_\mu + ieA_\mu$

and the field A (the "potential") must transform as $A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$

NOTE: The name "covariant derivative" is not causal: the charge space can indeed be thought as a curved space where the derivative must be "compensated" for the curvature. The field tensor plays the role of curvature tensor. Thus gauge theories have a geometric interpretation.

Quantum Electrodynamics (QED)

Lagrangian

$$L = \bar{\psi} i \gamma_{\mu} \left(\partial^{\mu} + igA^{\mu} \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

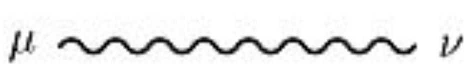
Fermion field

Photon field

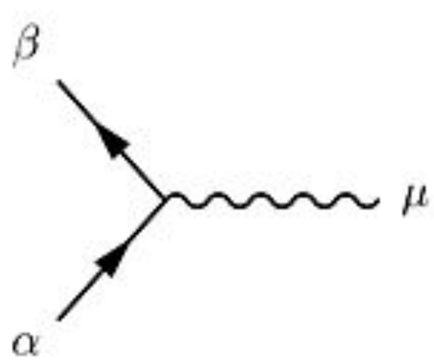
Gauge boson (photon)-fermion coupling
from U(1) gauge invariance

Basic Diagram Blocks

$$\rightarrow \quad \left(\frac{i}{\not{p} - m + i\varepsilon} \right)_{\beta\alpha}$$

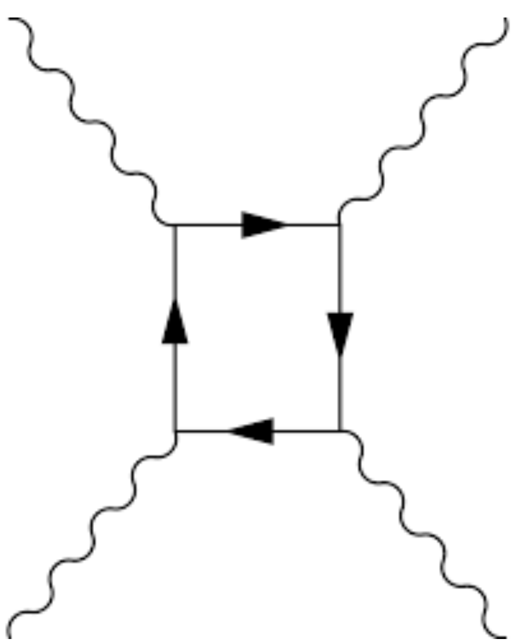


$$\rightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$



$$\rightarrow \quad -ie\gamma^{\mu}_{\beta\alpha}(2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3).$$

Higher order Loop Effects



Quantum Chromodynamics (QCD)

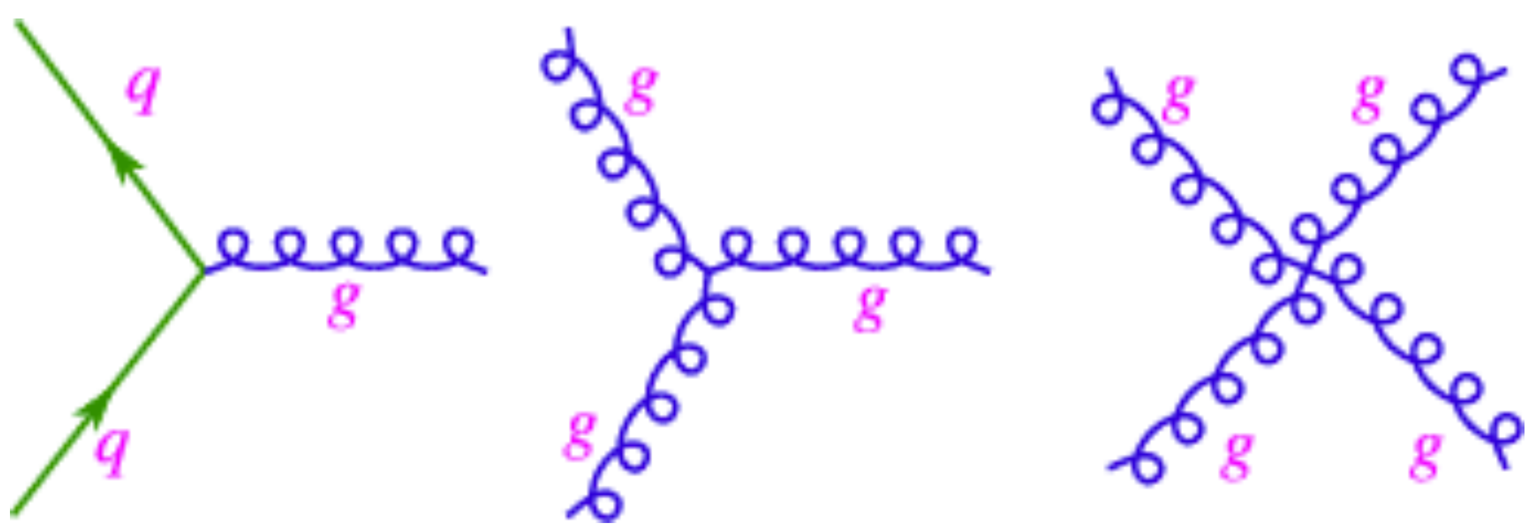
Lagrangian $\mathcal{L}_{QCD} = \bar{\psi}(i \not{D}_a T_a - m)\psi - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$

