5th problem sheet (20 points)
to be handed in on 19.01.2015 to the letterbox (foyer of Staudingerweg 7)

1. **Renormalisation of the fermion propagator**

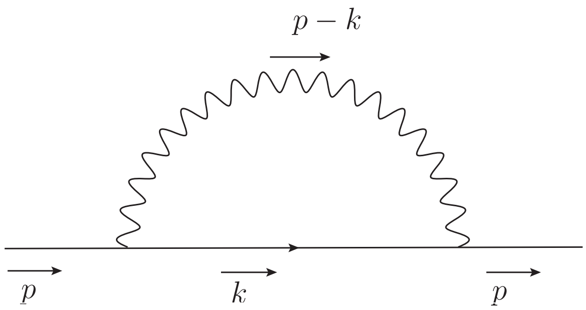
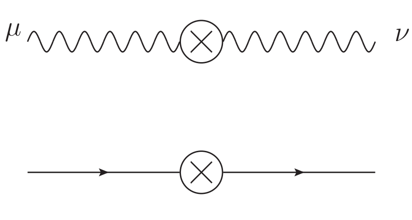


Figure 1: Loop correction to the fermion propagator on one-loop level.

(a) **(7P.)** Calculate the diagram scetched in figure 1:

- Write down the amplitude using Feynman rules, do not include the propagators of the external fermion lines.
- Introduce Feynman parameters to combine the denominator.
- Complete the square in the denominator, shifting $k \to \ell$. The denominator should become
$$[\ell^2 - \Delta + i\epsilon]^2 \,, \tag{1}$$
with
$$\ell = k - xp \qquad \text{and} \qquad \Delta = -x(1-x)p^2 + (1-x)m^2 \,. \tag{2}$$
- Rewrite the numerator in terms of ℓ, remember you can drop odd powers.
- Solve the momentum integral using Wick rotation and dimensional regularisation. You do not have to solve the Feynman parameter integral!

(b) **(2P.)** Now define the counterterms $Z_\psi - 1$ and $Z_m - 1$ (see figure 2) such that they cancel the divergence. Use the $\overline{MS}$ scheme to do so.



$$= -i(g^{\mu\nu}q^2 - q^\nu q^\nu)(Z_A - 1)$$

$$= i(\not{p}(Z_\psi - 1) - m(Z_m - 1))$$

Figure 2: Feynman rules for the counterterms of the fermion and photon propagators.

2. **Renormalisation of the photon propagator**

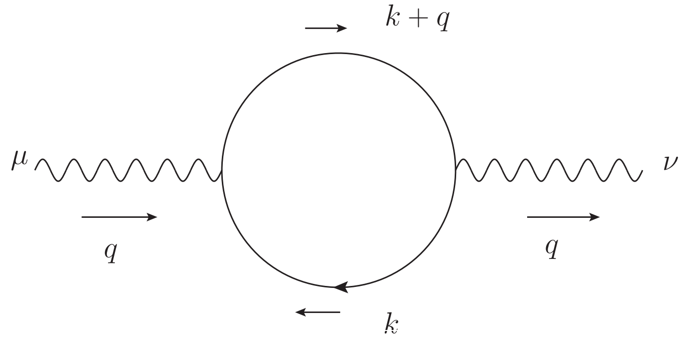


Figure 3: Loop correction to the photon propagator at one-loop level.

(a) **(9P.)** Calculate the diagram scetched in figure 3:

- Write down the amplitude using Feynman rules, do not include the external photon lines.
- Introduce Feynman parameters to combine the denominator.
- Complete the square in the denominator, shifting $k \to \ell$. The denominator should become

$$[\ell^2 - \Delta + i\epsilon]^2 \,, \tag{3}$$

with

$$\ell = k + qx \qquad \text{and} \qquad \Delta = m^2 - q^2 x(1-x) \,. \tag{4}$$

- Rewrite the numerator in terms of ℓ, remember you can drop odd powers.
- Solve the momentum integral using Wick rotation and dimensional regularisation. You do not have to solve the Feynman parameter integral!
Hint: You can write the amplitude as

$$i\Pi_2^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) i\Pi_2(q^2) \,, \tag{5}$$

thus extracting the dependence on the Lorentz indices. The counterterm is also written in the same form (see figure 2).

(b) **(2P.)** Now define the counterterm $Z_A - 1$ such that it cancels the divergence. Use again the $\overline{MS}$ scheme to do so.