Quantum Field Theory
Exercise 5

November 24, 2016

-to be handed in by 01.12.2016 (12:00 h) to the letterbox No. 37 ("relativistische QFT") in the foyer of Staudingerweg 7.

1. **Axial Current (30 points)**
   For a Dirac field, the transformations,

$$\psi(x) \to \psi'(x) = e^{i\alpha\gamma_5}\psi(x), \qquad \psi^\dagger(x) \to \psi^{\dagger\prime}(x) = \psi^\dagger(x)e^{-i\alpha\gamma_5},$$

   where α is here an arbitrary real parameter, are called chiral phase transformations.

   a)(15 points) Show that the Lagrangian density $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$ is invariant under chiral phase transformations in the zero-mass limit $m = 0$ only, and that the corresponding conserved current in this limit is the axial vector current $J^\mu_A = \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)$.

   b)(15 points) Derive the equations of motions for the fields

$$\psi_L(x) = \frac{1}{2}\left(\mathbb{1} - \gamma_5\right)\psi(x), \qquad \psi_R(x) = \frac{1}{2}\left(\mathbb{1} + \gamma_5\right)\psi(x),$$

   for non-vanishing mass, and show that they decouple in the limit $m = 0$.

2. **Fierz Transformations (35 points)**
   a) (10 points) Normalize the 16 matrices Γ^A (scalar $\mathbb{1}$, pseudoscalar γ_5, vector γ^μ, axial vector $\gamma^\mu\gamma_5$ and tensor $\sigma^{\mu\nu}$) to the convention

$$\mathrm{Tr}\left[\Gamma^A\Gamma^B\right] = 4\delta^{AB}.$$

   b) (10 points) We can define the so called *Fierz identity* as an equation

$$(\bar{u}_1\Gamma^A u_2)(\bar{u}_3\Gamma^B u_4) = \sum_{C,D} C^{AB}{}_{CD}\,(\bar{u}_1\Gamma^C u_4)(\bar{u}_3\Gamma^D u_2),$$

   with unknown coefficients $C^{AB}{}_{CD}$. Show that

$$C^{AB}{}_{CD} = \frac{1}{16}\mathrm{Tr}\left[\Gamma^C\Gamma^A\Gamma^D\Gamma^B\right].$$

c) (15 points) Work out explicitly the Fierz transformation laws for the product $(\bar{u}_1\gamma^\mu P_L u_2)(\bar{u}_3\gamma_\mu P_L u_4)$.

3. **Perturbation Theory in Quantum Mechanics (35 points)**
Consider a one-dimensional simple harmonic oscillator whose classical angular frequency is ω_0. For $t < 0$ it is known to be in the ground state. For $t > 0$ there is also a time-dependent potential

$$V(t) = F_0 x \cos(\omega t),$$

where F_0 is a constant.
a) (15 points) Calculate the field operator $\Psi_I(\mathbf{x}, t)$ in the interaction picture by rewriting the Schrödinger equation as an integral equation up to first order in the interaction Hamiltion H_{int}.
b) (15 points) Determine the probability that the system after time t is in the ground state.
c) (5 points) Is this procedure valid for $\omega \simeq \omega_0$?