Covariant derivative $D_a^\mu = \partial^\mu + igA_a^\mu$

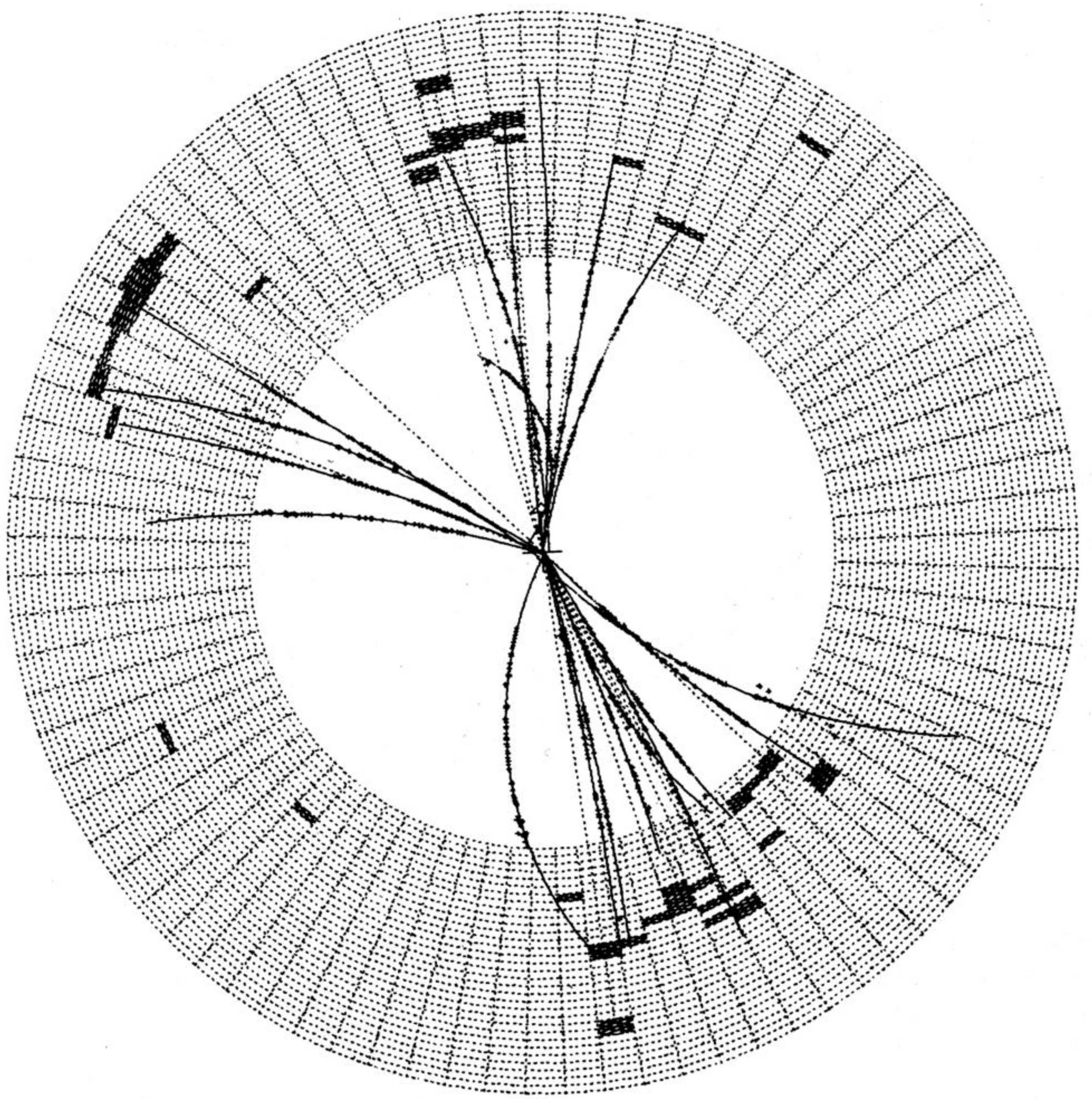
Gluon Field $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$

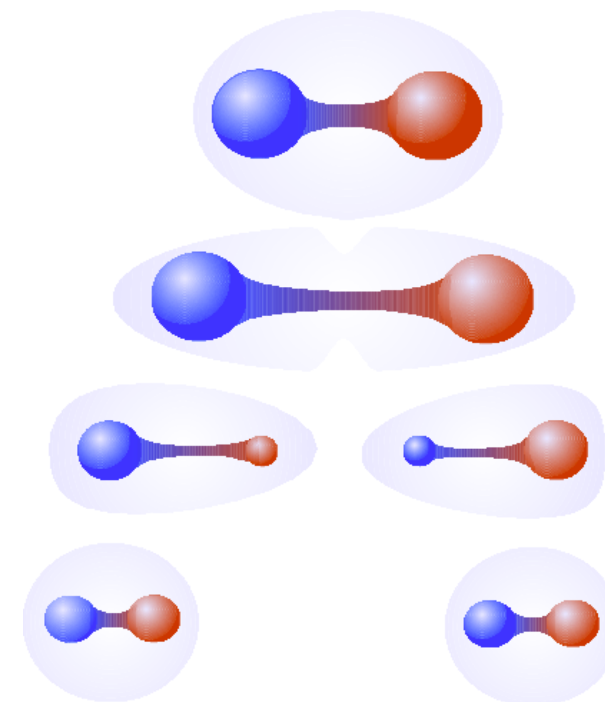
SU(3) Algebra $[T_a^{(F)}, T_b^{(F)}] = i f_{abc} T_c^{(F)}, \quad (T_a^{(A)})_{bc} = -i f_{abc}$

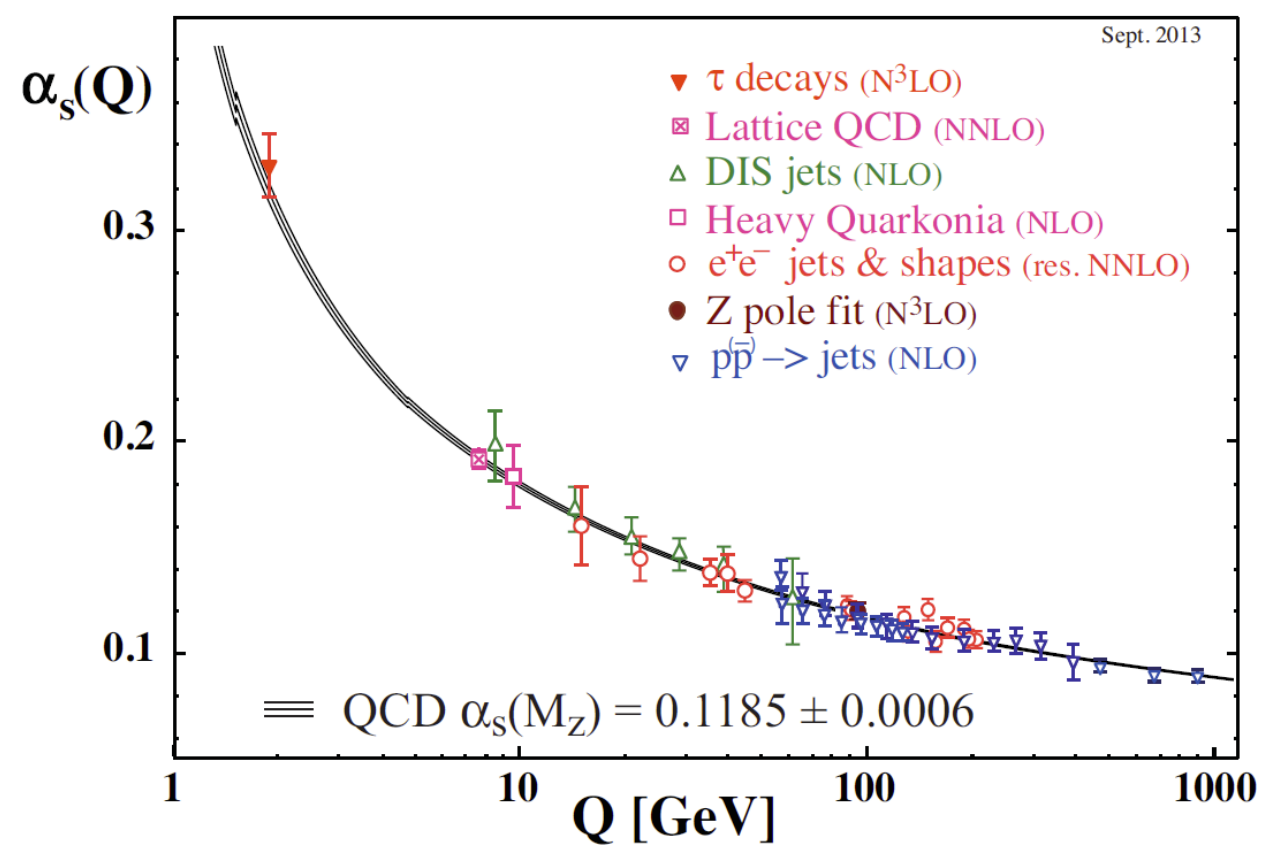
Basic Diagram Blocks (+ Propagators)



