problem sheet 7

to be handed by **Friday 15.7.2016 (12:00)** to the letterbox 37 (foyer of Staudingerweg 7)

1. **QCD Beta-Function (60 P.)**

   Calculate the one-loop contributions to the gauge boson self-energy, the fermion self-energy and the quark-gluon vertex correction to obtain the quantities Z_A, Z_ψ and $Z_{g\psi}$ respectively. Use dimensional regularisation and the $\bar{MS}$ renormalisation scheme. You can as usual concentrate on the divergent terms. Therefrom calculate the one-loop beta-function for QCD (equation 9.116 in the lecture notes):

   $$\beta \overset{\text{1-loop}}{\approx} g_0 \mu \frac{\partial}{\partial \mu}(Z_\psi - Z_{g\psi} + \frac{1}{2} Z_A)\,. \tag{1}$$

   You can find the contributing diagrams and the correct solution in chapter 9.6 of the lecture notes.

2. **Beta-Function of a complex scalar field in a non-abelian gauge theory (40 P.)**

   In the last exercises we considered a complex scalar field in a non-abelian gauge theory. When considering multiple complex scalars, the Lagrangian becomes

   $$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \bar{c}^a \partial^\mu D^{ac}_\mu c^c + (D^\mu \phi_i)^* D_\mu \phi_i - m^2 \phi_i^* \phi_i \tag{2}$$

   including a gauge fixing term and ghost fields for completeness.

   (a) (32 P.) To obtain the one loop beta-function of this theory, compute the contributions involving the scalars and combine them with the gauge boson and ghost contributions from Standard Model QCD (see lecture notes).

   (b) (8 P.) Under what conditions does this theory become asymptotically free?