

## A. Klenke, Probability Theory, 1st ed., Errata, 21.10.2019

p 6, line 8f	Replace $\mathcal{A}$ by $\mathcal{A}_I$ (five times).
p 8, line -14f	Replace $\tau \subset \Omega$ by $\tau \subset 2^\Omega$ .
p 11, line 6	Change “1.10” to “1.9”.
p 15, line 13	We have to assume $\mu(A_1 \cup \dots \cup A_n) < \infty$ .
p 18, line 22	Replace $a < b$ by $a \leq b$ .
p 20, line 5	Change this sentence to: Assume that there exist sets $\Omega_1, \Omega_2, \dots \in \mathcal{E}$ such that $\bigcup_{n=1}^{\infty} \Omega_n = \Omega$ and $\mu(\Omega_n) < \infty$ for all $n \in \mathbb{N}$ .
p 20, line -3	<p>Replace this sentence by the following paragraph: Now let <math>\Omega_1, \Omega_2, \dots \in \mathcal{E}</math> be a sequence such that <math>\bigcup_{n=1}^{\infty} \Omega_n = \Omega</math> and <math>\mu(\Omega_n) &lt; \infty</math> for all <math>n \in \mathbb{N}</math>. Let <math>E_n := \bigcup_{i=1}^n \Omega_i</math>, <math>n \in \mathbb{N}</math>, and <math>E_0 = \emptyset</math>. Hence <math>E_n = \biguplus_{i=1}^n (E_{i-1}^c \cap \Omega_i)</math>. For any <math>A \in \mathcal{A}</math> and <math>n \in \mathbb{N}</math>, we thus get</p> $\mu(A \cap E_n) = \sum_{i=1}^n \mu((A \cap E_{i-1}^c) \cap \Omega_i) = \sum_{i=1}^n \nu((A \cap E_{i-1}^c) \cap \Omega_i) = \nu(A \cap E_n).$
p 25, line 6	Erase the right hand side of the first line in the display formula.
p 25, line 15	Replace $a < b$ by $a \leq b$ .
p 27, line 18	Replace $[x, 0)$ by $(x, 0)$ . Add “for $x < 0$ ”.
p 27, lines -5, -4	Replace $F$ by $F_\mu$ (twice).
p 32, line 11	Append “ $A \subset \bigcup_{i=1}^{\infty} A_i$ and ”
p 51, line -4	Delete “with $k := \#J$ ”.
p 54, line 11	Replace Example 1.14 by Remark 1.14.
p 57, line 3	Replace display formula by $\mathbf{P} \left[ \bigcap_{j \in J} \{X_j \in A_j\} \right] = \prod_{j \in J} \mathbf{P}[X_j \in A_j]$ .
p 57, line -15	Erase (iii).
p 59, line 3	Replace $X^{-1}$ by $X_i^{-1}$ .
p 60, line -3	Replace “ring” by “semiring”.

p 73, line 28	The graph $(T, \sim)$ may contain circles. It is not too hard to see that, e.g., three trifurcation points can be pairwise directly connected. In order to save the argument one can do the following: Glue together all open edges that can be connected by an open path which traverses no trifurcation point. We thus obtain a graph consisting off the trifurcation points and these clumps of edges. In this graph, trifurcation points are the neighbours of the adjacent clumps of open edges but they are not direct neighbours of other trifurcation points. This graph now does not contain circles. Hence the remaining argument works (with minor changes).
p78, line 3	Replace by $\sum_{k=1}^{\infty} \mathbf{P}[X = k] \cdot k(k-1) \cdots (k-n+1)$
p 83, 3	change to “for all $i, k, n \in \mathbb{N}_0$ .”
p78, line 6	The $x_i$ have to have an accumulation point in $(0, 1)$ unless $\psi(z) < \infty$ for some $z > 1$ .
p89, lines 12, 13	Replace these two lines by: Clearly, $f^+ \leq g^+$ a.e., hence $(f^+ - g^+)^+ = 0$ a.e. By Theorem 4.8, we get $\int (f^+ - g^+)^+ d\mu = 0$ . Since $f^+ \leq g^+ + (f^+ - g^+)^+$ (not only a.e.), we infer from Lemma 4.6(i) and (iii)
	$\int f^+ d\mu \leq \int (g^+ + (f^+ - g^+)^+) d\mu = \int g^+ d\mu.$
	Similarly, we use $f^- \geq g^-$ a.e. to obtain
	$\int f^- d\mu \geq \int g^- d\mu.$
p98, line 13,16	Replace $\int_{\varepsilon}^{\infty} g(t) dt$ by $\int_0^{\infty} (g(\varepsilon) \wedge g(t)) dt$ .
p105, line -9	Change $c = 0$ to $c = -\mathbf{E}[Y]$ .
p108, line 9	Erase $\limsup_{n \rightarrow \infty}$ .
p109, line -5	$\tilde{S}_n$ instead of $S_n$ .
p115, line 19	Replace $Y_i$ by $Y_i(x)$ .
p115, line 21	Replace $Z_i$ by $Z_i(x)$ .

