

## A. Klenke, Probability Theory, 2nd edition, Errata, 21.10.2019

p 17, line 5	Replace $a < b$ by $a \leq b$ .
p 24, line 3	Replace $a < b$ by $a \leq b$ .
p 25, line -6	Replace $[x, 0)$ by $(x, 0)$ . Add “for $x < 0$ ”.
p 26, lines 7, 9	Replace $F$ by $F_\mu$ (twice).
p 74, line 27	Replace $=$ by $\geq$
p 89, lines 12, 13	<p>Replace these two lines by:</p> <p>Clearly, <math>f^+ \leq g^+</math> a.e., hence <math>(f^+ - g^+)^+ = 0</math> a.e. By Theorem 4.8, we get <math>\int (f^+ - g^+)^+ d\mu = 0</math>. Since <math>f^+ \leq g^+ + (f^+ - g^+)^+</math> (not only a.e.), we infer from Lemma 4.6(i) and (iii)</p> $\int f^+ d\mu \leq \int (g^+ + (f^+ - g^+)^+) d\mu = \int g^+ d\mu.$ <p>Similarly, we use <math>f^- \geq g^-</math> a.e. to obtain</p> $\int f^- d\mu \geq \int g^- d\mu.$
p134, line 10	Replace $\ f_n - f\ _p$ by $\ f_n - f\ _p^p$ .
p154, line 20	In the definition of $g$ : Erase minus sign.
p196, line 26	Replace $\mathbf{E}[X_s]$ by $X_s$ .
p234, line 5	Replace “ $\mathcal{E}_n =$ ” by “ $\mathcal{E}_n \supset$ ”.
p235, (12.4)	Replace $(N\Xi_N(A_l))^{m_l}$ by $(N\Xi_N(A_l))_{m_l}$ .
p236, line 14ff	Replace $Y_{-n}$ by $Y_n$ .
p261, line 17ff	Replace the definitions of $V_n$ and $W_{n+1}$ by $V_n := \bigcup_{i=1}^n U_i$ and $W_{n+1} := V_{N(\overline{W}_n)}$ , respectively.
p263, line -4	We also have to show that $F(-\infty) = 0$ in order that $F$ be a distribution function. This however follows from tightness just as in the lines -3ff.
p276, line 9	Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$ .
p276, line 16	Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$ .
p289, line -6	Replace $\omega \in \Omega$ by $\omega \in E$ .

p297, line 1	Replace $H(x)$ by $H_z(x)$ .
p297, line 6	Replace $h(y)$ by $h_z(y)$ .
p301, line 8	$\ f\ _2 = \ \varphi\ _2/(2\pi)^{d/2}$ .
p301, line -3	Factor $1/\sqrt{2\pi}$ in front of the integral is missing.
p304, line 18	Replace $(t/a)$ by $(t/\theta)$ .
p306, line -7	Replace $\varphi(t)$ by $\varphi_X(t)$ .
p316, lines 3, 4	Replace $h^n$ by $ h ^n$ (two instances).
p316, lines 13, 14	Replace $\sqrt{2\pi n}$ by $1/\sqrt{2\pi n}$ (two instances).
p318, line 1	$\mathbf{E}[X^{2k}] = (-1)^k u^{(2k)}(0)$ .
p335, lines 25, 27	Replace $\mu_n$ by $\nu_n$ .
p335, line 27	Replace $\nu$ by $\mu$ .
p341, line 11	The map $f_t$ is not continuous. At this point, we have to work with $g_{t,\varepsilon}(x) = e^{-itx} - 1 - itx\mathbf{1}_{\{ x <1-\varepsilon\}}$ instead of $g_t(x) = e^{itx} - 1 - itx\mathbf{1}_{ x <1}$ . We choose $\varepsilon > 0$ such that $\nu$ has no atoms at the points of discontinuity $-1 + \varepsilon$ and $1 - \varepsilon$ . By the Portemanteau Theorem (Theorem 13.16(iii)), we get convergence of the integrals. Finally, we let $\varepsilon \rightarrow 0$ .
p342, (16.16)	Replace $(0, \infty)$ by $\mathbb{R} \setminus \{0\}$ .
p344, (16.20)	Replace $i(c^+ - c^-)$ by $-i \operatorname{sign}(t)(c^+ - c^-)$ .
p353, line 11	Replace $\kappa_{t_{n+1}-t_n}$ by $\kappa_{t_{i+1}-t_i}$ .
p356, line 12	Replace $t \in \mathbb{N}_0$ by $t \in I$ (twice).
p408, line 13f	Replace $p$ by $r$ (three times).
p408, line 15	Replace $\varrho^k$ by $\rho^k$ .
p437, line 14	Replace $\varrho_i$ by $\varrho_k$ .
p438, lines 5,6,7	Replace $\infty$ by 0 (three times).
p448, line 14	Replace $\xrightarrow{n \rightarrow \infty}$ by $\xrightarrow{m \rightarrow \infty}$ .
p450, line 7	Of course, the convergence $A_n^\varepsilon \uparrow A_n^0$ holds only on the event $\{S_n \rightarrow \infty\}$ , which has probability 1.
p450, lines 9, 10	Replace $A_n^\varepsilon$ by $A_i^\varepsilon$ (twice).
p450, line 10	Replace $S_n \geq \frac{pn\varepsilon}{2}$ by $S_n \geq S^- + \frac{pn\varepsilon}{2}$ .
p450, line 11	Replace $\frac{pn\varepsilon}{2}$ by $\frac{p\varepsilon}{2}$ .

