Quantum Field Theory
Exercise 11

January 20, 2016

-to be handed in by 28.01.2016 (12:00 h) to the "Theoretische Physik 6a" letterbox (No. 37) in the foyer of Staudingerweg 7.

1. **Path Integral for a Free Complex Scalar Field (35 points)**

   Obtain the following form of a path integral $Z_0(J, J^*)$ for a free complex scalar field

   $$Z_0(J, J^*) = \text{Exp}\left[-\int d^4x \int d^4x' J^*(x')\Delta(x-x')J(x)\right], \tag{1}$$

   where $\Delta(x-x')$ is the Feynman propagator in position space. *Hint:* Note that in the case of a complex scalar field one has to introduce two source terms of the form $J^*(x)\phi(x) + J(x)\phi^*(x)$.

2. **Crossing Symmetry (20 points)**

   In the exercise 9 you derived the spin averaged squared matrix element for the process $e^+(p_2)\, e^-(p_1) \to \gamma(k_1)\, \gamma(k_2)$

   $$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = 2e^4\left[\frac{p_1k_2}{p_1k_1} + \frac{p_1k_1}{p_1k_2} + 2m^2\left(\frac{1}{p_1k_1} + \frac{1}{p_1k_2}\right) - m^4\left(\frac{1}{p_1k_1} + \frac{1}{p_1k_2}\right)^2\right]. \tag{2}$$

   Using the so called *crossing symmetry* (see literature : Srednicki,Peskin...), obtain the spin averaged squared matrix element for Compton scattering $e^-(p)\gamma(k) \to e^-(p')\gamma(k')$.

3. **Polarized $e^+e^- \to \mu^+\mu^-$ Cross Section (45 points)**

   Starting from the matrix element for $e^+e^- \to \mu^+\mu^-$ process

   $$i\mathcal{M}(e^-(p)e^+(p') \to \mu^-(k)\mu^+(k')) = \frac{ie^2}{(p+p')^2}\left(\bar{v}(p')\gamma^\mu u(p)\right)\left(\bar{u}(k)\gamma_\mu v(k')\right) \tag{3}$$

calculate the differential cross section

$$\frac{d\sigma}{d\Omega}(e^+_R e^-_L \to \mu^+_L \mu^-_R), \tag{4}$$

where indices L and R label left and right handed particles, respectively. Work in the massless limit ($m_e, m_\mu \to 0$). Express the differential cross section in terms of the incoming electron energy E and the angle between the incoming electron and the outgoing muon.