

A. Klenke, Probability Theory, 1st ed., Errata, 14.01.2023

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| p 6, line 8f | Replace \mathcal{A} by \mathcal{A}_I (five times). |
| p 8, line -14f | Replace $\tau \subset \Omega$ by $\tau \subset 2^\Omega$. |
| p 11, line 6 | Change “1.10” to “1.9”. |
| p 15, line 13 | We have to assume $\mu(A_1 \cup \dots \cup A_n) < \infty$. |
| p 18, line 22 | Replace $a < b$ by $a \leq b$. |
| p 20, line 5 | Change this sentence to: Assume that there exist sets $\Omega_1, \Omega_2, \dots \in \mathcal{E}$ such that $\bigcup_{n=1}^{\infty} \Omega_n = \Omega$ and $\mu(\Omega_n) < \infty$ for all $n \in \mathbb{N}$. |
| p 20, line -3 | <p>Replace this sentence by the following paragraph: Now let $\Omega_1, \Omega_2, \dots \in \mathcal{E}$ be a sequence such that $\bigcup_{n=1}^{\infty} \Omega_n = \Omega$ and $\mu(\Omega_n) < \infty$ for all $n \in \mathbb{N}$. Let $E_n := \bigcup_{i=1}^n \Omega_i$, $n \in \mathbb{N}$, and $E_0 = \emptyset$. Hence $E_n = \biguplus_{i=1}^n (E_{i-1}^c \cap \Omega_i)$. For any $A \in \mathcal{A}$ and $n \in \mathbb{N}$, we thus get</p> $\mu(A \cap E_n) = \sum_{i=1}^n \mu((A \cap E_{i-1}^c) \cap \Omega_i) = \sum_{i=1}^n \nu((A \cap E_{i-1}^c) \cap \Omega_i) = \nu(A \cap E_n).$ |
| p 25, line 6 | Erase the right hand side of the first line in the display formula. |
| p 25, line 15 | Replace $a < b$ by $a \leq b$. |
| p 27, line 18 | Replace $[x, 0)$ by $(x, 0)$. Add “for $x < 0$ ”. |
| p 27, lines -5, -4 | Replace F by F_μ (twice). |
| p 32, line 11 | Append “ $A \subset \bigcup_{i=1}^{\infty} A_i$ and ” |
| p 51, line -4 | Delete “with $k := \#J$ ”. |
| p 54, line 11 | Replace Example 1.14 by Remark 1.14. |
| p 57, line 3 | Replace display formula by $\mathbf{P} \left[\bigcap_{j \in J} \{X_j \in A_j\} \right] = \prod_{j \in J} \mathbf{P}[X_j \in A_j]$. |
| p 57, line -15 | Erase (iii). |
| p 59, line 3 | Replace X^{-1} by X_i^{-1} . |
| p 60, line -3 | Replace “ring” by “semiring”. |

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| p 73, line 28 | The graph (T, \sim) may contain circles. It is not too hard to see that, e.g., three trifurcation points can be pairwise directly connected. In order to save the argument one can do the following: Glue together all open edges that can be connected by an open path which traverses no trifurcation point. We thus obtain a graph consisting off the trifurcation points and these clumps of edges. In this graph, trifurcation points are the neighbours of the adjacent clumps of open edges but they are not direct neighbours of other trifurcation points. This graph now does not contain circles. Hence the remaining argument works (with minor changes). |
| p78, line 3 | Replace by $\sum_{k=1}^{\infty} \mathbf{P}[X = k] \cdot k(k-1) \cdots (k-n+1)$ |
| p 83, 3 | change to “for all $i, k, n \in \mathbb{N}_0$.” |
| p78, line 6 | The x_i have to have an accumulation point in $(0, 1)$ unless $\psi(z) < \infty$ for some $z > 1$. |
| p89, lines 12, 13 | Replace these two lines by: Clearly, $f^+ \leq g^+$ a.e., hence $(f^+ - g^+)^+ = 0$ a.e. By Theorem 4.8, we get $\int (f^+ - g^+)^+ d\mu = 0$. Since $f^+ \leq g^+ + (f^+ - g^+)^+$ (not only a.e.), we infer from Lemma 4.6(i) and (iii) |
| | $\int f^+ d\mu \leq \int (g^+ + (f^+ - g^+)^+) d\mu = \int g^+ d\mu.$ |
| | Similarly, we use $f^- \geq g^-$ a.e. to obtain |
| | $\int f^- d\mu \geq \int g^- d\mu.$ |
| p98, line 13,16 | Replace $\int_{\varepsilon}^{\infty} g(t) dt$ by $\int_0^{\infty} (g(\varepsilon) \wedge g(t)) dt$. |
| p105, line -9 | Change $c = 0$ to $c = -\mathbf{E}[Y]$. |
| p108, line 9 | Erase $\limsup_{n \rightarrow \infty}$. |
| p109, line -5 | \tilde{S}_n instead of S_n . |
| p115, line 19 | Replace Y_i by $Y_i(x)$. |
| p115, line 21 | Replace Z_i by $Z_i(x)$. |