p117, line 17	Change $\ell(k)$ to $\ell(e_k)$ .
p117, line 18	Change $l \geq k$ to $l > k$ .
p117, line -6	Prepend minus signs to both sides of the equation.
p121, line 17	Replace $ S_k /k \leq 2 S_k /k_{n+1}$ by $ S_k /l(k) \leq 2 S_k /l(k_{n+1})$ .
p124, line 1ff	In order that equality holds in line 5, (and not only $\leq$ ), we need to establish that $2^n \mathbf{P}[N_{2^{-n}} \geq 2] \xrightarrow{n \rightarrow \infty} \lambda$ . This can be inferred by the fact that for all $n \in \mathbb{N}$ and $\varepsilon > 0$ , we have <p style="text-align: center;"><math>\mathbf{P}[N_{2^{-n}} \geq 2] \geq \lfloor 2^{-n}/\varepsilon \rfloor \mathbf{P}[N_\varepsilon \geq 2] - \lfloor 2^{-n}/\varepsilon \rfloor^2 \mathbf{P}[N_\varepsilon \geq 2]^2,</math></p> which implies (by suitably letting $\varepsilon \rightarrow 0$ ) that $2^n \mathbf{P}[N_{2^{-n}} \geq 2] \geq \lambda - 2^{-n} \lambda^2 \xrightarrow{n \rightarrow \infty} \lambda$ .
p132, line 10	Replace $\ f_n - f\ _p$ by $\ f_n - f\ _p^p$ .
p140, lines 1/2	Replace $\{ f - f_{n'_k}  > g_k\} = \{ f - f_{n'_k}  > g\}$ by <p style="text-align: center;"><math> f - f_{n'_k}  = ( f - f_{n'_k}  - g)^+ + g_k</math></p> and $\int_{\{ f - f_{n'_k}  > g\}}  f - f_{n'_k}  d\mu$ by $\int ( f - f_{n'_k}  - g)^+ d\mu$ .
p153, line 4	In the definition of $\varrho$ : Erase minus sign.
p154, line 23,26	Replace $\Lambda^2$ by $L^2$ .
p158, Ex. 7.4.1	Not $F$ , but its inverse $F^{-1}$ is the continuous distribution function of a singular measure.
p175, line -3	Replace (vii) by (vi).
p172, line 17	Replace “ $X$ be a nonnegative” by “ $X > 0$ be a strictly positive”.
p175, line 20	$(Z_n)$ is decreasing, hence $Z$ is in fact the <i>limit</i> and not only limsup. Hence Fatou’s lemma can be applied. In line 24 replace $\mathbf{E}[Z_n]$ by $\mathbf{E}[Z_n   \mathcal{F}]$ .
p184, line 2	exchange $\mu_1$ and $\mu_2$
p185, (8.16)	Replace $\kappa_{Y, \mathcal{F}}$ by $\kappa_{X, \mathcal{F}}$ .

