problem sheet 6

to be handed by **Wednesday 5.7.2017 (12:00)** to the letterbox 37 (foyer of Staudingerweg 7)

1. **Kinetic Term of Non-Abelian Gauge Theories (40 P.)**

   In the lecture you deduced the kinetic term of non-abelian gauge theories

   $$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} \tag{1}$$

   calculating the commutator of covariant derivatives. Now derive the gauge kinetic term using the approach of a Wilson loop (compare chapter 9.1 of the lecture notes).

2. **Complex scalar field in a non-abelian gauge theory (20 P.)**

   Consider a complex scalar field ϕ in a non-abelian gauge theory. Write down the Lagrangian of the theory (you do not have to specify the self interaction of the scalar field). Then deduce the Feynman rules of this theory. (You can leave out the self interaction of the gauge bosons.)

3. **QCD Feynman rules (30 P.)**

   To derive the QCD Feynman rules, the relevant terms in the Lagrangian are:

   $$\begin{aligned}\mathcal{L}_{QCD} \supset & -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + -g f^c_{bc} A^c_\mu \left(\partial^\mu c^b \bar{c}^a\right) \\ & + \sum_q \bar{\psi}_q (i\gamma^\mu D_\mu - m_q)\psi_q \end{aligned} \tag{2}$$

   Use eq. 2 to derive the feynman rules for:

   - gluon-quark coupling **(5P.)**
   - gluon-ghost coupling **(5P.)**
   - 3 gluon coupling **(10P.)**
   - 4 gluon coupling **(10P.)**

4. **QCD Theta Term (10 P.)**

   The most general QCD Lagrangian contains another so called θ term:

   $$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a} \tag{3}$$

   where $\tilde{G}^{\mu\nu,a} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}$ is the dual gluon field-strength tensor.

   - argue that this term is gauge invariant **(5P.)**
   - argue that this term violates CP **(5P.)**