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| p117, line 17 | Change $\ell(k)$ to $\ell(e_k)$. |
| p117, line 18 | Change $l \geq k$ to $l > k$. |
| p117, line -6 | Prepend minus signs to both sides of the equation. |
| p121, line 17 | Replace $ S_k /k \leq 2 S_k /k_{n+1}$ by $ S_k /l(k) \leq 2 S_k /l(k_{n+1})$. |
| p124, line 1ff | In order that equality holds in line 5, (and not only \leq), we need to establish that $2^n \mathbf{P}[N_{2^{-n}} \geq 2] \xrightarrow{n \rightarrow \infty} \lambda$. This can be inferred by the fact that for all $n \in \mathbb{N}$ and $\varepsilon > 0$, we have <p style="text-align: center;">$\mathbf{P}[N_{2^{-n}} \geq 2] \geq [2^{-n}/\varepsilon] \mathbf{P}[N_\varepsilon \geq 2] - [2^{-n}/\varepsilon]^2 \mathbf{P}[N_\varepsilon \geq 2]^2,$</p> which implies (by suitably letting $\varepsilon \rightarrow 0$) that $2^n \mathbf{P}[N_{2^{-n}} \geq 2] \geq \lambda - 2^{-n} \lambda^2 \xrightarrow{n \rightarrow \infty} \lambda$. |
| p132, line 10 | Replace $\ f_n - f\ _p$ by $\ f_n - f\ _p^p$. |
| p140, lines 1/2 | Replace $\{ f - f_{n'_k} > g_k\} = \{ f - f_{n'_k} > g\}$ by <p style="text-align: center;">$f - f_{n'_k} = (f - f_{n'_k} - g)^+ + g_k$</p> and $\int_{\{ f - f_{n'_k} > g\}} f - f_{n'_k} d\mu$ by $\int (f - f_{n'_k} - g)^+ d\mu$. |
| p154, line 23,26 | Replace Λ^2 by L^2 . |
| p158, Ex. 7.4.1 | Not F , but its inverse F^{-1} is the continuous distribution function of a singular measure. |
| p175, line -3 | Replace (vii) by (vi). |
| p172, line 17 | Replace “ X be a nonnegative” by “ $X > 0$ be a strictly positive”. |
| p175, line 20 | (Z_n) is decreasing, hence Z is in fact the <i>limit</i> and not only limsup. Hence Fatou’s lemma can be applied. In line 24 replace $\mathbf{E}[Z_n]$ by $\mathbf{E}[Z_n \mathcal{F}]$. |
| p180, line -5 | append: “and which is such that $\kappa(\omega_1, E) < \infty$ for all $\omega_1 \in \Omega_1$ and $E \in \mathcal{E}$.” |
| p184, line 2 | exchange μ_1 and μ_2 |
| p185, (8.16) | Replace $\kappa_{Y, \mathcal{F}}$ by $\kappa_{X, \mathcal{F}}$. |

