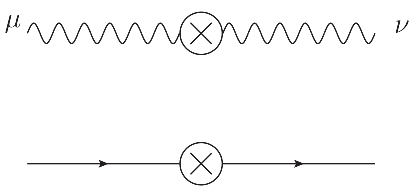
problem sheet 3
to be handed by Wednesday 24.5.2017 (12:00) to the letterbox 37 (foyer of Staudingerweg 7)

1. **Renormalisation of the fermion propagator (5 P.)**

   In the second exercise we derived the amplitude $\Sigma_2(p)$, the one-loop level correction to the fermion propagator. Use the result we obtained to define the counterterms $Z_\psi - 1$ and $Z_m - 1$ (see figure 1) such that they cancel the divergence. Use the $\overline{MS}$ scheme to do so.



$$= -i(g^{\mu\nu}q^2 - q^\nu q^\nu)(Z_A - 1)$$

$$= i(\not{p}(Z_\psi - 1) - m(Z_m - 1))$$

Figure 1: Feynman rules for the counterterms of the fermion and photon propagators.

2. **Renormalisation of the photon propagator (60 P.)**

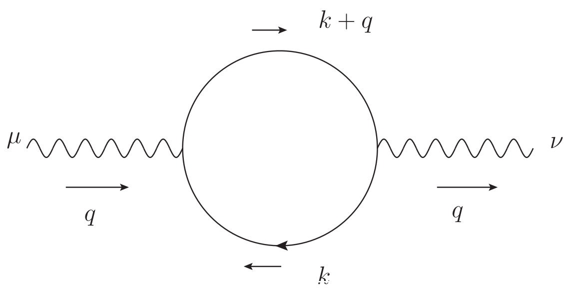


Figure 2: Loop correction to the photon propagator at one-loop level.

   (a) **(50 P.)** Calculate the diagram sketched in figure 2:

   - Write down the amplitude using Feynman rules, do not include the external photon lines.
   - Introduce Feynman parameters to combine the denominator.
   - Complete the square in the denominator, shifting $k \to \ell$. The denominator should become
   $$[\ell^2 - \Delta + i\epsilon]^2 \,, \tag{1}$$
   with
   $$\ell = k + qx \qquad \text{and} \qquad \Delta = m^2 - q^2 x(1-x) \,. \tag{2}$$
   - Rewrite the numerator in terms of ℓ, remember you can drop odd powers.
   - Solve the momentum integral using Wick rotation and dimensional regularisation. You do not have to solve the Feynman parameter integral! (Hint: Take a look at part (b).)

(b) **(5 P.)** The amplitude can be written as

$$i\Pi_2^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) i\Pi_2(q^2)\,, \tag{3}$$

thus extracting the dependence on the Lorentz indices. The counterterm has the same form (see figure 1). What dictates the structure of $\Pi^{\mu\nu}$?

(c) **(5 P.)** Now define the counterterm $Z_A - 1$ such that it cancels the divergence. Use the $\overline{MS}$ scheme to do so.

3. **Regularisation of the ϕ^4 Tadpole Diagram (35 P.)**

In chapter 4 of last years lecture you can find the ϕ^4 theory with its Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4\,. \tag{4}$$

It gives rise to a so-called tadpole diagram shown in figure 3.

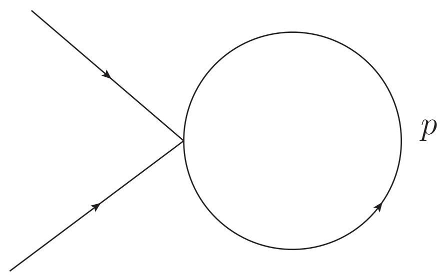


Figure 3: Tadpole diagram in ϕ^4 theory.

Calculate the divergent amplitude of the tadpole diagram (as before omit the external lines) and regularise the integral using

(a) an UV cutoff Λ,

(b) Pauli-Villars regularisation

- Introduce a heavy partner particle.
- Why does this fail for the tadpole diagram?
- Introduce an additional term to regularise the integral.
- Perform the regularisation.

(c) dimensional regularisation, shifting the spacetime dimensions from 4 to $d = 4-\varepsilon$.