QCD: Jets, Gluons, Confinement







Electroweak Unification

Lepton current (chiral) $J^{+}_{\mu} = \bar{\ell}\gamma_{\mu}(1-\gamma_5)\nu = 2\,\bar{\ell}_L\gamma_{\mu}\nu_L$

Left-handed doublet $\mathsf{L} \equiv \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L = \begin{pmatrix} L\,\nu \\ L\,\ell \end{pmatrix} = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$

Right-handed singlet $\mathsf{R} \equiv R\,\ell = \ell_R$

Gauge Fields

$$\begin{array}{ccc} SU(2)_L & \longrightarrow & W^1_{\mu}\,,\ W^2_{\mu}\,,\ W^3_{\mu} \\ U(1)_Y & \longrightarrow & B_{\mu}\,. \end{array}$$

$$\begin{aligned} W^i_{\mu\nu} &\equiv \partial_{\mu}W^i_{\nu} - \partial_{\nu}W^i_{\mu} + g\,\epsilon^{ijk}\,W^j_{\mu}W^k_{\nu}\,, \\ B_{\mu\nu} &\equiv \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}\,, \end{aligned}$$

Electroweak Unification

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^i_{\mu\nu} W^{i\ \mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\begin{aligned}
\mathcal{L}_{\text{leptons}} &= \bar{\mathsf{R}}\, i\, \not{\partial}\, \mathsf{R} + \bar{\mathsf{L}}\, i\, \not{\partial}\, \mathsf{L} \\
&= \bar{\ell}_R\, i\, \not{\partial}\, \ell_R + \bar{\ell}_L\, i\, \not{\partial}\, \ell_L + \bar{\nu}_L\, i\, \not{\partial}\, \nu_L \\
&= \bar{\ell}\, i\, \not{\partial}\, \ell + \bar{\nu}\, i\, \not{\partial}\, \nu\ .
\end{aligned}$$

$$\mathsf{L} \quad : \quad \partial_\mu + i\, \frac{g}{2}\, \tau^i\, W^i_\mu + i\, \frac{g'}{2}\, Y B_\mu\ ,$$

$$\mathsf{R} \quad : \quad \partial_\mu + i\, \frac{g'}{2}\, Y\ B_\mu\ ,$$

Goldstone Mechanism

Gauge bosons are massless. Higgs mechanism based on SSB provides masses to W and Z bosons. Let's start with the Goldstone mechanism.

Simple example: scalar field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$$

$$V(\phi^* \phi) = \mu^2 (\phi^* \phi) + \lambda (\phi^* \phi)^2$$

invariant under the global U(1) transformation $\phi \rightarrow \exp(-i\theta)\phi$

redefining $\phi = \dfrac{(\phi_1 + i\phi_2)}{\sqrt{2}}$ then $\mathcal{L} = \dfrac{1}{2}\left(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2\right) - V(\phi_1, \phi_2)$

which is (equivalently) invariant under the global SO(2) rotation

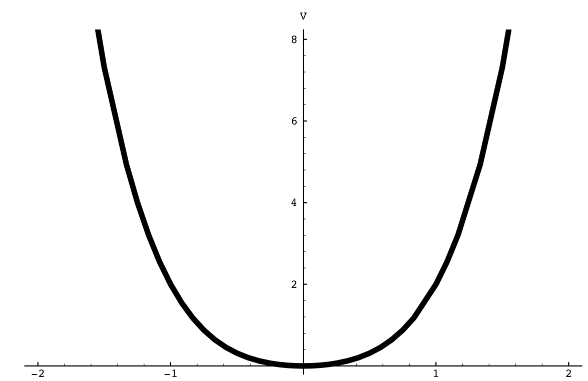
$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Goldstone Mechanism

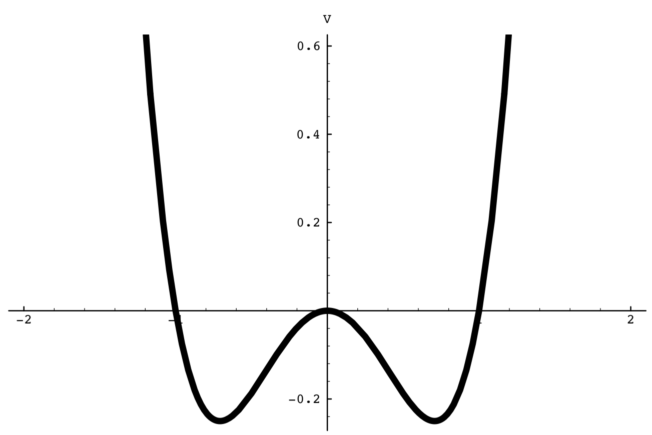
If $\mu^2 > 0$ then the minimum is at $\phi_1 = \phi_2 = 0$

For small oscillations around the minimum $\mathcal{L} = \sum_{i=1}^{2} \frac{1}{2} \left(\partial_\mu \phi_i \partial^\mu \phi_i - \mu^2 \phi_i^2 \right)$

and we have 2 scalar fields with equal masses $\mu^2 > 0$



If $\mu^2 < 0$ we have a continuum of vacua satisfying



$$< |\phi|^2 >= \frac{(< \phi_1 >^2 + < \phi_2 >^2)}{2} = \frac{-\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

The vacuum structure is also SO(2), but it is broken if we choose a particular one.

Goldstone Mechanism

Choosing $\begin{aligned} \phi_1 &= v \\ \phi_2 &= 0 \end{aligned}$ and redefining $\begin{aligned} \phi_1' &= \phi_1 - v \\ \phi_2' &= \phi_2 \,. \end{aligned}$

we have $$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_1' \partial^\mu \phi_1' - \frac{1}{2}(-2\mu^2)\phi_1'^{\,2} + \frac{1}{2}\partial_\mu \phi_2' \partial^\mu \phi_2' + \text{ interaction terms}$$

massive field massless field

Goldstone theorem: if an exact continuous GLOBAL symmetry is spontaneously broken (i.e. the vacuum is not invariant wrt to it), then the theory contains a massless scalar particle for each broken generator of the original group.

