6th problem sheet (20 points)
to be handed in on 02.02.2015 to the letterbox (foyer of Staudingerweg 7)

# Higgs Decays

1. **Higgs decay to fermions** $h^0 \to f\bar{f}$

   (a) **(3P.)** The decay width of a particle (A) decaying to two final state particles is given by

   $$\Gamma = \frac{1}{2m_A}\int d\Pi_2 |\mathcal{M}(m_A \to p_1 + p_2)|^2 \,. \tag{1}$$

   Show that for two final state particles that have the same mass and thus momenta $p_1 = (E, 0, 0, p)$ and $p_2 = (E, 0, 0, -p)$ (in the com frame) the angular dependence drops out and thus we get

   $$\Gamma = \frac{1}{8\pi}\frac{p}{m_A^2}|\mathcal{M}(m_A \to p_1 + p_2)|^2 \,, \tag{2}$$

   using that energy conservation gives $E = 1/2m_A$. Note that $p^2 = E^2 - m_{1,2}^2$.

   (b) **(5P.)** Calculate the decay rate for the process $h^0 \to f\bar{f}$. (The relevant Feynman rules are shown in figure 1.) Use the following expression for the Higgs VEV

   $$v^2 = \frac{m_W^2 \sin^2\theta_w}{\alpha_{em}\pi} \tag{3}$$

   to express the final result.

   For coloured final states (quarks) beware of the factor from the non-abelian gauge theory ($N_c = 3$)!

   The final expression should be

   $$\Gamma(h^0 \to f\bar{f}) = \frac{\alpha_{em} m_h}{8\sin^2\theta_w}\frac{m_f^2}{m_W^2}N_c(f)\left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2} \tag{4}$$

2. **Higgs decay to gluons** $h^0 \to gg$ **(10P.)**

   The Higgs does not couple directly to gluons, but it has a coupling via a quark loop. Why is the contribution from the top quark dominant? Calculate the amplitude using the same steps as in the last exercise sheet. Note that the trace of the generators $t$ of $SU(3)$ is given by

   $$\mathrm{tr}[t^a t^b] = \frac{1}{4}\mathrm{tr}[\lambda^a\lambda^b] = \frac{1}{2}\delta^{ab}\,, \tag{5}$$

   where $\lambda$ are the Gell-Mann matrices.

   Leave the polarisation vectors of the final state gluons in definite helicity states and use

   $$\epsilon^*(p_i)\cdot p_j = 0 \tag{6}$$

for $i, j = 1, 2$. The inner product of the polarisation vectors is only nonzero for the combinations

$$\epsilon_-^* \cdot \epsilon_+^* = \epsilon_+^* \cdot \epsilon_-^* = 1\,, \tag{7}$$

which you have to sum up, leaving you with a factor 2.

Now calculate the decay rate of the process using equation (2) and note that you have to include an additional factor $\frac{1}{2}$, since the final state gluons are identical particles.

You should get

$$\Gamma(h^0 \to gg) = \frac{\alpha_{em} m_h}{8 \sin^2\theta_w} \frac{m_h^2}{m_W^2} \frac{\alpha_s^2}{9\pi^2} |\sum_q I_f(\tau_q)|^2\,, \tag{8}$$

where

$$\tau_q = \left(\frac{m_h}{m_q}\right)^2\,, \tag{9}$$

$$I_f(\tau_q) = 3\int_0^1 dx \int_0^{1-x} dy \frac{1-4xy}{1-xy\tau_q}\,. \tag{10}$$

3. **Higgs decay to photons $h^0 \to \gamma\gamma$ (5P.)**

This decay channel has two types of contributions at leading order (one-loop level): In one type of diagram fermions propagate in the loop, in the other the W boson (and related would-be Goldstone bosons).

