Quantum Field Theory
Exercise 5

November 17, 2015

-to be handed in by 26.11.2015 (12:00 h) to the "Theoretische Physik 6a" letterbox (No. 37) in the foyer of Staudingerweg 7.

1. **The Quantized Dirac Field (40 points)**

   Express the following quantities in terms of creation and annihilation operators:

   (a)(15 points) momentum $\mathbf{P} = -i\int d^3x\, \psi^\dagger \nabla \psi$,

   (b)(15 points) charge $Q = \int d^3x\, \psi^\dagger \psi$.

   In addition, calculate:

   (c)(10 points) $\left[\mathbf{P},\, a^\dagger(\mathbf{p}', s')\, a(\mathbf{p}', s')\right]$.

2. **Axial Current (30 points)**
   For a Dirac field, the transformations,

   $$\psi(x) \to \psi'(x) = e^{i\alpha\gamma_5}\psi(x), \qquad \psi^\dagger(x) \to \psi^{\dagger\prime}(x) = \psi^\dagger(x)e^{-i\alpha\gamma_5},$$

   where α is here an arbitrary real parameter, are called chiral phase transformations.

   (a)(15 points) Show that the Lagrangian density $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$ is invariant under chiral phase transformations in the zero-mass limit $m = 0$ only, and that the corresponding conserved current in this limit is the axial vector current $J^\mu_A = \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)$.

   (b)(15 points) Deduce the equations of motions for the fields

   $$\psi_L(x) = \frac{1}{2}\left(\mathbb{1} - \gamma_5\right)\psi(x), \qquad \psi_R(x) = \frac{1}{2}\left(\mathbb{1} + \gamma_5\right)\psi(x),$$

   for non-vanishing mass, and show that they decouple in the limit $m = 0$.

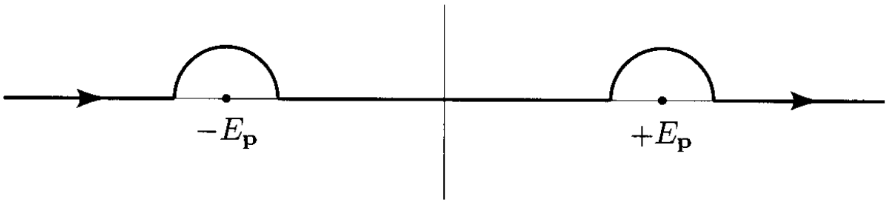

Figure 1: Integration curve for the retarded Green's function.

3. **Green's Function for the Klein-Gordon Field (30 points)**

Starting solely from the definition of the Green's function for the Klein-Gordon field

$$\left(\Box + m^2\right) \Delta(x-y) = -i\delta^{(4)}(x-y), \tag{1}$$

express the retarded Green's function in the form

$$\Delta_R(x-y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\mathbf{k}}} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \left(e^{-iE_{\mathbf{k}}(x^0-y^0)} - e^{iE_{\mathbf{k}}(x^0-y^0)}\right) \Theta\left(x^0-y^0\right), \tag{2}$$

where Θ is a Heaviside function.
Hint: Use the Residue theorem. The integration curve is given in fig. 1.

Try to interpret the significance of the retarded Green's function in quantum field theory in comparison with the classical field theory! Why do we use Feynman propagator for quantifying propagation amplitudes, rather than any of three other Green's functions?