Quantum Field Theory
Exercise 8

December 15, 2016

-to be handed in by 22.12.2016 (12:00 h) to the letterbox No. 37 ("relativistische QFT") in the foyer of Staudingerweg 7.

1. **Two-Loop Corrections to the Fermion Propagator (20 points)**
   Given a Yukawa theory

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_\psi)\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_\phi^2\phi^2 - g\bar{\psi}\psi\phi - v\phi^3$$

   with a Dirac fermion ψ and a scalar field ϕ, draw the Feynman diagram of all two-loop corrections to the fermion propagator.

2. **Fermion–Scalar Scattering (80 points)**
   Using the Yukawa theory of the lecture,

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_\psi)\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_\phi^2\phi^2 - g\bar{\psi}\psi\phi,$$

   the aim of this exercise is to compute the fermion–scalar scattering process. Let the incoming and outgoing momenta of the fermion be p and p', and k and k' for the scalar, respectively.

   a)(6 points) Draw all tree-level Feynman diagrams of such a fermion-scalar scattering.

   b)(30 points) Compute the matrix element $i\mathcal{M}$ for the sum of diagrams in a).

   c)(30 points) Using the result of part b) compute now the spin summed matrix element squared $\sum_s |\mathcal{M}|^2$. Remember that $\sum_s u^s(p)\bar{u}^s(p) = p\!\!\!/ + m$.
   Hint: The results of exercise 3.4 will probably come in handy.

   d)(14 points) Express the result of part c) in the three so-called Mandelstam variables

$$\begin{aligned} s &= (p+k)^2, \\ t &= (p-p')^2, \text{ and} \\ u &= (p-k')^2. \end{aligned}$$