

Quantum Field Theory

Exercise 10

January 14, 2016

-to be handed in by 21.01.2016 (12:00 h) to the "Theoretische Physik 6a" letterbox (No. 37) in the foyer of Staudingerweg 7.

1. e^+e^- pair annihilation into photons (100 points)

$$e^+(p_2) e^-(p_1) \rightarrow \gamma(k_1) \gamma(k_2)$$

a) (80 points)

Show that the squared matrix element (summed over initial spins and averaged over the outgoing polarisations) is equal to

$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = 2e^4 \left[\frac{p_1 k_2}{p_1 k_1} + \frac{p_1 k_1}{p_1 k_2} + 2m^2 \left(\frac{1}{p_1 k_1} + \frac{1}{p_1 k_2} \right) - m^4 \left(\frac{1}{p_1 k_1} + \frac{1}{p_1 k_2} \right)^2 \right]. \quad (1)$$

The relevant Feynman diagrams are shown in 1.

Hint: First add the matrix elements corresponding to each diagram and then construct the squared matrix element.

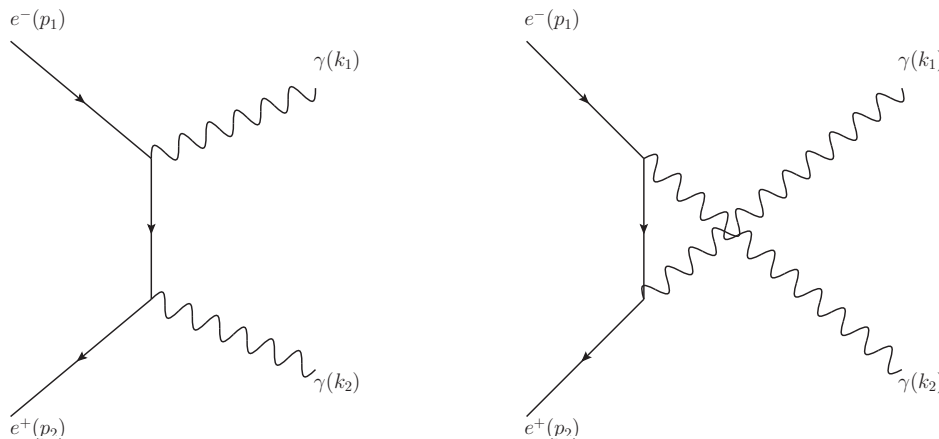


Figure 1: Tree-level Feynman diagram for $e^+(p_2) e^-(p_1) \rightarrow \gamma(k_1) \gamma(k_2)$ process.

b) (10 points) Calculate the differential cross section in the center of mass frame (the kinematics are shown in Figure 2). It should give

$$\frac{d\sigma}{d \cos \theta} = \frac{\pi \alpha^2}{2Ep} \left[\frac{E^2 + p^2 \cos^2 \theta}{m^2 + p^2 \sin^2 \theta} + \frac{2m^2}{m^2 + p^2 \sin^2 \theta} - \frac{2m^4}{(m^2 + p^2 \sin^2 \theta)^2} \right]. \quad (2)$$

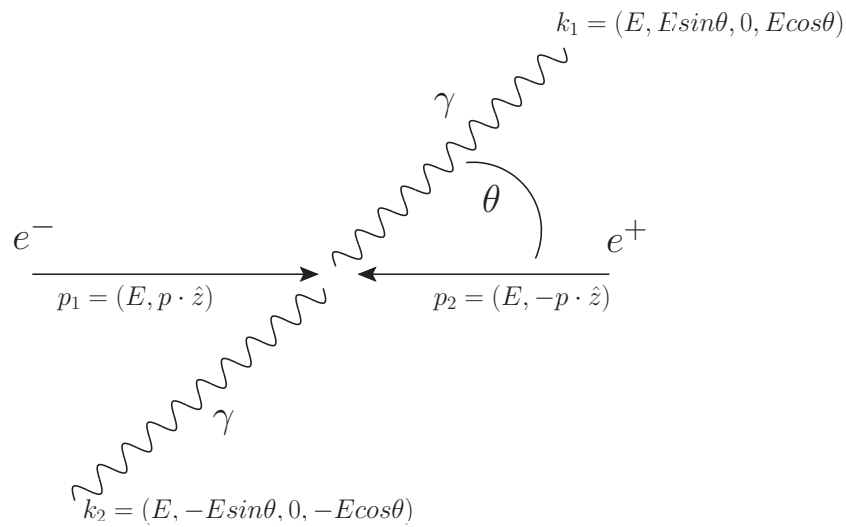


Figure 2: Parametrisation of the momenta in the com system.

c) (5 points)

What is the high energy limit ($E^2 \gg m^2$) of the differential cross section?

d) (5 points)

Explain the origin of the divergence of the differential cross section in the high energy limit!