p192, line -7	Replace $\{\tau \leq t\}$ by $\{\tau_K \leq t\}$ .
p193, line 1	We assume that $I \subset [0, \infty)$ is closed under addition (at least for (ii) and (iii)).
p196, line 27	Replace $\mathbf{E}[X_s]$ by $X_s$ .
p213, line 6	Change $\langle X \rangle$ to $\langle X \rangle_\tau$ .
p218, line 15	$\mathbf{E}[ X_n ^p] < \infty$ (bracket ] missing).
p225, line 9	exchange + and -.
p227, line 15	After “events” append: “with $A_n \in \mathcal{F}_n$ for all $n \in \mathbb{N}$ .”
p227, line 17	Replace $X_n = \sum_{k=1}^{\infty} (\mathbf{1}_{A_n} - \mathbf{P}[A_n   \mathcal{F}_{n-1}])$ by $X_n = \sum_{k=1}^n (\mathbf{1}_{A_k} - \mathbf{P}[A_k   \mathcal{F}_{k-1}])$ .
p228, line 13	Replace $Z^n$ by $Z_n$ .
p232, line 21	Replace $i < k$ by $i \leq k$ .
p234, line 13	Replace “ $\mathcal{E}_n =$ ” by “ $\mathcal{E}_n \supset$ ”.
p234, line 17	In order to avoid trivial cases, assume, for example, $E = \{0, 1\}$ , $X_1, X_2, \dots$ independent with $\mathbf{P}[X_n] \in (0, 1)$ for all $n \in \mathbb{N}$ and $B = \{1\}$ .
p234, line 25	For $A \in \mathcal{E}_n$ , there exists a measurable $B \subset E^{\mathbb{N}}$ such that $B^\varrho = B$ for all $\varrho \in S_n$ . Define $F = \mathbf{1}_B$ .
p235, (12.4)	Replace $(N\Xi_N(A_l))^{m_l}$ by $(N\Xi_N(A_l))_{m_l}$ .
p236, line 20ff	Replace $Y_{-n}$ by $Y_n$ .
p237 (12.5)	Replace $\prod_{l=1}^n$ by $\prod_{l=1}^k$ .

p248, line17	<p>Since <math>\tau</math> is not a semiring, Thm 1.65 cannot be used directly. A more subtle (and hopefully correct) proof for outer regularity is the following:</p> <p>First assume <math>B \subset E</math> is closed and let <math>\varepsilon &gt; 0</math>. Let <math>B_\delta := \{x \in E : d(x, B) &lt; \delta\}</math> be the open <math>\delta</math>-neighbourhood of <math>B</math>. As <math>B</math> is closed, we have <math>\bigcap_{\delta&gt;0} B_\delta = B</math>. Since <math>\mu</math> is upper semicontinuous, there is a <math>\delta &gt; 0</math> such that <math>\mu(B_\delta) \leq \mu(B) + \varepsilon</math>.</p> <p>Not let <math>B \in \mathcal{E}</math> and <math>\varepsilon &gt; 0</math>. Consider <math>\mathcal{A} := \{V \cap C : V \subset E \text{ open, } C \subset E \text{ closed}\}</math>. We have <math>\mathcal{E} = \sigma(\mathcal{A})</math> and <math>\mathcal{A}</math> is a semiring. By Theorem 1.65, there are mutually disjoint sets <math>A_n = V_n \cap C_n \in \mathcal{A}</math>, <math>n \in \mathbb{N}</math>, such that <math>B \subset A := \bigcup_{n=1}^{\infty} A_n</math> and <math>\mu(A) \leq \mu(B) + \varepsilon/2</math>. As shown above, for any <math>n \in \mathbb{N}</math>, there is an open set <math>W_n \supset C_n</math> such that <math>\mu(W_n) \leq \mu(C_n) + \varepsilon 2^{-n-1}</math>. Hence also <math>U_n := V_n \cap W_n</math> is open. Let <math>B \subset U := \bigcup_{n=1}^{\infty} U_n</math>. We conclude that <math>\mu(U) \leq \mu(A) + \sum_{n=1}^{\infty} \varepsilon 2^{-n-1} \leq \mu(B) + \varepsilon</math>.</p>
p262, line -10	Replace $\mu$ by $\mu_n$ .
p261, line -7	We also have to show that $F(-\infty) = 0$ in order that $F$ be a distribution function. This however follows from tightness just as in the lines -6ff.
p264, line -3	Replace $\mathcal{E}$ by $\mathcal{U}$ .
p266, line -3	Erase $\alpha(\bigcup_{i=1}^n A_i) =$ .
p274, line 6	Replace $A_1, \dots, A_n$ by $A_1, \dots, A_N$ .
p274, line 10f	Delete "respectively a semiring" (twice).
p274, line 14	Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$ .
p274, line 21	Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$ .
p280, line 9	Replace $\kappa_1 \otimes \kappa$ by $\kappa_1 \otimes \kappa_2$ .
p280, line -5	Replace $\bigotimes_{k=0}^i \mathcal{A}_k$ by $\bigotimes_{k=1}^i \mathcal{A}_k$ .
p281, line -1	Replace $\varphi_k$ by $\varphi_n$ .
p286, line 24	Replace $P_{n+1} _{\bar{A}^n}$ by $P_{n+1} _{\bar{A}^n}$ .
p288, line 17	Replace $\omega \in \Omega$ by $\omega \in E$ .