Higgs Mechanism

The Higgs mechanism works in a similar way to the Goldstone's, but in this case we require that the lagrangian is initially invariant under a LOCAL gauge transformation. In this case we have to introduce the covariant derivative and the gauge field transformations

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + iqA_\mu \qquad\qquad A_\mu \longrightarrow A'_\mu = A_\mu - \partial_\mu \alpha(x)$$

With the parameterization $\phi = \exp\left(i\frac{\phi'_2}{v}\right)\frac{(\phi'_1 + v)}{\sqrt{2}} \simeq \frac{1}{\sqrt{2}}(\phi'_1 + v + i\phi'_2) = \phi' + \frac{v}{\sqrt{2}}$

the lagrangian becomes

$$\begin{aligned}\mathcal{L} \quad = \quad & \frac{1}{2}\partial_\mu\phi'_1\partial^\mu\phi'_1 - \frac{1}{2}(-2\mu^2){\phi'_1}^2 + \frac{1}{2}\partial_\mu\phi'_2\partial^\mu\phi'_2 + \text{ interact.} \\ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu + qvA_\mu\partial^\mu\phi'_2 \ .\end{aligned}$$

with one massive scalar field (the Higgs boson) and a massive gauge boson.

Higgs Mechanism

For removing the last "bad" term mixing propagators for the gauge field and the scalar field we can choose the gauge $\alpha(x) = -\frac{1}{qv}\phi'_2(x)$

In this way, the field redefinition becomes

$$\phi = \exp\left[iq\left(-\frac{\phi'_2}{qv}\right)\right]\exp\left(i\frac{\phi'_2}{v}\right)\frac{(\phi'_1+v)}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\phi'_1+v)$$

and the final lagrangian (now without Goldstone massless field) becomes

$$\begin{aligned}\mathcal{L} \quad &= \quad \frac{1}{2}\partial_\mu\phi'_1\partial^\mu\phi'_1 - \frac{1}{2}(-2\mu^2)\phi_1'^{\,2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A^{\mu\,\prime} \\ &+\frac{1}{2}q^2\left(\phi'_1+2v\right)\phi'_1 A'_\mu A^{\mu\,\prime} - \frac{\lambda}{4}\phi_1'^{\,3}\left(\phi'_1+4v\right)\ .\end{aligned}$$

We started with 2 dof of charged scalar fields and two dof of the gauge field A (4 doc in total) and we ended up with one scalar field (1 dof) and a massive gauge field (3 dof) therefore, one scalar field was "eaten up" by A while gaining its mass!

The Standard Model

$$\mathcal{L}_{SM} = \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}$$

Kinetic energies / self interactions of the gauge bosons

$$+ \bar{L}\gamma^{\mu}(i\partial_{\mu} - \frac{1}{2}g\tau W_{\mu} - \frac{1}{2}g'YB_{\mu})L + \bar{R}\gamma^{\mu}(i\partial_{\mu} - \frac{1}{2}g'YB_{\mu})R$$

Kinetic energies and electroweak interactions of fermions

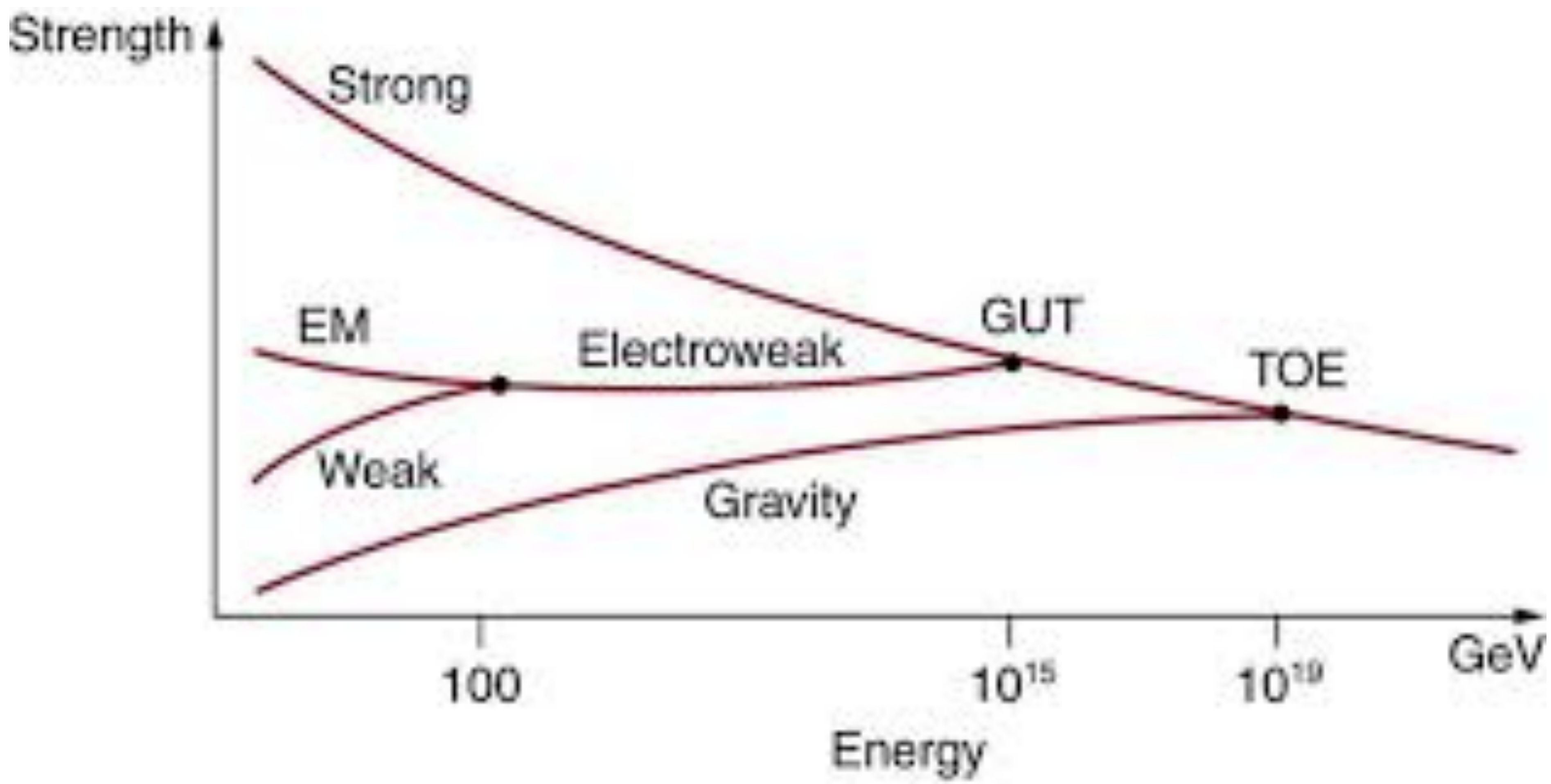
$$+ \frac{1}{2}|(i\partial_{\mu} - \frac{1}{2}g\tau W_{\mu} - \frac{1}{2}g'YB_{\mu})\phi|^{2} - V(\phi)$$

