Quantum Field Theory
Exercise 10

Janurary 19, 2016

-to be handed in by 26.01.2017 (12:00 h) to the letterbox No. 37 (“relativistische QFT”) in the foyer of Staudingerweg 7.

1. e^+e^- **pair annihilation into photons (100 points)**

$$e^+(p_2)\,e^-(p_1) \to \gamma(k_1)\,\gamma(k_2)$$

a) (80 points)
Show that the squared matrix element (summed over initial spins and averaged over the outgoing polarisations) is equal to

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = 2e^4\left[\frac{p_1k_2}{p_1k_1}+\frac{p_1k_1}{p_1k_2}+2m^2\left(\frac{1}{p_1k_1}+\frac{1}{p_1k_2}\right)-m^4\left(\frac{1}{p_1k_1}+\frac{1}{p_1k_2}\right)^2\right]. \tag{1}$$

The relevant Feynman diagrams are shown in 1.
Hint: First add the matrix elements corresponding to each diagram and then construct the squared matrix element.

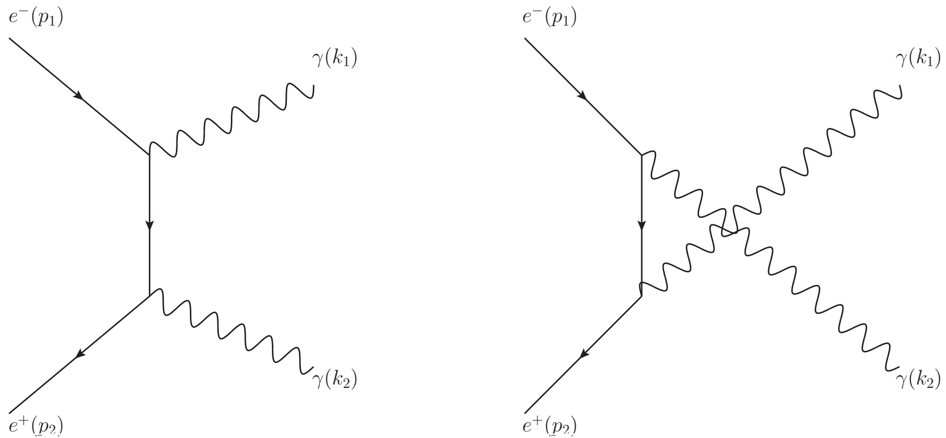

Figure 1: Tree-level Feynman diagram for $e^+(p_2)\,e^-(p_1) \to \gamma(k_1)\,\gamma(k_2)$ process.

b) (10 points) Calculate the differential cross section in the center of mass frame (the kinematics are shown in Figure 2). It should give

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2Ep}\left[\frac{E^2+p^2\cos^2\theta}{m^2+p^2\sin^2\theta}+\frac{2m^2}{m^2+p^2\sin^2\theta}-\frac{2m^4}{(m^2+p^2\sin^2\theta)^2}\right]. \tag{2}$$

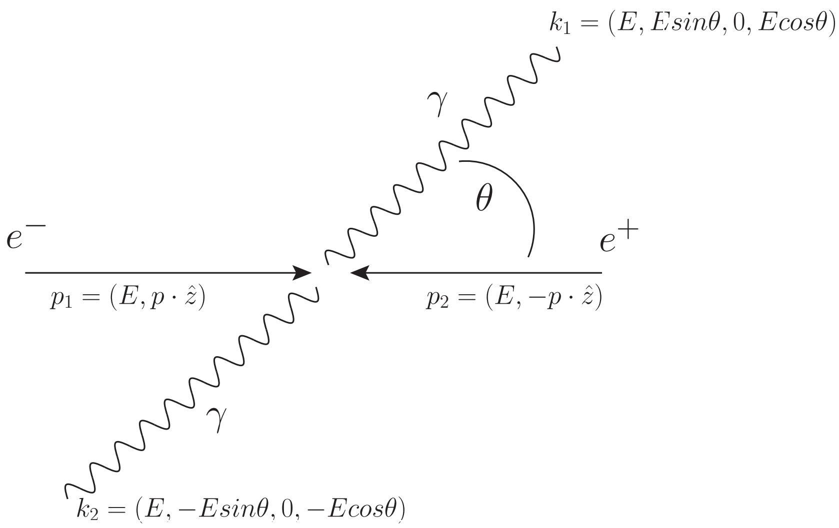

Figure 2: Parametrisation of the momenta in the com system.

c) (5 points)
What is the high energy limit ($E^2 \gg m^2$) of the differential cross section?

d) (5 points)
Explain the origin of the divergence of the differential cross section in the high energy limit!