<p>p450, line 12ff.</p>	<p>The argument could be given in some more detail. For example: There is an <math>\varepsilon &gt; 0</math> such that <math>\mathbf{P}[X_1 &lt; -2\varepsilon] &gt; \varepsilon</math>. Let <math>L := \liminf_{n \rightarrow \infty} S_n</math>. As shown above, we have <math>\mathbf{P}[L = \infty] = 0</math>. The event <math>\{L &gt; -\infty\}</math> is invariant and hence has probability 0 or 1. We assume <math>\mathbf{P}[L &gt; -\infty] = 1</math> and construct a contradiction. Inductively, define stopping times <math>\tau_1 := \inf\{k \in \mathbb{N} : S_k &lt; L + \varepsilon\}</math> and</p> $\tau_{n+1} := \inf\{k > \tau_n : S_k < L + \varepsilon\} \quad \text{for } n \in \mathbb{N}.$ <p>By assumption, we have <math>\tau_n &lt; \infty</math> almost surely for all <math>n</math>. Let <math>\mathbb{F} = (\mathcal{F}_n)_{n \in \mathbb{N}_0} = \sigma((X_n)_{n \in \mathbb{N}})</math> be the filtration generated by <math>X = (X_n)_{n \in \mathbb{N}}</math>. Let <math>\mathcal{F}_{\tau_n}</math> be the <math>\sigma</math>-algebra of <math>\tau_n</math>-past. Define the events <math>A_n := \{X_{\tau_{n+1}} &lt; -2\varepsilon\}</math>, <math>n \in \mathbb{N}</math>. On <math>A_n</math>, we have <math>S_{\tau_{n+1}} &lt; L - \varepsilon</math>. Clearly, <math>A_n</math> is independent of <math>\mathcal{F}_{\tau_n}</math> and thus</p> $\mathbf{P}[A_n   \mathcal{F}_{\tau_n}] = \mathbf{P}[A_n] > \varepsilon.$ <p>By the conditional version of the Borel-Cantelli Lemma (see Exercise 11.2.7), we infer</p> $\mathbf{P}\left[\limsup_{n \rightarrow \infty} A_n\right] = 1.$ <p>This shows that almost surely <math>S_{\tau_{n+1}} &lt; L - \varepsilon</math> infinitely often and this in turn contradicts the assumption that <math>L</math> be finite. Consequently, we have <math>\mathbf{P}[\liminf S_n = -\infty] = 1</math>. The statement for <math>\limsup S_n</math> is similar.</p>
<p>p460, line -6</p>	<p>Replace <i>Chebyshev's</i> by <i>Markov's</i>.</p>
<p>p489, line -2.</p>	<p>Replace “<math>a = \lceil (t + s)n \rceil - (t + s)n</math> and <math>a = sn - \lfloor sn \rfloor</math>” by “<math>a = (t + s)n - \lfloor (t + s)n \rfloor</math> and <math>a = \lceil sn \rceil - sn</math>”.</p>
<p>p502, line -2,-1</p>	<p>Replace <math>\tau_n \leq s</math> by <math>\tau_n &gt; s</math> (three times).</p>
<p>p539, line 5</p>	<p><math>I^\beta(x) = \beta \cdot (F^\beta(x) - \inf_{y \in \mathcal{M}_1(\Sigma)} F^\beta(y))</math>.</p>

p551, line 20	$X_i := (\int \delta_{\phi_i(x)} X(dx)) \Big _{(0,\infty)} = (X \circ \phi_i^{-1}) \Big _{(0,\infty)}$
p551, line 23	Replace $G_1 \geq G_2$ by $G_1 \leq G_2$ .
p558, line -3	Replace $X^{n,1} = (X_{I_1^n}, X_2, \dots$ by $\hat{X}^{n,1} = (X_{I_1^n}, X_1^n, X_2^n, \dots$
p559, line 2	Replace $X^{n,1}$ by $\hat{X}^{n,1}$ .
p576, line 18	Replace $\mathcal{P}_T$ by $\mathcal{P}_T^n$ .
p577, line -5	Replace $F(X_t) - F(X_0)$ by $F(X_T) - F(X_0)$ .
p577, line -3	Replace $\langle X \rangle_t$ by $\langle X \rangle_T$ .
p581, line -1	Replace $\sigma_s^{i,l}$ by $\sigma_s^{l,i}$ .
p582, (25.17)	Replace $\int_0^t$ by $\int_0^T$ (three times).
p582, line 17	Replace $F$ by $(F(W_t))_{t \geq 0}$ .
p587, line 7	Replace $d = 2$ by $d \leq 2$ .
p587, line 11	On the r.h.s. replace $\ W_t\  < r$ by $\ W_t\  \leq s$ .
p596, line 18	In the right inequality on the right hand side the factor $K$ is missing.
p599, line 17	Replace $\mathbf{1}_{(0,\infty)}$ by $\mathbf{1}_{[0,\infty)}$ .
p599, line -6	Replace $\int_0^1$ by $\int_0^t$ .