W,Z, Gamma, Higgs masses and couplings

$$+ g''(\bar{q}\gamma^{\mu}T_{a}q)G^{a}_{\mu} + (G_{1}\bar{L}\phi R + G_{2}\bar{L}\phi_{c}R + h.c.)$$

Quark-gluon interaction | Fermion masses and coupling to the Higgs field

GUTs



Strength
Strong
EM
Electroweak
GUT
TOE
Weak
Gravity
100
10^{15}
10^{19}
GeV
Energy

Scalar Fields

Action $$S = \int dx^4 \sqrt{-g} \underbrace{\left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]}_{\text{Lagrangian}}$$

Possible Potential $$V = \frac{1}{2} m^2 \phi^2 + \frac{\lambda_\phi}{4} \phi^4$$

Equations of motion $$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad \Rightarrow \quad \partial^\mu \partial_\mu \phi - \frac{dV}{d\phi} = 0$$

If V=0 the wave equation is recovered $$\partial^\mu \partial_\mu \phi = -\ddot{\phi} + \nabla^2 \phi = 0$$

Scalar Fields

Energy Momentum Tensor

$$T^{\mu\nu} = -\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi + g^{\mu\nu} \mathcal{L}$$

Energy Density

$$\rho = T^{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\nabla\phi^2 + V(\phi) \quad ,$$

Avg. Pressure

$$P = \frac{1}{3}(P^{11} + P^{22} + P^{33}) = \frac{1}{6}\dot{\phi}^2 - \frac{1}{2}\nabla\phi^2 + V(\phi)$$

If homogeneous and isotropic:

$$\boxed{P = -\rho}$$

Distances

Considering the FLRW metric for radial distances and fixed time $t, \theta, \phi = const.$

$$\underbrace{d(t)}_{\text{proper distance}} = \int_0^r \frac{a\,dr}{\sqrt{1-kr^2}} = a(t)S_k^{-1}(r) = \frac{a_1}{1+z}S_k^{-1} = \frac{d^c}{1+z}$$

comoving distance (present time): d^c

For a light ray

Not function of time

$$dt = -a(t)\frac{dr}{\sqrt{1-kr^2}} \Rightarrow \int_{t_1}^{t_0}\frac{dt}{a(t)} = \int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \frac{d}{a} = \frac{d^c}{a_0}$$

In terms of red-shift

$$d^c(z) = a_0\int_{t_1}^{t_0}\frac{dt}{a(t)} = \int\frac{dt}{x} = \int_{\frac{1}{1+z}}^{1}\frac{dx}{x}\frac{1}{dx/dt} = \int_0^z \frac{dz'}{H(z')}$$

$$x = a/a_0 \qquad dx/x = da/a = -dz/(1+z) \qquad H = \dot{a}/a = \dot{x}/x$$

Distances

The proper distance as a function of the red-shift is therefore

$$d(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}$$

which is the proper distance at the time the light left the astrophysical object.
The comoving distance light has travelled since the beginning of the Universe is

$$d(z) = \int_0^\infty \frac{dz'}{H(z')}$$

The "sphere" defined by the last equation is called the "particle horizon" and it represents the maximum distance we can observe.
The "event horizon" instead characterizes how far in the future light can travel.
The "Hubble distance" 1/H is also called "horizon".

Hubble Horizon

Hubble Radius (or Hubble Horizon)

It is the distance from the observer at which the recession velocity of a galaxy would equal the speed of light.
In other words, the Hubble radius is the radius of the observable Universe.
If the Hubble constant is about 70 km/s/Mpc, the Hubble radius is about 14 Bly.

$$v = H \cdot R \Rightarrow \boxed{R_H = \frac{c}{H}}$$

Particle Horizon

Particle Horizon

The particle horizon (or cosmological horizon), is the maximum distance from which particles could have traveled to the observer in the age of the universe. Its magnitude nowadays defines the size of the observable universe.

Using the conformal time $d\eta = dt/a(t)$

the FLRW metric becomes $ds^2 = a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_k^2(\chi)d\Omega^2\right]$

and considering only radial rays, light follow straight lines $\chi = \pm\eta + C$

The maximum distance a light ray can travel is

$$\Delta\eta = r_{max} = \int_0^t \frac{dt'}{a(t')} = \int_{a_0}^{a_1} \frac{da}{a\dot{a}} = \int_{\ln a_0}^{\ln a_1} \boxed{\frac{1}{aH}} d\ln a = \int_{\ln a_0}^{\ln a_1} R_{cH} d\ln a$$

Comoving Hubble radius

In an accelerating universe, if two particles are separated by a distance greater than the Hubble radius, they will never be able to communicate. If they are outside of each other's particle horizon, they could have never communicated.

The Horizon Problem

Light rays $d\tau^2 = dt^2 - a^2(t)dr^2 = 0 \quad \Rightarrow \quad \int_0^{r_{max}} dr = \int_0^t \frac{dt'}{a(t')}$

In a radiation-dominated universe $a \sim \sqrt{t}$

Horizon:
distance travelled while the Universe is expanding

$$D_H = a(t) \int_0^{r_{max}} dr = a(t) \int_0^t \frac{dt'}{a(t')} = \sqrt{t} \int_0^t \frac{dt'}{\sqrt{t'}} = 2t$$

The horizon is getting larger faster than the expansion of the Universe.
Some portions of the Universe did never communicate between each other.