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| p192, line -7 | Replace $\{\tau \leq t\}$ by $\{\tau_K \leq t\}$. |
| p193, line 1 | We assume that $I \subset [0, \infty)$ is closed under addition (at least for (ii) and (iii)). |
| p196, line 27 | Replace $\mathbf{E}[X_s]$ by X_s . |
| p213, line 6 | Change $\langle X \rangle$ to $\langle X \rangle_\tau$. |
| p218, line 15 | $\mathbf{E}[X_n ^p] < \infty$ (bracket] missing). |
| p225, line 9 | exchange + and -. |
| p227, line 15 | After “events” append: “with $A_n \in \mathcal{F}_n$ for all $n \in \mathbb{N}$.” |
| p227, line 17 | Replace $X_n = \sum_{k=1}^{\infty} (\mathbf{1}_{A_n} - \mathbf{P}[A_n \mathcal{F}_{n-1}])$ by $X_n = \sum_{k=1}^n (\mathbf{1}_{A_k} - \mathbf{P}[A_k \mathcal{F}_{k-1}])$. |
| p228, line 13 | Replace Z^n by Z_n . |
| p232, line 21 | Replace $i < k$ by $i \leq k$. |
| p234, line 13 | Replace “ $\mathcal{E}_n =$ ” by “ $\mathcal{E}_n \supset$ ”. |
| p234, line 17 | In order to avoid trivial cases, assume, for example, $E = \{0, 1\}$, X_1, X_2, \dots independent with $\mathbf{P}[X_n] \in (0, 1)$ for all $n \in \mathbb{N}$ and $B = \{1\}$. |
| p234, line 25 | For $A \in \mathcal{E}_n$, there exists a measurable $B \subset E^{\mathbb{N}}$ such that $B^\varrho = B$ for all $\varrho \in S_n$. Define $F = \mathbf{1}_B$. |
| p235, (12.4) | Replace $(N\Xi_N(A_l))^{m_l}$ by $(N\Xi_N(A_l))_{m_l}$. |
| p236, line 20ff | Replace Y_{-n} by Y_n . |
| p237 (12.5) | Replace $\prod_{l=1}^n$ by $\prod_{l=1}^k$. |

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| p248, line17 | <p>Since τ is not a semiring, Thm 1.65 cannot be used directly. A more subtle (and hopefully correct) proof for outer regularity is the following:</p> <p>First assume $B \subset E$ is closed and let $\varepsilon > 0$. Let $B_\delta := \{x \in E : d(x, B) < \delta\}$ be the open δ-neighbourhood of B. As B is closed, we have $\bigcap_{\delta>0} B_\delta = B$. Since μ is upper semicontinuous, there is a $\delta > 0$ such that $\mu(B_\delta) \leq \mu(B) + \varepsilon$.</p> <p>Not let $B \in \mathcal{E}$ and $\varepsilon > 0$. Consider $\mathcal{A} := \{V \cap C : V \subset E \text{ open, } C \subset E \text{ closed}\}$. We have $\mathcal{E} = \sigma(\mathcal{A})$ and \mathcal{A} is a semiring. By Theorem 1.65, there are mutually disjoint sets $A_n = V_n \cap C_n \in \mathcal{A}$, $n \in \mathbb{N}$, such that $B \subset A := \bigcup_{n=1}^{\infty} A_n$ and $\mu(A) \leq \mu(B) + \varepsilon/2$. As shown above, for any $n \in \mathbb{N}$, there is an open set $W_n \supset C_n$ such that $\mu(W_n) \leq \mu(C_n) + \varepsilon 2^{-n-1}$. Hence also $U_n := V_n \cap W_n$ is open. Let $B \subset U := \bigcup_{n=1}^{\infty} U_n$. We conclude that $\mu(U) \leq \mu(A) + \sum_{n=1}^{\infty} \varepsilon 2^{-n-1} \leq \mu(B) + \varepsilon$.</p> |
| p262, line -10 | Replace μ by μ_n . |
| p261, line -7 | We also have to show that $F(-\infty) = 0$ in order that F be a distribution function. This however follows from tightness just as in the lines -6ff. |
| p264, line -3 | Replace \mathcal{E} by \mathcal{U} . |
| p266, line -3 | Erase $\alpha(\bigcup_{i=1}^n A_i) =$. |
| p274, line 6 | Replace A_1, \dots, A_n by A_1, \dots, A_N . |
| p274, line 10f | Delete "respectively a semiring" (twice). |
| p274, line 14 | Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$. |
| p274, line 21 | Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$. |
| p280, line 9 | Replace $\kappa_1 \otimes \kappa$ by $\kappa_1 \otimes \kappa_2$. |
| p280, line -5 | Replace $\bigotimes_{k=0}^i \mathcal{A}_k$ by $\bigotimes_{k=1}^i \mathcal{A}_k$. |
| p281, line -1 | Replace φ_k by φ_n . |
| p286, line 24 | Replace $P_{n+1} _{\tilde{A}^n}$ by $P_{n+1} _{\tilde{A}^n}$. |
| p288, line 17 | Replace $\omega \in \Omega$ by $\omega \in E$. |