p289, line -4	Replace $\otimes_{k=i}^n$ by $\otimes_{k=i}^{n-1}$ .
p289, line -2f	Replace $A_{l+1}$ by $A_{j_{l+1}}$ (twice).
p290, line 2	Replace $f_{l+1}(\omega_{l+1})$ by $f_{l-1}(\omega_{l-1})$ .
p290, line 3	Replace $f(\omega_{l+1})$ by $f_{l+1}(\omega_{l+1})$ .
p290, line 17	Replace $\mu \otimes \kappa$ by $\int \mu(dx) \kappa(x, \cdot)$ .
p295, line 3	Replace $H(x)$ by $H_z(x)$ .
p295, line 8	Replace $h(y)$ by $h_z(y)$ .
p299, line 6	$\ f\ _2 = \ \varphi\ _2 / (2\pi)^{d/2}$ .
p302, line 21	Replace $(t/a)$ by $(t/\theta)$ .
p303, line -1	Factor $1/\sqrt{2\pi}$ in front of the integral is missing.
p303, line 11f	Replace $+$ by $-$ (four instances).
p305, line 11	Replace $\varphi(t)$ by $\varphi_X(t)$ .
p314, lines 2, 3	Replace $h^n$ by $ h ^n$ (two instances).
p314, lines 12, 13	Replace $\sqrt{2\pi n}$ by $1/\sqrt{2\pi n}$ (two instances).
p315, line 18	Replace $\varphi^{(2n)}(0)$ by $(-1)^n \varphi^{(2n)}(0)$ .
p315, line -3	$\mathbf{E}[X^{2k}] = (-1)^k u^{(2k)}(0)$ .
p316, line 4	$\mathbf{E}[X^{2n}] = (-1)^n u^{(2n)}(0) = (-1)^n \varphi^{(2n)}(0)$ .
p328, line -5	Replace $\theta^{-1} = r = k$ by $\theta = r = k/2$ .
p329, line -2	Replace display formula by $\varphi_{rv}(t) = \exp\left(r \sum_{k=1}^{\infty} \frac{((1-p)e^{it})^k - (1-p)^k}{k}\right) = p^r (1 - (1-p)e^{it})^{-r}.$
p331, lines -3, -1	Replace $\mu_n$ by $\nu_n$ .
p331, line -1	Replace $\nu$ by $\mu$ .

p333, line 13	Replace $h(t)$ by $h(x)$ .
p333, line -6	Replace $u(1)$ by $2u(1)$ .
p333, line -3	Replace $t \wedge 1$ by $t \vee 1$ .
p336, line -6	Here and in the remaining proof the sign of $\bar{\psi}(0)$ is wrong. Replace $\bar{\psi}(0) \leq 0$ by $\bar{\psi}(0) \geq 0$ .
p336, line -3	$\bar{\psi}(0) > 0$ .
p337, line 9	$\bar{\psi}_n(0) > 0$ and $\tilde{\nu}_n(dx) = (h(x)/\bar{\psi}_n(0))\nu_n(dx)$ .
p337, line 10	Replace $-\bar{\psi}(t)/\bar{\psi}(0)$ by $\bar{\psi}(t)/\bar{\psi}(0)$ .
p337, line 13, 15	Erase the minus sign.
p337, line 15	Replace $t \wedge 1$ by $t \vee 1$ .
p337, line 16	The map $f_t$ is not continuous. At this point, we have to work with $g_{t,\varepsilon}(x) = e^{-itx} - 1 - itx \mathbf{1}_{\{ x  < 1-\varepsilon\}}$ instead of $g_t(x) = e^{-itx} - 1 - itx \mathbf{1}_{ x  < 1}$ . We choose $\varepsilon > 0$ such that $\nu$ has no atoms at the points of discontinuity $-1 + \varepsilon$ and $1 - \varepsilon$ . By the Portemanteau Theorem (Theorem 13.16(iii)), we get convergence of the integrals. Finally, we let $\varepsilon \rightarrow 0$ .
p337, line -5	Insert the factor $\bar{\psi}_n(0)$ before the integral $\int f_t(x) \tilde{\nu}_n(dx)$ .
p338, (16.16)	Replace $(0, \infty)$ by $\mathbb{R} \setminus \{0\}$ .
p340, line 10	The relation $nb = n^{1/\alpha}b$ is wrong since it does not pay respect to the change due to passing from $\nu$ to $\nu \circ m_{n^{1/\alpha}}^{-1}$ . We first have to compute the explicit form of $\nu$ . Then we can compute the correct scaling relation (here without detailed derivation):
	$nb = bn^{1/\alpha} - (c^+ - c^-) \begin{cases} (1 - \alpha)^{-1}(n^{1/\alpha} - n), & \text{if } \alpha \neq 1, \\ n \log(n), & \text{if } \alpha = 1. \end{cases}$
	Consequently, we get $b = (c^+ - c^-)/(1 - \alpha)$ in the case $\alpha \neq 1$ . No changes are necessary for the case $\alpha = 1$ .
p340, (16.18)	Replace $i(c^+ - c^-)$ by $-i \operatorname{sign}(t)(c^+ - c^-)$ .

