Quantum Field Theory
Exercise 6

December 1, 2016

-to be handed in by 8.12.2016 (12:00 h) to the letterbox No. 37 ("relativistische QFT") in the foyer of Staudingerweg 7.

1. **Wick's Theorem (30 points)**
   Using Wick's theorem evaluate the following expressions and show the results diagrammatically:

   (a) **(15 points)** $\langle 0|T(\phi^4(x)\phi^4(y))|0\rangle$

   (b) **(15 points)** $\langle 0|T(\bar{\psi}(x)\psi(x)\bar{\psi}(y)\psi(y))|0\rangle$

   Note that the contraction for the Dirac field is defined as $\overbrace{\psi(x)\bar{\psi}}(y) = S_F(x-y)$, where $S_F(x-y)$ is the Feynman propagator. Keep also in mind that fermion fields anticommute!

2. **Symmetry factor (20 points)**
   Compute the symmetry factor for each diagram shown in fig. 1.

3. **ϕ^3 Theory (50 points)**
   The Lagrangian of ϕ^3 theory reads

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\phi^3. \tag{1}$$

   (a) **(5 points)** What are the position space Feynman rules for this theory?

   Hint: Compare eq. 1 to ϕ^4 theory (cf. lecture notes). What could in principle be different?

   (b) **(20 points)** Starting from $\langle 0|T\phi(x_1)\phi(x_2)...\phi(x_n)\text{Exp}(-i\int d^4z\mathcal{H}_\text{I})|0\rangle$, where $\mathcal{H}_\text{I}$ is the interacting Hamilton density, determine all *connected* Feynman diagrams up to order λ^2 with up to $n=3$ external vertices.

   (c) **(20 points)** Using the Feynman rules in position space, write down the amplitude corresponding to the diagram shown in fig. 2. Apply a Fourier transform to the propagator terms. In order to solve the remaining integral over momentum space, apply the so called *Wick rotation* given by

$$k^0 = ik_E^0, \qquad \mathbf{k} = \mathbf{k}_E,$$

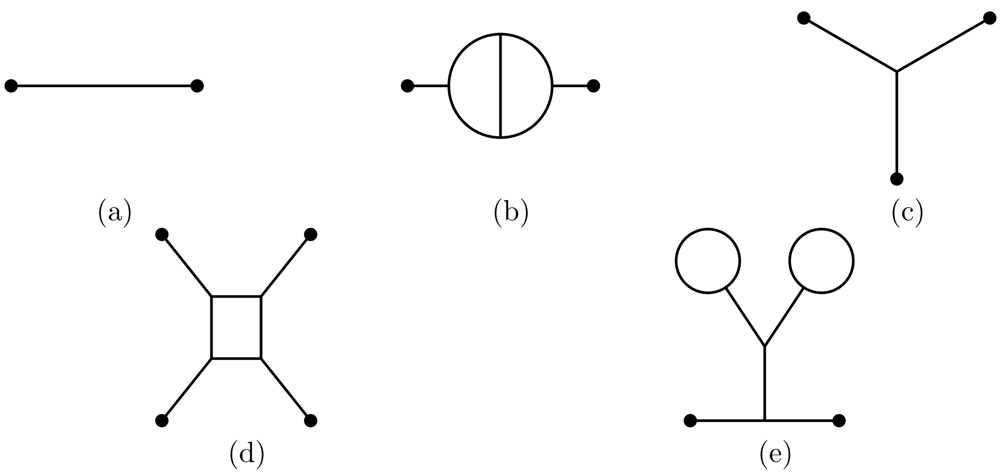

Figure 1

where the subscript E denotes components in Euclidean space. Note that the Wick rotation effectively transforms the problem to Euclidean space (metric tensor $\eta_E^{\nu\mu} = \text{diag}(1,1,1,1)$). Since the integral is not finite, perform a cut-off regularization, i.e. change the upper boundary of the radial integral from $+\infty$ to Λ (cut-off energy scale). The final result should contain the coupling λ, the mass of the scalar particle m and the aforementioned cut-off scale Λ.

Hint: note that the surface "area" of a four-dimensional unit sphere is $2\pi^2$.

(d) **(5 points)** Draw the potential of the field ϕ. Can you determine the ground state energy? Discuss the validity of this theory.

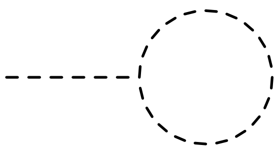

Figure 2