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| p289, line -4 | Replace $\otimes_{k=i}^n$ by $\otimes_{k=i}^{n-1}$. |
| p289, line -2f | Replace A_{l+1} by $A_{j_{l+1}}$ (twice). |
| p290, line 2 | Replace $f_{l+1}(\omega_{l+1})$ by $f_{l-1}(\omega_{l-1})$. |
| p290, line 3 | Replace $f(\omega_{l+1})$ by $f_{l+1}(\omega_{l+1})$. |
| p290, line 17 | Replace $\mu \otimes \kappa$ by $\int \mu(dx) \kappa(x, \cdot)$. |
| p295, line 3 | Replace $H(x)$ by $H_z(x)$. |
| p295, line 8 | Replace $h(y)$ by $h_z(y)$. |
| p299, line 6 | $\ f\ _2 = \ \varphi\ _2 / (2\pi)^{d/2}$. |
| p302, line 21 | Replace (t/a) by (t/θ) . |
| p303, line -1 | Factor $1/\sqrt{2\pi}$ in front of the integral is missing. |
| p303, line 11f | Replace $+$ by $-$ (four instances). |
| p305, line 11 | Replace $\varphi(t)$ by $\varphi_X(t)$. |
| p314, lines 2, 3 | Replace h^n by $ h ^n$ (two instances). |
| p314, lines 12, 13 | Replace $\sqrt{2\pi n}$ by $1/\sqrt{2\pi n}$ (two instances). |
| p315, line 18 | Replace $\varphi^{(2n)}(0)$ by $(-1)^n \varphi^{(2n)}(0)$. |
| p315, line -3 | $\mathbf{E}[X^{2k}] = (-1)^k u^{(2k)}(0)$. |
| p316, line 4 | $\mathbf{E}[X^{2n}] = (-1)^n u^{(2n)}(0) = (-1)^n \varphi^{(2n)}(0)$. |
| p319, line -3 | Replace $L_n(\varepsilon)$ by $\varepsilon^{-2} L_n(\varepsilon)$. |
| p320, line 9 | Replace εt by $\varepsilon t $. |
| p328, line -5 | Replace $\theta^{-1} = r = k$ by $\theta = r = k/2$. |
| p329, line -2 | Replace display formula by |
| | $\varphi_{r\nu}(t) = \exp\left(r \sum_{k=1}^{\infty} \frac{((1-p)e^{it})^k - (1-p)^k}{k}\right) = p^r (1 - (1-p)e^{it})^{-r}.$ |
| p331, lines -3, -1 | Replace μ_n by ν_n . |
| p331, line -1 | Replace ν by μ . |

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| p333, line 13 | Replace $h(t)$ by $h(x)$. |
| p333, line -6 | Replace $u(1)$ by $2u(1)$. |
| p333, line -3 | Replace $t \wedge 1$ by $t \vee 1$. |
| p336, line -6 | Here and in the remaining proof the sign of $\bar{\psi}(0)$ is wrong. Replace $\bar{\psi}(0) \leq 0$ by $\bar{\psi}(0) \geq 0$. |
| p336, line -3 | $\bar{\psi}(0) > 0$. |
| p337, line 9 | $\bar{\psi}_n(0) > 0$ and $\tilde{\nu}_n(dx) = (h(x)/\bar{\psi}_n(0))\nu_n(dx)$. |
| p337, line 10 | Replace $-\bar{\psi}(t)/\bar{\psi}(0)$ by $\bar{\psi}(t)/\bar{\psi}(0)$. |
| p337, line 13, 15 | Erase the minus sign. |
| p337, line 15 | Replace $t \wedge 1$ by $t \vee 1$. |
| p337, line 16 | The map f_t is not continuous. At this point, we have to work with $g_{t,\varepsilon}(x) = e^{-itx} - 1 - itx \mathbf{1}_{\{ x < 1-\varepsilon\}}$ instead of $g_t(x) = e^{-itx} - 1 - itx \mathbf{1}_{ x < 1}$. We choose $\varepsilon > 0$ such that ν has no atoms at the points of discontinuity $-1 + \varepsilon$ and $1 - \varepsilon$. By the Portemanteau Theorem (Theorem 13.16(iii)), we get convergence of the integrals. Finally, we let $\varepsilon \rightarrow 0$. |
| p337, line -5 | Insert the factor $\bar{\psi}_n(0)$ before the integral $\int f_t(x) \tilde{\nu}_n(dx)$. |
| p338, (16.16) | Replace $(0, \infty)$ by $\mathbb{R} \setminus \{0\}$. |
| p340, line 10 | The relation $nb = n^{1/\alpha}b$ is wrong since it does not pay respect to the change due to passing from ν to $\nu \circ m_{n^{1/\alpha}}^{-1}$. We first have to compute the explicit form of ν . Then we can compute the correct scaling relation (here without detailed derivation): |
| | $nb = bn^{1/\alpha} - (c^+ - c^-) \begin{cases} (1 - \alpha)^{-1}(n^{1/\alpha} - n), & \text{if } \alpha \neq 1, \\ n \log(n), & \text{if } \alpha = 1. \end{cases}$ |
| | Consequently, we get $b = (c^+ - c^-)/(1 - \alpha)$ in the case $\alpha \neq 1$. No changes are necessary for the case $\alpha = 1$. |
| p340, (16.18) | Replace $i(c^+ - c^-)$ by $-i \operatorname{sign}(t)(c^+ - c^-)$. |

