4th problem sheet (20 points)
to be handed in on 05.01.2015 to the letterbox (foyer of Staudingerweg 7)

1. **Pair annihilation e^+e^- into photons**

$$e^+(p_2)\,e^-(p_1) \to \gamma(k_1)\,\gamma(k_2)$$

(a) **(9P.)** Show that the squared amplitude (averaged over initial spins and summed over the outgoing polarisations) is equal to

$$\frac{1}{4}\overline{|\mathcal{M}|^2} = 2e^4\left[\frac{p_1k_2}{p_1k_1} + \frac{p_1k_1}{p_1k_2} + 2m^2\left(\frac{1}{p_1k_1} + \frac{1}{p_1k_2}\right) - m^4\left(\frac{1}{p_1k_1} + \frac{1}{p_1k_2}\right)^2\right] \quad (1)$$

(b) **(6P.)** Calculate the differential cross section in the center of mass frame (the kinematics are shown in Figure 1). It should give

$$\frac{d\sigma}{d\cos\theta} = \frac{2\pi\alpha^2}{s}\left(\frac{E}{p}\right)\left[\frac{E^2 + p^2\cos^2\theta}{m^2 + p^2\sin^2\theta} + \frac{2m^2}{m^2 + p^2\sin^2\theta} - \frac{2m^4}{(m^2 + p^2\sin^2\theta)^2}\right] \quad (2)$$

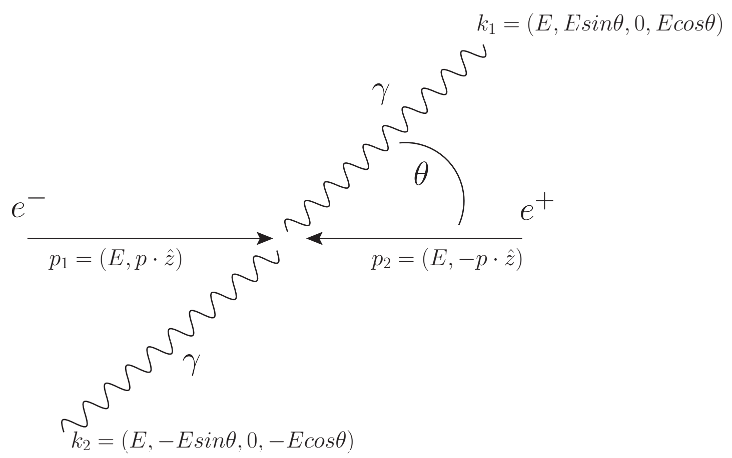

Figure 1: Parametrisation of the momenta in the com system.

(c) **(2P.)** What is the high energy limit ($E^2 \gg m^2$) of the differential cross section?

(d) **(3P.)** Explain where the divergence of the differential cross section in the high energy limit comes from. How is it regularised?