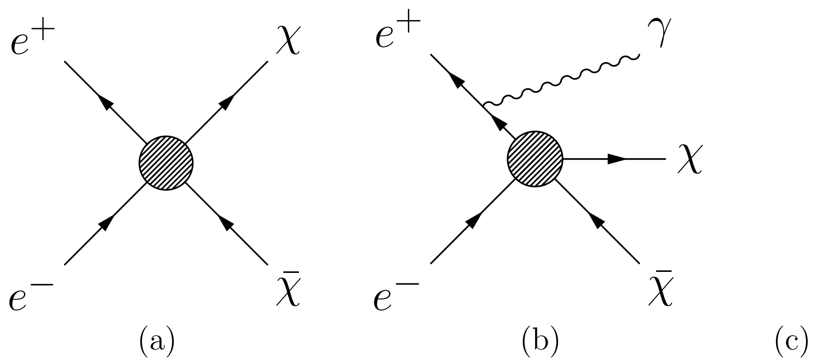

Figure 1: (a) Effective dark matter–Standard Model interaction. (b) One of the two diagrams for dark matter + mono-photon production at an e^+e^- collider.

1. **Mono-photon signals of dark matter at an e^+e^- collider**

   Consider (Dirac) fermionic dark matter (χ) interacting with electrons and positrons (e) through an effective vertex of the form
$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda^2}(\bar{e}e)(\bar{\chi}\chi) \tag{1}$$
   (see figure 1a).

   (a) Show that the differential cross section $d\sigma/dE_\gamma$ for the process $e^+e^- \to \gamma\bar{\chi}\chi$ (figure 1b) in the approximation of vanishing electron mass ($m_e = 0$) and vanishing dark matter mass ($m_\chi = 0$). is given by
$$\frac{d\sigma}{dE_\gamma} = \frac{8\pi^2\alpha E_\gamma}{s\Lambda^4}(s - 2\sqrt{s}E_\gamma)\int d\cos\theta\, \frac{1}{\sin^2\theta}\left[2\frac{\sqrt{s}}{E_\gamma} - \frac{s}{E_\gamma^2} + 2\cos^2\theta\right] \tag{2}$$
   Here, E_γ is the photon energy, θ is the angle of the photon relative to the beam axis, and $\sqrt{s}$ is the center of mass energy. Remember that there are two contributing Feynman diagrams!
   Hints: Express the spin-averaged squared matrix element in terms of $\sqrt{s}$, E_γ, and the invariant mass of the two DM particles, m_{12}^2. For the integral over the 3-body phase space $d\Phi_3$, use the decomposition
$$\begin{aligned} d\Phi_3(\sqrt{s}; k_1, k_2, k_\gamma) &= (2\pi)^4\delta^{(4)}(\sqrt{s} - k_1 - k_2 - k_\gamma)\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}\frac{d^3k_\gamma}{(2\pi)^3 2E_\gamma} \\ &= \frac{dm_{12}^2}{2\pi}\, d\Phi_2(\sqrt{s}; k_{12}, k_\gamma)\, d\Phi_2(k_{12}; k_1, k_2)\,. \end{aligned} \tag{3}$$
   Here, k_1, k_2 and E_1, E_2 are the 4-momenta and energies of the DM particles, respectively, k_γ is the photon 4-momentum, and $k_{12} \equiv k_1 + k_2$. Note that $m_{12}^2 = k_{12}^2$. In the second line of equation (3), Φ_2 denotes the two-body phase space, which is defined as
$$d\Phi_2(k_{12}; k_1, k_2) \equiv (2\pi)^4\delta^{(4)}(\sqrt{k_{12}^2} - k_1 - k_2)\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}\,. \tag{4}$$
   Note that Φ_3 and Φ_2 are Lorentz invariant!

(b) Plot $d\sigma/dE_\gamma$ as a function of E_γ for $\Lambda = 1$ TeV, and taking into account only photons with $4^\circ \leq \theta \leq 176^\circ$. Explain the singularity at $E_\gamma = 0$ and why it is not a problem in practice.

(c) The main Standard Model background to $e^+e^- \to \gamma\bar{\chi}\chi$ is $e^+e^- \to \gamma Z$, followed by the decay $Z \to \nu\nu$. What E_γ distribution do you expect (qualitatively) for this background? At what value of E_γ will the background peak? Compare the expected background distribution to the expected signal distribution.

(d) In a particular LEP search for monophoton events, $N^{\mathrm{obs}} = 1\,518$ events have been observed. The expected number of Standard Model background events is $N^{\mathrm{bg}} = 1\,562$. Compute a lower 95% confidence level (CL) limit on the number N^{sig} of signal events. A particular value of N^{sig} is ruled out at 95% CL if the probability of obtaining $\leq N^{\mathrm{obs}}$ events from a Poisson distribution with expectation value $N^{\mathrm{bg}} + N^{\mathrm{sig}}$ is less than 5%.