The Horizon Problem

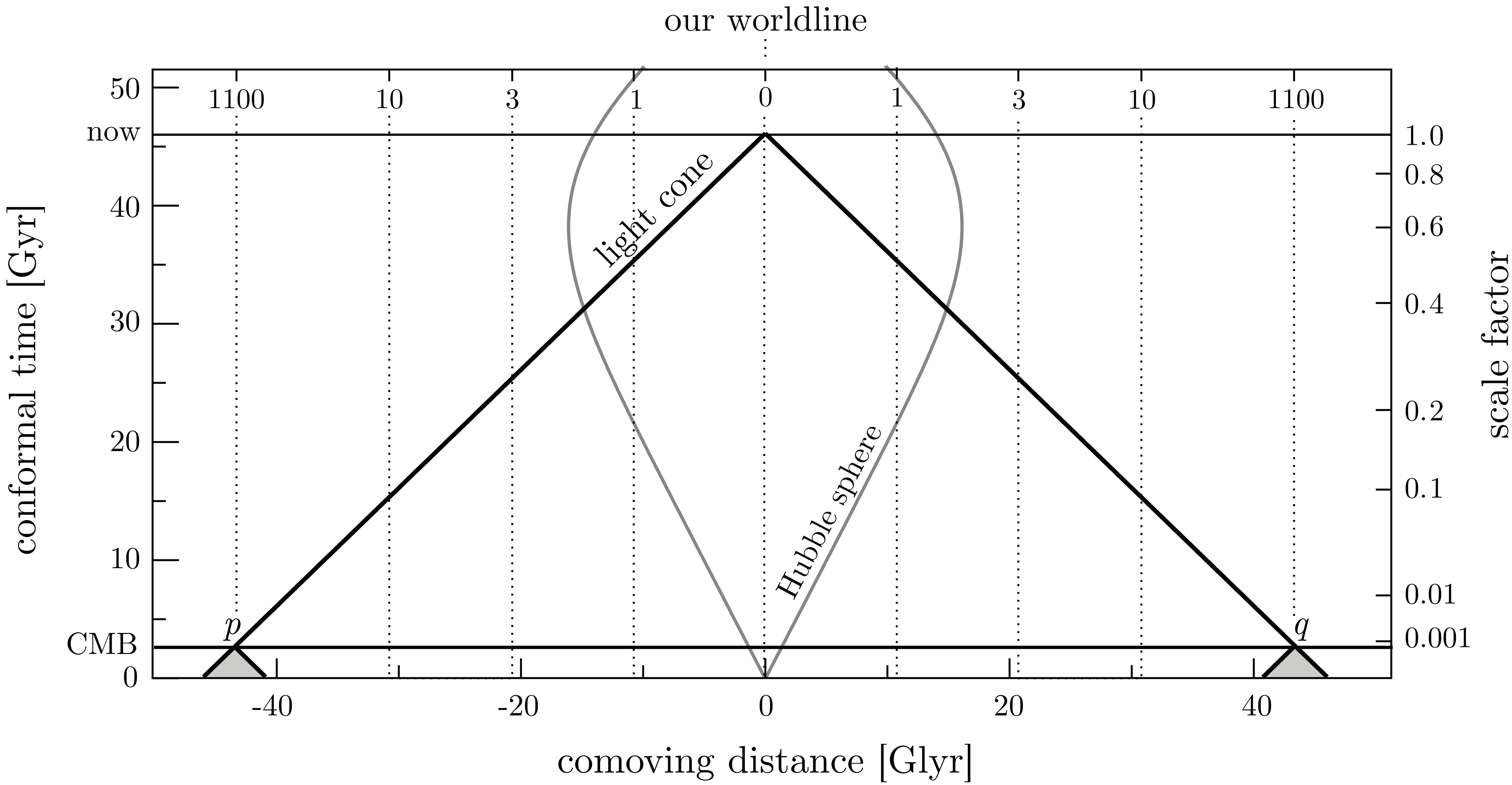


figure from Daniel Baumann

The Flatness Problem

Consider the Friedmann equation
$$\left(\frac{1}{\Omega}-1\right)\boxed{\rho_c a^2} = -\frac{3k}{8\pi G}$$

constant

Must increase for compensating for the density drop

With only matter and radiation, density drops faster than scale factor

Since the Planck time, the density x a^2 should have decreased by a factor 10^{60}.
This means that 1/Omega-1 should have increased 10^{60} times.
Today, we measure a nearly flat Universe, so Omega=1.
This means that at the beginning, the Universe should have been fine-tuned to flatness at the 10^{60} level!

Is this really a signal that something is wrong or that it requires an explanation?

Inflation as a possible solution

Inflation condition $\ddot{a} > 0$

the Friedmann acceleration equation $\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$

implies

$$\ddot{a} > 0 \iff \frac{\dot{H}}{H^2} < 1 \iff \rho + 3P < 0 \iff \frac{d}{dt}\left(\frac{1}{aH}\right) < 0$$

"special" fluid needed

$1 + 3w < 0 \Rightarrow w < -1/3$

The comoving Hubble radius is decreasing!

—> Solution of the horizon problem

Solution of the Horizon Problem

Considering only one matter/rad. component $H = \frac{\dot{a}}{a} = H_0 \sqrt{\Omega} a^{-\frac{3}{2}(1+w)}$

if $w \neq -1$ then $a(t) \propto t^{2/3(1+w)}$

if $w = -1$ then $a(t) = e^{Ht}$

In terms of conformal time $a(\eta) \propto \begin{cases} \eta^{\frac{2}{1+3w}} & w \neq -1 \\ -\frac{1}{\eta} & w = -1 \end{cases}$

and therefore $\eta \propto \frac{2}{(1+3w)} a^{\frac{1}{2}(1+3w)}$

Since inflation requires $1 + 3w < 0$ then if a—> the conformal time goes to -infinity.

This means that inflation introduces much more conformal time between the decoupling time and the initial singularity.

