1. **Charge conjugation and seesaw mechanism**
   Proof the following relations:
   (a) $(\psi^c)^c = \psi$,
   (b) $\overline{\psi_1^c}\psi_2 = \overline{\psi_2^c}\psi_1$,
   (c) Show that

$$-\frac{1}{2}\overline{n^c}Mn + h.c. = -m_D\overline{\nu_L}N_R - \frac{1}{2}m_M\overline{(N_R)^c}N_R + h.c.\,, \tag{1}$$

   i.e. that eqs. (*) and (**) from the lecture are equivalent. Here, $n = (\nu_L, (N_R)^c)^T$ and

$$M = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}. \tag{2}$$

   (d) Compute the eigenvalues and eigenvectors of M approximately and use them to prove that, indeed, an effective mass term of the form $-\frac{1}{2}m_\nu\overline{(\nu_L')^c}\nu_L'$, with $m_\nu = m_D^2/m_M$, is generated.

2. **Majorana mass terms**
   (a) Why is a Majorana mass term for neutrinos, i.e. a term of the form

$$\mathcal{L} \supset \frac{1}{2}m\,\overline{(\nu_L)^c}\,\nu_L + h.c.\,, \tag{3}$$

   forbidden in the Standard Model?
   (b) Extensions of the Standard Model containing an $SU(2)_L$ triplet Higgs field Φ with hypercharge 2 can generate a mass term of the form (3). Explain how this works.
   Hint: It is useful to write Φ in the form

$$\Phi = \begin{pmatrix} \frac{\phi^+}{\sqrt{2}} & \phi^{++} \\ \phi^0 & \frac{\phi^+}{\sqrt{2}} \end{pmatrix}, \tag{4}$$

   which transforms under an $SU(2)_L$ rotation U_L according to

$$\Phi \to U_L\Phi U_L^\dagger\,. \tag{5}$$

3. **Neutrinoless double electron capture**
   Neutrinoless double electron capture has been proposed as an alternative to neutrinoless double beta decay for measuring (Majorana) neutrino mass. (Sujkowski Wycech, arXiv:hep-ph/0312040)
   (a) Draw the Feynman diagram corresponding to neutrinoless double electron capture.
   (b) What would be the experimental signature?
   (c) Discuss how a coincidence trigger can help to reduce backgrounds.