Quantum Field Theory
Exercise 7

December 3, 2015

-to be handed in by 10.12.2015 (12:00 h) to the Theoretische Physik 6a letterbox (No. 37) in the foyer of Staudingerweg 7.

1. **Creation of Scalar Particles by a Classical Source (50 points)**
Creation of Klein-Gordon particles by a classical source can be described by the Hamiltonian

$$H = H_0 + \int d^3x(-j(t, \mathbf{x})\phi(x)),$$

where H_0 is the free Klein-Gordon Hamiltonian, $\phi(x)$ is the Klein-Gordon field, and $j(x)$ is a scalar function.
a) (10 points) Show that the probability that the source creates no particles is given by

$$P(0) = \left| \langle 0| T \left\{ \text{Exp} \left[i \int d^4x\, j(x)\, \phi_I(x) \right] \right\} |0\rangle \right|^2 .$$

b) (10 points) Evaluate the term in $P(0)$ of order j^2, and show that $P(0) = 1 - \lambda + \mathcal{O}(j^4)$ where

$$\lambda = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} |\tilde{j}(p)|^2,$$

with $\tilde{j}(p) = \int d^4x\, j(x)\, e^{ipx}$.

c) (10 points) Represent the term computed in part b) as a Feynman diagram. Now represent the whole perturbation series for $P(0)$ in terms of Feynman diagrams. Show that the series exponentiates, so that it can be summed exactly : $P(0) = e^{-\lambda}$.

d) (10 points) Compute the probability that the source creates one particle. Perform this computation first to $\mathcal{O}(j)$ and then to all orders.

e) (10 points) Obtain the following expression for the probability that the source produces n particles

$$P(n) = \frac{1}{n!}\lambda^n e^{-\lambda}.$$

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2. **Decay of a Scalar Particle (50 points)**

Consider the following Lagrangian involving two real scalar fields Φ and ϕ, and fermion f :

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2}m_\Phi^2 \Phi^2 + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m_\phi^2 \phi^2 - \mu \Phi \phi \phi + \bar{f}(i\partial\!\!\!/ - m_f)f - y\bar{f}\Phi f.$$

Using the formula for the decay rate

$$d\Gamma = \frac{1}{2m_{\mathcal{A}}} \left(\prod_f \frac{d^3\mathbf{p}_f}{(2\pi)^3} \frac{1}{2E_f} \right) |\mathcal{M}(m_{\mathcal{A}} \to \{p_f\}|^2 \, (2\pi)^4 \delta^{(4)}(p_{\mathcal{A}} \to -\sum p_f),$$

where indices $\mathcal{A}$ and f denote quantities that correspond to the parent particle and the decay products, respectively, obtain the expression for the lifetime of a particle Φ by considering its decays shown in fig. 1.
The decays in fig. 1 are kinematically allowed as we assume $m_\Phi > 2m_\phi, 2m_f$.
Hint : In obtaining the squared matrix element $|\mathcal{M}|^2$ for diagram b) in fig. 1 sum over the spins of final state particles. It is helpful to write the expression in spinor indices!
The Standard model (SM) of particles and interactions consists of one scalar particle - Higgs boson that can decay into fermions (quarks and leptons) via diagram b) at leading order in perturbation theory. Being the only scalar particle, diagram a) does not have an analog in the SM, but there are many theories beyond the Standard model whose particle content has more than one scalar, making the decay into two other scalar particles possible.

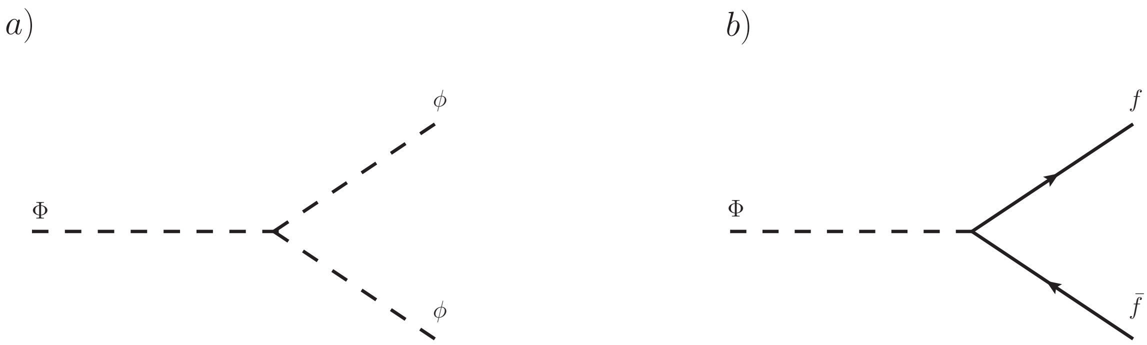

Figure 1: a) Decay of a scalar particle Φ into two ϕ particles. b) Decay of a scalar particle Φ into a pair of fermions.