Inflation and the Horizon Problem

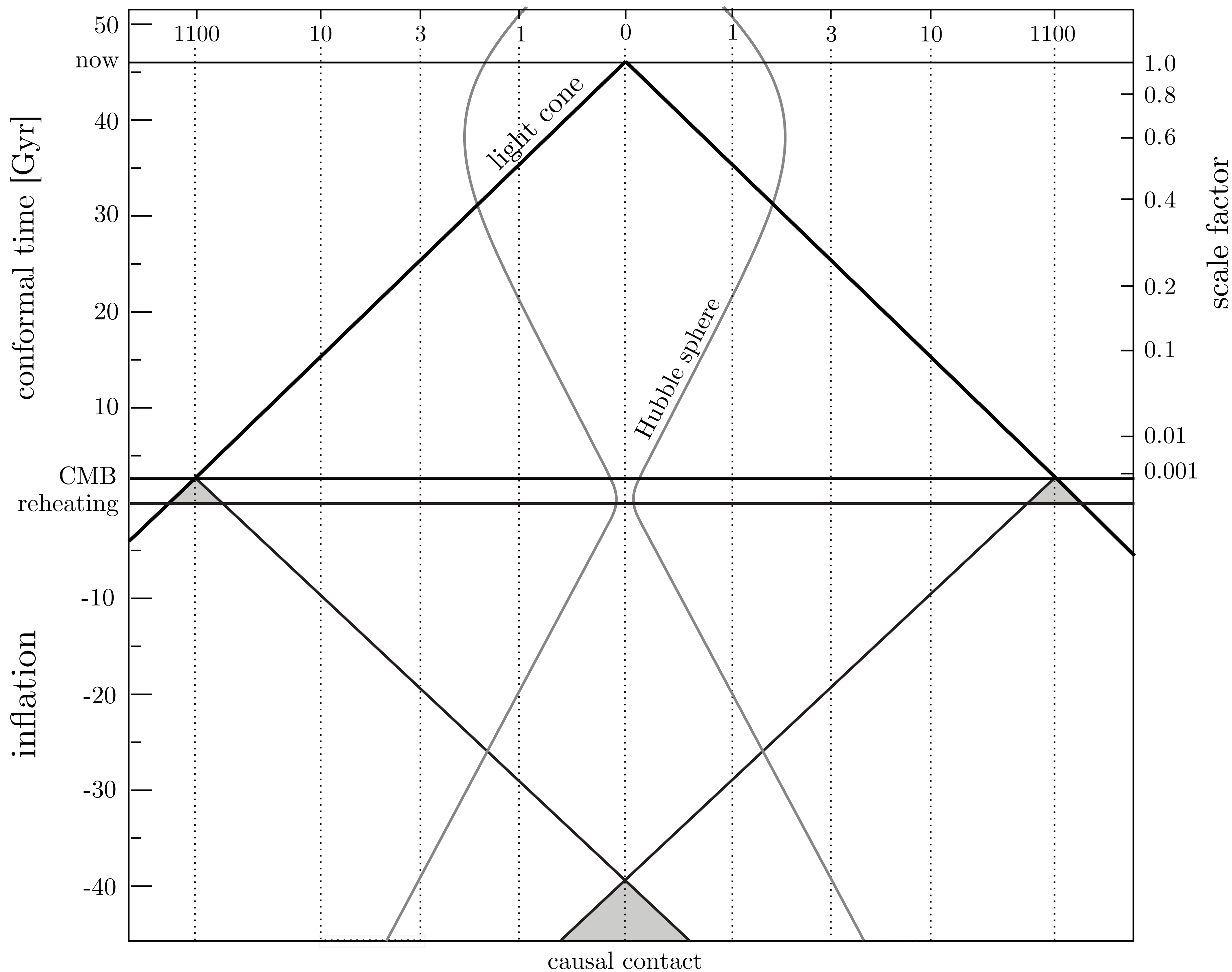


figure from Daniel Baumann

Solution to the Flatness Problem

During inflation $\dot{H} < H^2$ and approximating $\dot{H} \sim 0$

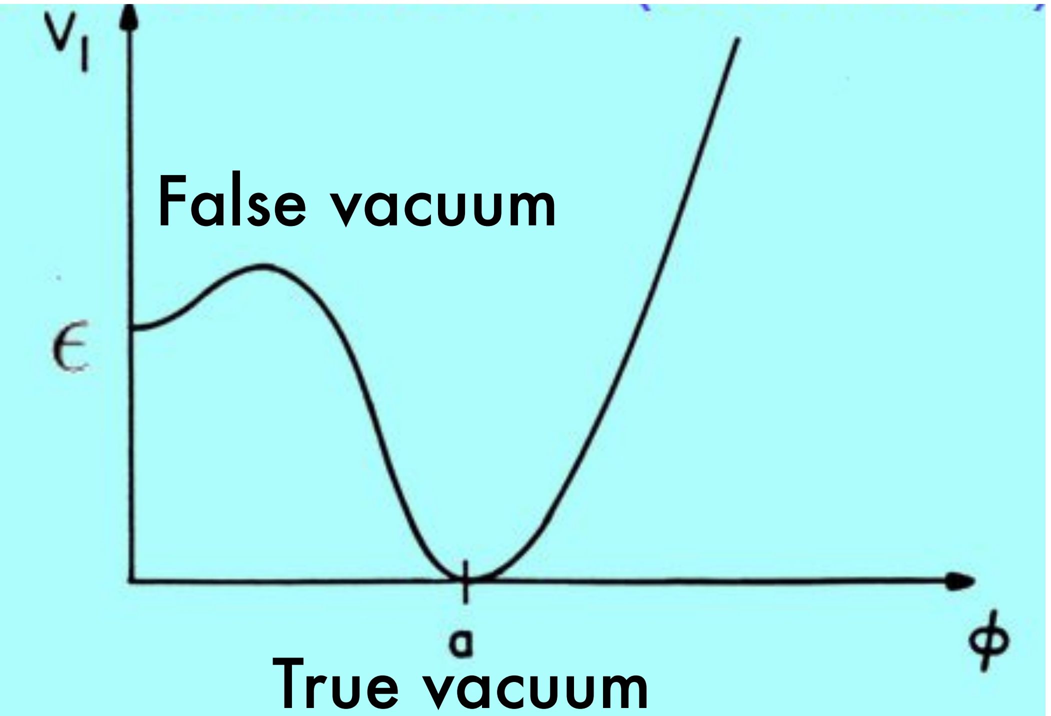
and combining the two Friedmann equations we can obtain an equation for the evolution of the density parameter

$$\frac{d\Omega}{d(\ln a)} = (1 + 3w)\Omega(\Omega - 1)$$

An analysis of this equation shows that if 1+3w<0, $\Omega = 1$ is an attractor solution for the evolution of the parameter. If 1+3w>0 it is instead an unstable fixed point. Therefore, inflation naturally drives the Universe towards an apparent flatness.

NOTE: Inflation does not imply k=0, but just that locally the Universe looks flat.

"Old Inflation"



Guth's Inflation:

- First order phase transition
- A scalar field (correct equation of state) initially trapped in a false vacuum starts to "fall down" into the true one.
- Bubbles of true vacuum enucleate in a "sea" of false vacuum.
- The energy difference between true and false vacuum acts like a cosmological constant, driving inflation.

- Condensed matter analogy: the false vacuum is like a superheated fluid and the true vacuum is the "vapour" phase. During the transition, bubble enucleation happens.

- PROBLEM: the fast expansion of the Universe never allows the bubbles to merge and thermalize. Collision of bubbles might also lead to large anisotropies.

- This scenario fails in two requirements for inflation: thermalization and reheating.

"New Inflation"

Equation of motion of a scalar field in the FRW metric + space homogeneity

$$\frac{1}{\sqrt{-g}}\partial_t(-g\partial_t\phi) + \sqrt{-g}\frac{\partial V}{\partial \phi} = 0$$

$$\boxed{\ddot{\phi} + 3H\dot{\phi} - \cancel{\frac{\nabla^2\phi}{a^2}} + \frac{dV}{d\phi} = 0}$$

$$3H\dot{\phi} \longrightarrow \text{Hubble "friction"}$$

Energy-Momentum tensor

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \partial^\mu\partial^\nu\phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

If the field varies slowly as a function of the space-time coordinates

$\boxed{T^{\mu\nu} \approx g^{\mu\nu}V(\phi)}$ $\longrightarrow$ structure similar to a cosmological constant

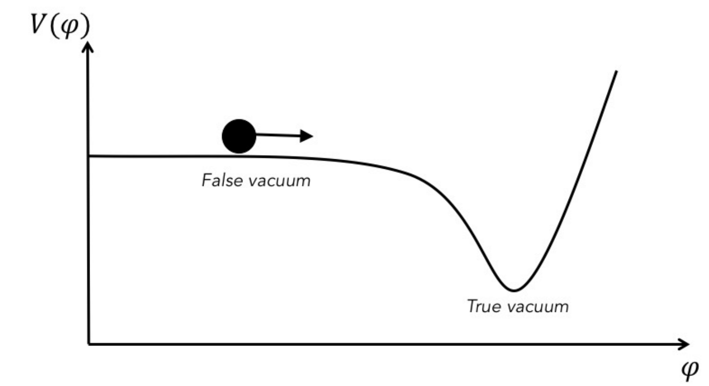
Difference: V is not constant but varies slowly, moving towards its minimum and thus stopping inflation.