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| p344, (17.12) | Replace $\sum_{i=0}^{n-1}$ by $\sum_{i=1}^n$. |
| p347, line 13 | Replace $\kappa_{t_{n+1}-t_n}$ by $\kappa_{t_{i+1}-t_i}$. |
| p353, line 3 | Change I to E . |
| p350, line -9 | Replace $t \in \mathbb{N}_0$ by $t \in I$ (twice). |
| p357, line 5 | Replace “With this convention” by “Finally, we assume that”. |
| p357, line -7 | Define $p = I$ if $\lambda = 0$. |
| p358, line 10 | Replace \tilde{p}_{t+s} by p_{t+s} . |
| p358, line 15 | Replace \int_0^t by \int_0^s (twice). |
| p359, line -2 | replace P_x^Y by \mathbf{P}_x^Y . |
| p363, line -4 | We also agree that $0/0 = 0$ and $0 \cdot \infty = 0$. |
| p363, line -2 | Before “state” insert “non-absorbing”. |
| p364, line 7 | We assume that $x \neq y$. |
| p366, line 15 | For the right term a factor 4^n is missing. |
| p373, line 15 | Replace “If X ” by “If any state” |
| p373, line 17f | Replace $\mu p^n(x)$ by $\mu p^n(\{x\})$ (twice) and $\mu(x)$ by $\mu(\{x\})$. |
| p375, line 19f | Replace p by \tilde{p} (four times). |
| p377, line 20f | $\mathbf{E}_8[\tau_8] = \frac{17}{8}$. |
| p384, line 4 | Only for $d = 1$, $\mu_1 \preceq \mu_2$ is equivalent to $F_1 \geq F_2$. Also, only for $d = 1$, F is a distribution function. The statement of p 383, line -2 remains true nevertheless (see Thm 3.3.5 of [116]). |
| p387, line 14 | Replace $p(x, \hat{y}^k)$ by $p_k(x, \hat{y}^k)$. |

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| p387, line 22 | <p>“Assume that L is sufficiently large...”: This does not work in this generality. A simple way out is the following: Since X is irreducible and aperiodic, there is an $N \in \mathbb{N}$ such that $p^N(0, x) > 0$ for all $x \in \{-1, 0, 1\}$. For the random walk $X'_n := X_{nN}$, $n \in \mathbb{N}$, the proof works with $L = 1$. We obtain a coupling of the random walk X at times $0, N, 2N, \dots$. Finally, fill the gaps by suitable random variables such that we recover the original random walk.</p> <p>The drawback of this proof is that (X, Y) is not Markov, in general. However, this is not necessary for the conclusion of Corollary 18.15. Hence, in Definition 18.10 we would like to drop the requirement that the coupling be Markov.</p> |
| p388, line 15 | first two terms: modulus signs are missing. |
| p388, line 28 | $Z := ((\tilde{X}_n, \tilde{Y}_n))_{n \in \mathbb{N}_0}$ (tildes missing). |
| p391, line -12 | Some additional assumption on q has to be made, e.g., symmetry. Or more generally, that $q(x, y) > 0$ iff $q(y, x) > 0$ and that q is not reversible with respect to π (instead that p is not the uniform distribution). |
| p400, line 20f | Replace p by r (three times). |
| p400, line 22 | Replace ϱ^k by ρ^k . |
| p407, line 1 | Instead of $F'_{A'}(x, y) > 0$ for all x , we only have $F'_{A'}(x_0, y) > 0$ which is not enough to apply Thm 19.6. The <i>proof</i> of Thm 19.6 however shows that we can relax the assumption of Thm 19.6: If $f(x_0) = \sup f(B_{x_0})$, then $f(x_0) = f(y)$ for all $y \in B_{x_0}$. This is the statement of the formula in line 3. |
| p411, line 5 | $\sum_{l=k}^{n-1}$ instead of $\sum_{l=k-1}^{n-1}$. |
| p415, line -3 | Replace $2D(A_1)$ by $4D(A_1)$. |
| p421 (19.11), | Instead of the effective resistances, there should be the resistances in the network that is reduced to the three points $0, 1$ and x . |
| p423, line 1 | Exercise 19.5.1 instead of 17.5.1. |
| p428, line -4f | Erase first two sentences. |
| p429, line -1 | Replace ϱ_i by ϱ_k . |
| p433, line 9 | Replace $c > 0$ by $c \in \mathbb{R}$. |

