Quantum Field Theory
Exercise 7

December 8, 2016

-to be handed in by 15.12.2016 (12:00 h) to the letterbox No. 37 ("relativistische QFT") in the foyer of Staudingerweg 7.

1. **Creation of Scalar Particles by a Classical Source (50 points)**
Creation of Klein-Gordon particles by a classical source can be described by the Hamiltonian

$$H = H_0 + \int d^3x\,(-j(t,\boldsymbol{x})\phi(x)),$$

where H_0 is the free Klein-Gordon Hamiltonian, $\phi(x)$ is the Klein-Gordon field, and $j(x)$ is a c-number scalar function.

   (a) **(10 points)** Show that the probability that the source creates no particles is given by

$$P(0) = \left|\langle 0|T\Big\{\exp[i\int d^4x\,j(x)\phi_I(x)]\Big\}|0\rangle\right|^2.$$

   (b) **(10 points)** Evaluate the term in $P(0)$ of order j^2, and show that $P(0) = 1 - \lambda + \mathcal{O}(j^4)$, where

$$\lambda = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}}|\tilde{j}(p)|^2$$

   with $\tilde{j}(p) = \int d^4x\,j(x)e^{ipx}$.

   (c) **(10 points)** Represent the term computed in part (b) as a Feynman diagram. Now represent the whole perturbation series for $P(0)$ in terms of Feynman diagrams. Show that the series exponentiates, so that it can be summed exactly: $P(0) = e^{-\lambda}$.

   (d) **(10 points)** Compute the probability that the source creates one particle of momentum k. Perform this computation first to $\mathcal{O}(j)$ and then to all orders, using the trick from part (c).

   (e) **(10 points)** Obtain the following expression for the probability that the source produces n particles:

$$P(n) = \frac{1}{n!}\lambda^n e^{-\lambda}$$

2. **Decay of a Scalar Particle (50 points)**
Consider the following Lagrangian involving two real scalar fields Φ and ϕ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \mu\Phi\phi\phi.$$

What does the last term correspond to? Provided that $M < 2m$, calculate the lifetime of the Φ to lowest order in μ.

Hint: Note that the decay rate is given by

$$d\Gamma = \frac{1}{2M}\left(\prod_{f=\phi_1,\phi_2} \frac{d^3\boldsymbol{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)\left|\mathcal{M}(M \to \{p_{\phi_1}, p_{\phi_2}\})\right|^2 (2\pi)^4\delta^{(4)}\big(p_\Phi \to - \sum_{f=\phi_1,\phi_2} p_f\big).$$