p344, (17.12)	Replace $\sum_{i=0}^{n-1}$ by $\sum_{i=1}^n$ .
p347, line 13	Replace $\kappa_{t_{n+1}-t_n}$ by $\kappa_{t_{i+1}-t_i}$ .
p353, line 3	Change $I$ to $E$ .
p350, line -9	Replace $t \in \mathbb{N}_0$ by $t \in I$ (twice).
p357, line 5	Replace “With this convention” by “Finally, we assume that”.
p357, line -7	Define $p = I$ if $\lambda = 0$ .
p358, line 10	Replace $\tilde{p}_{t+s}$ by $p_{t+s}$ .
p358, line 15	Replace $\int_0^t$ by $\int_0^s$ (twice).
p359, line -2	replace $P_x^Y$ by $\mathbf{P}_x^Y$ .
p363, line -4	We also agree that $0/0 = 0$ and $0 \cdot \infty = 0$ .
p363, line -2	Before “state” insert “non-absorbing”.
p364, line 7	We assume that $x \neq y$ .
p366, line 15	For the right term a factor $4^n$ is missing.
p373, line 15	Replace “If $X$ ” by “If any state”
p373, line 17f	Replace $\mu p^n(x)$ by $\mu p^n(\{x\})$ (twice) and $\mu(x)$ by $\mu(\{x\})$ .
p375, line 19f	Replace $p$ by $\tilde{p}$ (four times).
p377, line 20f	$\mathbf{E}_8[\tau_8] = \frac{17}{8}$ .
p384, line 4	Only for $d = 1$ , $\mu_1 \preceq \mu_2$ is equivalent to $F_1 \geq F_2$ . Also, only for $d = 1$ , $F$ is a distribution function. The statement of p 383, line -2 remains true nevertheless (see Thm 3.3.5 of [116]).
p387, line 14	Replace $p(x, \hat{y}^k)$ by $p_k(x, \hat{y}^k)$ .



p387, line 22	<p>“Assume that <math>L</math> is sufficiently large...”: This does not work in this generality. A simple way out is the following: Since <math>X</math> is irreducible and aperiodic, there is an <math>N \in \mathbb{N}</math> such that <math>p^N(0, x) &gt; 0</math> for all <math>x \in \{-1, 0, 1\}</math>. For the random walk <math>X'_n := X_{nN}</math>, <math>n \in \mathbb{N}</math>, the proof works with <math>L = 1</math>. We obtain a coupling of the random walk <math>X</math> at times <math>0, N, 2N, \dots</math>. Finally, fill the gaps by suitable random variables such that we recover the original random walk.</p> <p>The drawback of this proof is that <math>(X, Y)</math> is not Markov, in general. However, this is not necessary for the conclusion of Corollary 18.15. Hence, in Definition 18.10 we would like to drop the requirement that the coupling be Markov.</p>
p388, line 15	first two terms: modulus signs are missing.
p388, line 28	$Z := ((\tilde{X}_n, \tilde{Y}_n))_{n \in \mathbb{N}_0}$ (tildes missing).
p391, line -12	Some additional assumption on $q$ has to be made, e.g., symmetry. Or more generally, that $q(x, y) > 0$ iff $q(y, x) > 0$ and that $q$ is not reversible with respect to $\pi$ (instead that $p$ is not the uniform distribution).
p400, line 20f	Replace $p$ by $r$ (three times).
p400, line 22	Replace $\varrho^k$ by $\rho^k$ .
p407, line 1	Instead of $F'_{A'}(x, y) > 0$ for all $x$ , we only have $F'_{A'}(x_0, y) > 0$ which is not enough to apply Thm 19.6. The <i>proof</i> of Thm 19.6 however shows that we can relax the assumption of Thm 19.6: If $f(x_0) = \sup f(B_{x_0})$ , then $f(x_0) = f(y)$ for all $y \in B_{x_0}$ . This is the statement of the formula in line 3.
p411, line 5	$\sum_{l=k}^{n-1}$ instead of $\sum_{l=k-1}^{n-1}$ .
p415, line -3	Replace $2D(A_1)$ by $4D(A_1)$ .
p421 (19.11),	Instead of the effective resistances, there should be the resistances in the network that is reduced to the three points $0, 1$ and $x$ .
p423, line 1	Exercise 19.5.1 instead of 17.5.1.
p428, line -4f	Erase first two sentences.
p429, line -1	Replace $\varrho_i$ by $\varrho_k$ .
p433, line 9	Replace $c > 0$ by $c \in \mathbb{R}$ .