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| p435, line 9 | Replace L^1 by \mathcal{L}^1 . |
| p440, line 14 | Replace $\xrightarrow{n \rightarrow \infty}$ by $\xrightarrow{m \rightarrow \infty}$. |
| p442, line 12 | Of course, the convergence $A_n^\varepsilon \uparrow A_n^0$ holds only on the event $\{S_n \rightarrow \infty\}$, which has probability 1. |
| p442, lines 14, 15 | Replace A_n^ε by A_i^ε (twice). |
| p442, line 15 | Replace $S_n \geq \frac{pn\varepsilon}{2}$ by $S_n \geq S^- + \frac{pn\varepsilon}{2}$. |
| p442, line 16 | Replace $\frac{pn\varepsilon}{2}$ by $\frac{p\varepsilon}{2}$. |
| p442, line 17ff. | <p>The argument could be given in some more detail. For example: There is an $\varepsilon > 0$ such that $\mathbf{P}[X_1 < -2\varepsilon] > \varepsilon$. Let $L := \liminf_{n \rightarrow \infty} S_n$. As shown above, we have $\mathbf{P}[L = \infty] = 0$. The event $\{L > -\infty\}$ is invariant and hence has probability 0 or 1. We assume $\mathbf{P}[L > -\infty] = 1$ and construct a contradiction. Inductively, define stopping times $\tau_1 := \inf\{k \in \mathbb{N} : S_k < L + \varepsilon\}$ and</p> $\tau_{n+1} := \inf\{k > \tau_n : S_k < L + \varepsilon\} \quad \text{for } n \in \mathbb{N}.$ <p>By assumption, we have $\tau_n < \infty$ almost surely for all n. Let $\mathbb{F} = (\mathcal{F}_n)_{n \in \mathbb{N}_0} = \sigma((X_n)_{n \in \mathbb{N}})$ be the filtration generated by $X = (X_n)_{n \in \mathbb{N}}$. Let \mathcal{F}_{τ_n} be the σ-algebra of τ_n-past. Define the events $A_n := \{X_{\tau_n+1} < -2\varepsilon\}$, $n \in \mathbb{N}$. On A_n, we have $S_{\tau_n+1} < L - \varepsilon$. Clearly, A_n is independent of \mathcal{F}_{τ_n} and thus</p> $\mathbf{P}[A_n \mathcal{F}_{\tau_n}] = \mathbf{P}[A_n] > \varepsilon.$ <p>By the conditional version of the Borel-Cantelli Lemma (see Exercise 11.2.7), we infer</p> $\mathbf{P}\left[\limsup_{n \rightarrow \infty} A_n\right] = 1.$ <p>This shows that almost surely $S_{\tau_n+1} < L - \varepsilon$ infinitely often and this in turn contradicts the assumption that L be finite. Consequently, we have $\mathbf{P}[\liminf S_n = -\infty] = 1$. The statement for $\limsup S_n$ is similar.</p> |