"New Inflation"

A slowly-varying field can be achieved with a large Hubble parameter

Slow-roll approximation $\ddot{\phi} \approx 0 \Longrightarrow \dot{\phi} \approx const$

$$\dot{\phi} = -\frac{dV/d\phi}{3H} \quad \Rightarrow \quad dt = \frac{3H(dV/d\phi)}{d\phi}$$



Friedmann acceleration equation during inflation: $H^2 = \frac{8\pi G}{3}V(\phi)$

With the above equations we can calculate the number of e-foldings

$$N = \int d(\ln a) = \int H dt = 8\pi G \int d\phi \frac{V(\phi)}{dV/d\phi}$$

"New Inflation"

For fixing ideas, we can take $V \sim g\phi^n/n$

and the e-foldings become

$$N = \frac{4\pi G}{n}(\phi_1^2 - \phi_2^2) \approx \frac{4\pi G}{n}\phi_1^2 = \frac{4\pi}{nM_P^2}\phi_1^2$$

where we assumed $\phi_2 \ll \phi_1$ at the end of inflation.

The largest scales in the CMB are created at N=60 before the end of inflation.
This means that N>60 for fixing the horizon problem and

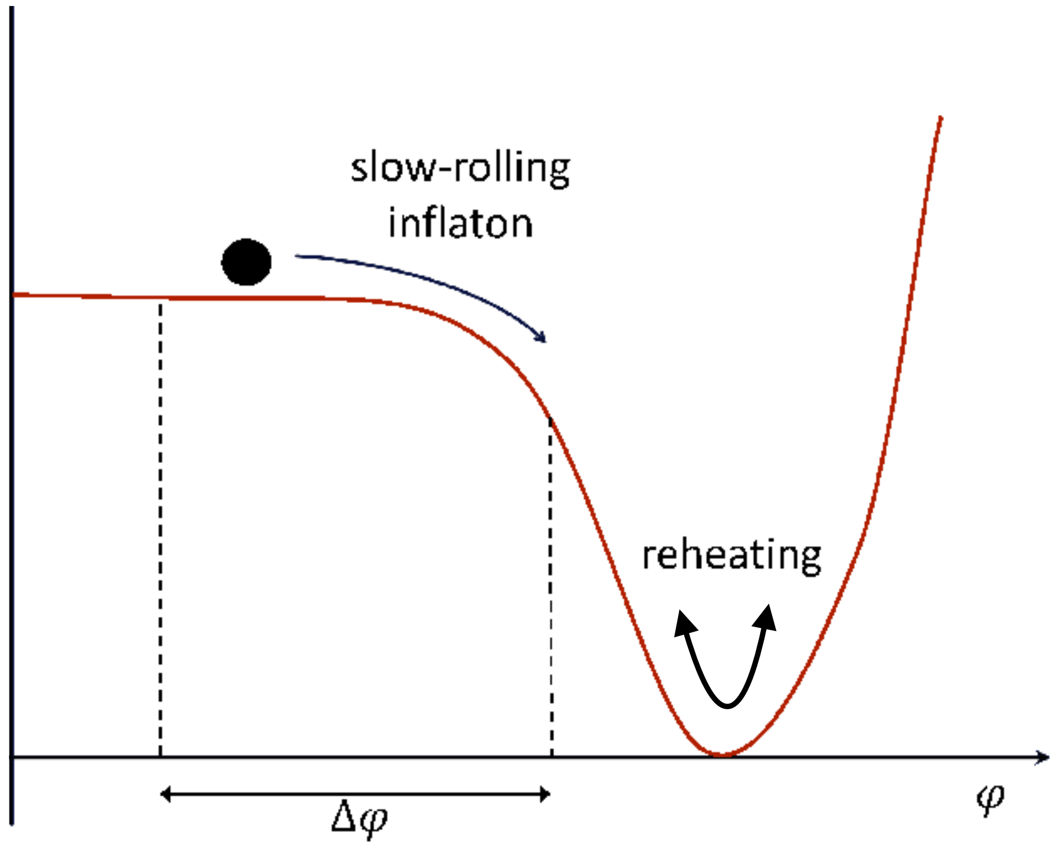
$$\phi_1^2 > 5.6 \cdot n \cdot M_P^2$$

6.9 Reheating

During inflation most of the energy density in the universe is in the inflaton potential. The inflationary phase ends when the potential becomes steep and the inflaton field gains kinetic energy. The energy of the inflaton has to be transferred to the SM particles. This process is called **reheating** and corresponds to the start of the classical hot Big Bang. After reaching the minimum, the inflation starts to oscillate into it. Let's assume $V(\phi) = m^2\phi^2$ in the neighborhood of the minimum. With homogeneity we have

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \quad . \tag{6.34}$$

The Universe expands and the expansion scale will become larger than the oscillation period of the inflaton. This situation is described by $H^{-1} \ll m^{-1}$ and it means that we can disregard the Hubble friction term in Eq. 6.34 and have oscillations of frequency m.



Double-Field Inflation

Consider the potential $V(\phi,\chi) = \frac{1}{2}(a\phi^2 - v^2)\chi^2 + \frac{b}{4}\chi^4 + V(\phi)$

with two inflation fields χ and ϕ

At beginning the phi field is small and slow-rolls towards the chi=0 valley.

When $\phi^2 < v^2/a$ the second field acquires a non-zero VEV $\chi^2 \sim v^2/b$

and the phi mass becomes large $m^2_{eff} \sim (a/b)v^2$

The large mass drives the inflation towards the equilibrium point phi=0.

Starobinski R^2 Inflation

Consider a modification of RG with a R^2 term (motivated by quantum corrections)

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(1-\frac{R}{6m^2})$$

In standard RG $R = -8\pi G T^\mu_\mu$ and therefore R is not a real degree of freedom.

In R^2 gravity, the Ricci scalar satisfies the equation

$$\ddot{R} + 3H\dot{R} + m^2(R + 8\pi G T^\mu_\mu) = 0$$

or in terms of the Hubble constant

This looks like the equation of a scalar field (called the "scalaron") which in terms of the Hubble parameter looks like

$$R = -6\dot{H} - 12H^2$$

The scalaron can drive an inflationary phase and its subsequent decay generates the other particles.

Predictions from Inflation

Provide a mechanism for primordial perturbations generation: quantum fluctuations stretched by the exponential expansion.

Solves the horizon problem and produces a flat Universe

Predicts an almost scale-invariant perturbation spectrum $P(k) \propto k^n \quad n = 1$

Predicts primordial gravitational waves (–> B-modes in the CMB)