p435, line 9	Replace $L^1$ by $\mathcal{L}^1$ .
p440, line 14	Replace $\xrightarrow{n \rightarrow \infty}$ by $\xrightarrow{m \rightarrow \infty}$ .
p442, line 12	Of course, the convergence $A_n^\varepsilon \uparrow A_n^0$ holds only on the event $\{S_n \rightarrow \infty\}$ , which has probability 1.
p442, lines 14, 15	Replace $A_n^\varepsilon$ by $A_i^\varepsilon$ (twice).
p442, line 15	Replace $S_n \geq \frac{pn\varepsilon}{2}$ by $S_n \geq S^- + \frac{pn\varepsilon}{2}$ .
p442, line 16	Replace $\frac{pn\varepsilon}{2}$ by $\frac{p\varepsilon}{2}$ .
p442, line 17ff.	<p>The argument could be given in some more detail. For example: There is an <math>\varepsilon &gt; 0</math> such that <math>\mathbf{P}[X_1 &lt; -2\varepsilon] &gt; \varepsilon</math>. Let <math>L := \liminf_{n \rightarrow \infty} S_n</math>. As shown above, we have <math>\mathbf{P}[L = \infty] = 0</math>. The event <math>\{L &gt; -\infty\}</math> is invariant and hence has probability 0 or 1. We assume <math>\mathbf{P}[L &gt; -\infty] = 1</math> and construct a contradiction. Inductively, define stopping times <math>\tau_1 := \inf\{k \in \mathbb{N} : S_k &lt; L + \varepsilon\}</math> and</p> $\tau_{n+1} := \inf\{k > \tau_n : S_k < L + \varepsilon\} \quad \text{for } n \in \mathbb{N}.$ <p>By assumption, we have <math>\tau_n &lt; \infty</math> almost surely for all <math>n</math>. Let <math>\mathbb{F} = (\mathcal{F}_n)_{n \in \mathbb{N}_0} = \sigma((X_n)_{n \in \mathbb{N}})</math> be the filtration generated by <math>X = (X_n)_{n \in \mathbb{N}}</math>. Let <math>\mathcal{F}_{\tau_n}</math> be the <math>\sigma</math>-algebra of <math>\tau_n</math>-past. Define the events <math>A_n := \{X_{\tau_n+1} &lt; -2\varepsilon\}</math>, <math>n \in \mathbb{N}</math>. On <math>A_n</math>, we have <math>S_{\tau_n+1} &lt; L - \varepsilon</math>. Clearly, <math>A_n</math> is independent of <math>\mathcal{F}_{\tau_n}</math> and thus</p> $\mathbf{P}[A_n   \mathcal{F}_{\tau_n}] = \mathbf{P}[A_n] > \varepsilon.$ <p>By the conditional version of the Borel-Cantelli Lemma (see Exercise 11.2.7), we infer</p> $\mathbf{P}\left[\limsup_{n \rightarrow \infty} A_n\right] = 1.$ <p>This shows that almost surely <math>S_{\tau_n+1} &lt; L - \varepsilon</math> infinitely often and this in turn contradicts the assumption that <math>L</math> be finite. Consequently, we have <math>\mathbf{P}[\liminf S_n = -\infty] = 1</math>. The statement for <math>\limsup S_n</math> is similar.</p>