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| p443, line -6 | Replace τ^{-n} by τ^{-i} . |
| p448, line -1 | Replace ϱ^γ by $\varrho^{-\gamma}$. |
| p450, line -2 | Replace <i>Chebyshev's</i> by <i>Markov's</i> . |
| p452, line 10 | Replace $(n+1)(1-\gamma)$ by $n_0(1-\gamma)$. |
| p458, line 11 | Delete the second integral sign. |
| p459, line -5 | Replace $A_N = \bigcap_{n \geq n_0}$ by $A_N = \liminf_{n \rightarrow \infty} A_{N,n}$. In the first display formula on the subsequent page replace the first " \leq " by " $=$ ". |
| p459, line -4 | Replace $B_{\tau^n} + t$ by $B_{\tau^{n+t}}$. |
| p461, (21.16) | Replace τ_n by τ^n . |
| p462, line 2 | Replace τ_n by τ^n . |
| p463, line 1 | Replace X_t by \tilde{X}_t . |
| p466, line 17ff | $k = 1, \dots, 2^{n-1}$ instead of $k = 1, \dots, 2^n$. In the display: change $2^{n/2}$ to $2^{(n-1)/2}$ (twice) and 2^{n+1} to 2^n (twice). |
| p466, line 22 | change to $\xi_{0,1}, (\xi_{n,k})_{n \in \mathbb{N}, k=1, \dots, 2^{n-1}}$ and $X^n := \xi_{0,1} B_{0,1} + \sum_{m=1}^n \sum_{k=1}^{2^{m-1}} \xi_{m,k} B_{m,k}$. |
| p467, line 10ff | change 2^n to 2^{n-1} (three times) and 2^{n+1} to 2^n (once). |
| p467, line -1 | Replace I^2 by $I(f)^2$. |
| p475, line12 | $U_{[nt]}^K$ and $T_{[nt]}^K$ instead of $U_{[nt]}^{K,n}$ and $T_{[nt]}^{K,n}$. |
| p475, line -7 | Replace $\frac{N}{\varepsilon^2}$ by $\frac{N}{\varepsilon^2 \sigma^2}$. |
| p476, (21.35) | Replace $\frac{n(n-1)}{2}$ by $3n(n-1)$ (twice). |
| p476, line 16. | Replace " $a = \lceil (t+s)n \rceil - (t+s)n$ and $a = sn - \lfloor sn \rfloor$ " by " $a = (t+s)n - \lfloor (t+s)n \rfloor$ and $a = \lceil sn \rceil - sn$ ". |
| p476, (21.36) | Replace $3t^2$ by $18t^2$ (three times) and $3\sqrt{N}$ by $18\sqrt{N}$. |
| p486, line 14 | Replace $V_T^1(\langle F, G \rangle_T)$ by $V_T^1(\langle F, G \rangle)$. |
| p487, line 6 | Insert a 2 in front of the second sum. |
| p488, line-1 | Replace \mathbb{R}^d by \mathbb{R}^3 . |

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| p490, line -7 | Replace $(a_{k+1} - a_0)$ by $(a_{k+1} - a_k)$. |
| p491, line 10 | Replace $\sum_{t \in \mathcal{P}_{s,T}^n}$ by $\sum_{t \in \mathcal{P}_{s',T}^n}$. |
| p491, line 14 | Replace $\sum_{t \in \mathcal{P}_{s,T}^n}$ by $\sum_{t \in \mathcal{P}_{s',T}^n}$ and \mathcal{F}_s by $\mathcal{F}_{s'}$. |
| p491, line 15 | Replace $M_T - M_s$ by $M_T - M_{s'}$ and \mathcal{F}_s by $\mathcal{F}_{s'}$. |
| p493, line 20 | Replace $M_{\tau_0 \wedge \tau_n \wedge t}^2$ by $M_{\tau_0 \wedge \tau_n \wedge t}^2$. |
| p499, line 8 | If $m = 0$ then $\theta = \delta_{(-1,0)}$ is a possible choice. In the remainder of the proof assume $m > 0$. |
| p500, line 6 | $\{\tau \leq t\} = \bigcap_{\substack{u,v \in \mathbb{Q} \\ u < 0 \leq v}} (\{\Xi \in (-\infty, u] \times [v, \infty)\} \cap \{\tau_{u,v} \leq t\}) \in \mathcal{F}_t$. |
| p502, line 1 | Replace $\mathbf{E}[X_\infty]$ by $\mathbf{E}[X_\infty^2]$. |
| p506, line 8 | Replace $\frac{1}{\sqrt{2\pi n}}$ by $\frac{1}{x\sqrt{2\pi n}}$. |
| p512, line -6ff | The rate function is finite only if the random variable X_1 can assume arbitrarily large and small values. Otherwise I can, e.g., look like in (23.6) and need not even be continuous. More precisely, the proof needs the following changes: |
| p512, line -2f | Replace \lim by \liminf . |
| p512, line -1 | Erase “ $= -I(x)$ ”. |
| p513, line 1 | change \lim to \liminf and $=$ to \geq . |
| p513, line 16 | “ $x \geq 0, x \in U$, such that $I(x) < \infty$ ”. |
| p513, line 17 | “ $(x - \varepsilon, x + \varepsilon) \subset U$ ”. |
| p513, line 18 | Replace \lim by \liminf and “ $= I(x - \varepsilon)$ ” by “ $\geq I(x)$ ”. (Strict inequality holds since I is convex and since $I(x) < \infty$ is assumed.) |
| p513, line 21,22,23 | Replace \lim by \liminf . |
| p513, line 25 | Replace the display formula and the subsequent text line by $\liminf_{n \rightarrow \infty} \frac{1}{n} \log P_n(U) \geq -\inf I(U).$ |
| p518, line 2 | Replace $\inf_{\mu} I$ by $\inf_{\mu} I_{\mu}$. |
| p518, line 4 | Replace \geq by \leq . |