- Calculate the contribution from the fermion loop process, which can be carried out complete parallel to the $h^0 \to gg$ decay. Remember to include the factor of the electric charges of the propagating fermions $Q_f$ and the color factor for the quarks.
- The calculation of the W boson loop consists of 13 different diagrams, including apart from the W boson also ghosts and would-be Goldstone bosons. We will give the solution of this contribution:

$$i\mathcal{M}(h^0 \to (\text{W-loop}) \to \gamma\gamma) = \frac{\alpha_{em} m_h}{8 \sin^2\theta_w} \frac{m_h^2}{m_W^2} \frac{\alpha_{em}^2}{18\pi^2} I_W(\tau_W)\,, \tag{11}$$

where

$$\tau_W = \left(\frac{m_h}{m_W}\right)^2\,, \tag{12}$$

$$I_W(\tau_W) = \frac{1}{\tau_W}\left(6I_1(\tau_W) - 8I_2(\tau_W) + \tau_W(I_1(\tau_W) - I_2(\tau_W)) + I_3(\tau_W)\right)\,, \tag{13}$$

$$I_1(\tau_W) = \int_0^1 dx \log(1 - x(1-x)\tau_W)\,, \tag{14}$$

$$I_2(\tau_W) = 2\int_0^1 dx \int_0^{1-x} dy \log(1 - xy\tau_W)\,, \tag{15}$$

$$I_3(\tau_W) = \int_0^1 dx \int_0^{1-x} dy \frac{(8 - 3x + y + 4xy)\tau_W}{1 - xy\tau_W}\,. \tag{16}$$

The full expression including both contributions from the fermion and W boson loops reads

$$\Gamma(h^0 \to \gamma\gamma) = \frac{\alpha_{em}^3 m_h^3}{144\pi^2 m_W^2 \sin^2\theta_w} |\sum_f Q_f^2 N_c(f) I_f(\tau_f) - I_W(\tau_W)|^2 , \tag{17}$$

where $I_W(\tau_W)$ is defined just above and $I_f(\tau_f)$ is defined in equation (10), substituting $\tau_q$ with $\tau_f$, which is defined analogously. Why is there a relative minus sign between the two contributions?

4. **Total decay width and branching fraction of the Higgs boson**

   (a) **(5P.)** Evaluate the Feynman integrals in the above calculations (numerically) with Mathematica/Maple for $m_h = 125$ GeV and add up all contributions to get the total decay width of the Higgs boson. For $h^0 \to f\bar{f}$ you have to include only the following fermions: $b\bar{b}$, $\tau^+\tau^-$ and $c\bar{c}$ (why?). For $h^0 \to gg$ and $h^0 \to \gamma\gamma$ via fermion loop only consider the propagation of the top quark.

   (b) **(2P.)** Now calculate the branching ratios of the Higgs decaying to $b\bar{b}$, $\tau^+\tau^-$, $c\bar{c}$, $gg$ and $\gamma\gamma$. Why is the decay channel $h^0 \to \gamma\gamma$ still so important, i.e. the first channel where the Higgs was seen?

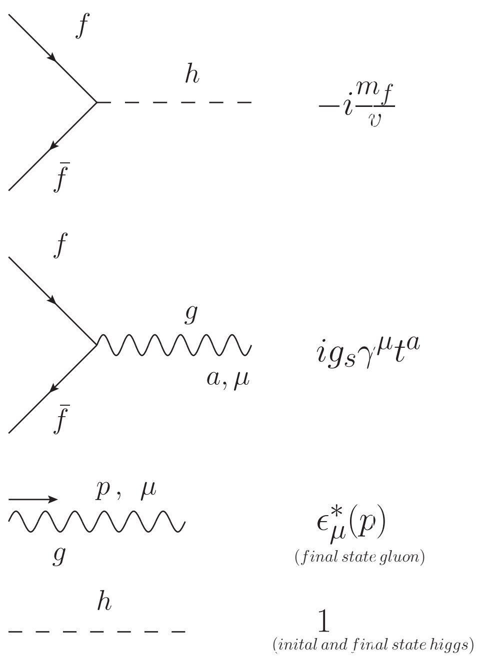


Figure 1: For these calculations relevant (new) SM Feynman rules.