p443, line -6	Replace $\tau^{-n}$ by $\tau^{-i}$ .
p448, line -1	Replace $\varrho^\gamma$ by $\varrho^{-\gamma}$ .
p450, line -2	Replace <i>Chebyshev's</i> by <i>Markov's</i> .
p452, line 10	Replace $(n+1)(1-\gamma)$ by $n_0(1-\gamma)$ .
p458, line 11	Delete the second integral sign.
p459, line -5	Replace $A_N = \bigcap_{n \geq n_0}$ by $A_N = \liminf_{n \rightarrow \infty} A_{N,n}$ . In the first display formula on the subsequent page replace the first " $\leq$ " by " $=$ ".
p459, line -4	Replace $B_{\tau^n} + t$ by $B_{\tau^{n+t}}$ .
p461, (21.16)	Replace $\tau_n$ by $\tau^n$ .
p462, line 2	Replace $\tau_n$ by $\tau^n$ .
p463, line 1	Replace $X_t$ by $\tilde{X}_t$ .
p466, line 17ff	$k = 1, \dots, 2^{n-1}$ instead of $k = 1, \dots, 2^n$ . In the display: change $2^{n/2}$ to $2^{(n-1)/2}$ (twice) and $2^{n+1}$ to $2^n$ (twice).
p466, line 22	change to $\xi_{0,1}, (\xi_{n,k})_{n \in \mathbb{N}, k=1, \dots, 2^{n-1}}$ and $X^n := \xi_{0,1} B_{0,1} + \sum_{m=1}^n \sum_{k=1}^{2^{m-1}} \xi_{m,k} B_{m,k}$ .
p467, line 10ff	change $2^n$ to $2^{n-1}$ (three times) and $2^{n+1}$ to $2^n$ (once).
p467, line -1	Replace $I^2$ by $I(f)^2$ .
p475, line12	$U_{[nt]}^K$ and $T_{[nt]}^K$ instead of $U_{[nt]}^{K,n}$ and $T_{[nt]}^{K,n}$ .
p475, line -7	Replace $\frac{N}{\varepsilon^2}$ by $\frac{N}{\varepsilon^2 \sigma^2}$ .
p476, (21.35)	Replace $\frac{n(n-1)}{2}$ by $3n(n-1)$ (twice).
p476, line 16.	Replace " $a = \lceil (t+s)n \rceil - (t+s)n$ and $a = sn - \lfloor sn \rfloor$ " by " $a = (t+s)n - \lfloor (t+s)n \rfloor$ and $a = \lceil sn \rceil - sn$ ".
p476, (21.36)	Replace $3t^2$ by $18t^2$ (three times) and $3\sqrt{N}$ by $18\sqrt{N}$ .
p486, line 14	Replace $V_T^1(\langle F, G \rangle_T)$ by $V_T^1(\langle F, G \rangle)$ .
p487, line 6	Insert a 2 in front of the second sum.
p488, line-1	Replace $\mathbb{R}^d$ by $\mathbb{R}^3$ .

p490, line -7	Replace $(a_{k+1} - a_0)$ by $(a_{k+1} - a_k)$ .
p491, line 10	Replace $\sum_{t \in \mathcal{P}_{s,T}^n}$ by $\sum_{t \in \mathcal{P}_{s',T}^n}$ .
p491, line 14	Replace $\sum_{t \in \mathcal{P}_{s,T}^n}$ by $\sum_{t \in \mathcal{P}_{s',T}^n}$ and $\mathcal{F}_s$ by $\mathcal{F}_{s'}$ .
p491, line 15	Replace $M_T - M_s$ by $M_T - M_{s'}$ and $\mathcal{F}_s$ by $\mathcal{F}_{s'}$ .
p493, line 20	Replace $M_{\tau_0 \wedge \tau_n \wedge t}^2$ by $M_{\tau_0 \wedge \tau_n \wedge t}^2$ .
p499, line 8	If $m = 0$ then $\theta = \delta_{(-1,0)}$ is a possible choice. In the remainder of the proof assume $m > 0$ .
p500, line 6	$\{\tau \leq t\} = \bigcap_{\substack{u,v \in \mathbb{Q} \\ u < 0 \leq v}} (\{\Xi \in (-\infty, u] \times [v, \infty)\} \cap \{\tau_{u,v} \leq t\}) \in \mathcal{F}_t$ .
p502, line 1	Replace $\mathbf{E}[X_\infty]$ by $\mathbf{E}[X_\infty^2]$ .
p506, line 8	Replace $\frac{1}{\sqrt{2\pi n}}$ by $\frac{1}{x\sqrt{2\pi n}}$ .
p512, line -6ff	The rate function is finite only if the random variable $X_1$ can assume arbitrarily large and small values. Otherwise $I$ can, e.g., look like in (23.6) and need not even be continuous. More precisely, the proof needs the following changes:
p512, line -2f	Replace $\lim$ by $\liminf$ .
p512, line -1	Erase “ $= -I(x)$ ”.
p513, line 1	change $\lim$ to $\liminf$ and $=$ to $\geq$ .
p513, line 16	“ $x \geq 0, x \in U$ , such that $I(x) < \infty$ ”.
p513, line 17	“ $(x - \varepsilon, x + \varepsilon) \subset U$ ”.
p513, line 18	Replace $\lim$ by $\liminf$ and “ $= I(x - \varepsilon)$ ” by “ $\geq I(x)$ ”. (Strict inequality holds since $I$ is convex and since $I(x) < \infty$ is assumed.)
p513, line 21,22,23	Replace $\lim$ by $\liminf$ .
p513, line 25	Replace the display formula and the subsequent text line by $\liminf_{n \rightarrow \infty} \frac{1}{n} \log P_n(U) \geq -\inf I(U).$
p518, line 2	Replace $\inf I$ by $\inf_{\mu} I_{\mu}$ .
p518, line 4	Replace $\geq$ by $\leq$ .