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| p520, line 12 | Replace $\lim_{\varepsilon \rightarrow 0}$ by $\limsup_{\varepsilon \rightarrow 0}$. |
| p520, line 12f | Replace $x \in I$ by $x \in E$ and $\phi(x) - \delta$ by $\phi(x) + \delta$. |
| p522, line 5 | $I^\beta(x) = \beta \cdot (F^\beta(x) - \inf_{y \in \mathcal{M}_1(\Sigma)} F^\beta(y))$. |
| p529, line -2 | Replace $2^{-n}\mu(A)$ by $1 - \exp(2^{-n}\mu(A))$. |
| p530, line 1 | Replace $2^{-n}\mu(A)$ by $1 - \exp(2^{-n}\mu(A))$. |
| p533, line 15 | The $(Y_x)_{x \in E}$ have to be independent of X . |
| p534, line 7 | Change $X^\kappa(A)$ to X^κ . |
| p534, line -4 | Change $\nu \in E$ to $\nu \in \mathcal{M}_1(E)$. |
| p536, line 20 | Define Δ'_n as $\Delta'_n := \{(x_1, \dots, x_{n-1}) \in (0, 1)^{n-1} : \sum_{i=1}^{n-1} x_i < 1\}$. |
| p536, line 24 | Replace Δ'_{n-1} by Δ'_n . |
| p536, line -1 | Replace (s_j/s) by s_j . |
| p537, line 15 | Replace $n - 2$ by $n - 1$. |
| p539, line 10 | Replace $X^{n,1} = (X_{I_1}^n, X_2, \dots)$ by $\hat{X}^{n,1} = (X_{I_1}^n, X_1^n, X_2^n, \dots)$. |
| p539, line 14 | Replace $X^{n,1}$ by $\hat{X}^{n,1}$. |
| p541, line 2 | Replace PD by GEM. |
| p541, line 3 | Replace Theorem 25 by Theorem 3.2. |
| p546, line 18 | Erase "a.s." |
| p548, lines 3,6 | Replace $I^W(H^{(t)})$ by $I_\infty^W(H^{(t)})$. |
| p555, line 19 | Replace \mathcal{P}_T by \mathcal{P}_T^n . |
| p556, line 2 | Replace $F(M_s)$ by $F'(M_s)$. |
| p556, line-10 | Add "and $M_0 = 0$ ". |
| p556, line -8 | Replace $F(X_t) - F(X_0)$ by $F(X_T) - F(X_0)$. |
| p556, line -6 | Replace $\langle X \rangle_t$ by $\langle X \rangle_T$. |
| p558, line -2 | Replace " $= T$ " by " $\leq T$ ". |

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| p560, line 11 | Replace $\sigma_s^{i,l}$ by $\sigma_s^{l,i}$. |
| p560, (25.16) | Replace \int_0^t by \int_0^T (three times). |
| p561, line 5 | Replace F by $(F(W_t))_{t \geq 0}$. |
| p561, line 17 | Replace (26.3) by (26.17). |
| p565, line -7 | Replace $d = 2$ by $d \leq 2$. |
| p565, line -3 | On the r.h.s. replace $\ W_t\ < r$ by $\ W_t\ \leq s$. |
| p574, line 15 | In the right inequality on the right hand side the factor K is missing. |
| p574, line 18 | ... and [51, Theorem 5.3.1]... |
| p574, line 18 | ... and [51, Theorem 5.3.1]... |
| p577, line 8 | Replace $\mathbf{1}_{(0,\infty)}$ by $\mathbf{1}_{[0,\infty)}$. |
| p577, line 15 | Replace \int_0^1 by \int_0^t . |