p520, line 12	Replace $\lim_{\varepsilon \rightarrow 0}$ by $\limsup_{\varepsilon \rightarrow 0}$ .
p520, line 12f	Replace $x \in I$ by $x \in E$ and $\phi(x) - \delta$ by $\phi(x) + \delta$ .
p522, line 5	$I^\beta(x) = \beta \cdot (F^\beta(x) - \inf_{y \in \mathcal{M}_1(\Sigma)} F^\beta(y))$ .
p529, line -2	Replace $2^{-n}\mu(A)$ by $1 - \exp(2^{-n}\mu(A))$ .
p530, line 1	Replace $2^{-n}\mu(A)$ by $1 - \exp(2^{-n}\mu(A))$ .
p533, line 15	The $(Y_x)_{x \in E}$ have to be independent of $X$ .
p534, line 7	Change $X^\kappa(A)$ to $X^\kappa$ .
p534, line -4	Change $\nu \in E$ to $\nu \in \mathcal{M}_1(E)$ .
p536, line 20	Define $\Delta'_n$ as $\Delta'_n := \{(x_1, \dots, x_{n-1}) \in (0, 1)^{n-1} : \sum_{i=1}^{n-1} x_i < 1\}$ .
p536, line 24	Replace $\Delta'_{n-1}$ by $\Delta'_n$ .
p536, line -1	Replace $(s_j/s)$ by $s_j$ .
p537, line 15	Replace $n - 2$ by $n - 1$ .
p539, line 10	Replace $X^{n,1} = (X_{I_1}^n, X_2, \dots)$ by $\hat{X}^{n,1} = (X_{I_1}^n, X_1^n, X_2^n, \dots)$ .
p539, line 14	Replace $X^{n,1}$ by $\hat{X}^{n,1}$ .
p546, line 18	Erase "a.s."
p548, lines 3,6	Replace $I^W(H^{(t)})$ by $I_\infty^W(H^{(t)})$ .
p555, line 19	Replace $\mathcal{P}_T$ by $\mathcal{P}_T^n$ .
p556, line 2	Replace $F(M_s)$ by $F'(M_s)$ .
p556, line-10	Add "and $M_0 = 0$ ".
p556, line -8	Replace $F(X_t) - F(X_0)$ by $F(X_T) - F(X_0)$ .
p556, line -6	Replace $\langle X \rangle_t$ by $\langle X \rangle_T$ .
p558, line -2	Replace " $= T$ " by " $\leq T$ ".

p560, line 11	Replace $\sigma_s^{i,l}$ by $\sigma_s^{l,i}$ .
p560, (25.16)	Replace $\int_0^t$ by $\int_0^T$ (three times).
p561, line 5	Replace $F$ by $(F(W_t))_{t \geq 0}$ .
p561, line 17	Replace (26.3) by (26.17).
p565, line -7	Replace $d = 2$ by $d \leq 2$ .
p565, line -3	On the r.h.s. replace $\ W_t\  < r$ by $\ W_t\  \leq s$ .
p574, line 15	In the right inequality on the right hand side the factor $K$ is missing.
p574, line 18	... and [51, Theorem 5.3.1]...
p574, line 18	... and [51, Theorem 5.3.1]...
p577, line 8	Replace $\mathbf{1}_{(0,\infty)}$ by $\mathbf{1}_{[0,\infty)}$ .
p577, line 15	Replace $\int_0^1$ by $\int_0^t$ .