

A. Klenke, Probability Theory, 2nd edition, Errata, 14.01.2023

p 17, line 5	Replace $a < b$ by $a \leq b$.
p 24, line 3	Replace $a < b$ by $a \leq b$.
p 25, line -6	Replace $[x, 0)$ by $(x, 0)$. Add “for $x < 0$ ”.
p 26, lines 7, 9	Replace F by F_μ (twice).
p 74, line 27	Replace $=$ by \geq
p 89, lines 12, 13	<p>Replace these two lines by:</p> <p>Clearly, $f^+ \leq g^+$ a.e., hence $(f^+ - g^+)^+ = 0$ a.e. By Theorem 4.8, we get $\int (f^+ - g^+)^+ d\mu = 0$. Since $f^+ \leq g^+ + (f^+ - g^+)^+$ (not only a.e.), we infer from Lemma 4.6(i) and (iii)</p> $\int f^+ d\mu \leq \int (g^+ + (f^+ - g^+)^+) d\mu = \int g^+ d\mu.$ <p>Similarly, we use $f^- \geq g^-$ a.e. to obtain</p> $\int f^- d\mu \geq \int g^- d\mu.$
p134, line 10	Replace $\ f_n - f\ _p$ by $\ f_n - f\ _p^p$.
p181, line 2	append: “and which is such that $\kappa(\omega_1, E) < \infty$ for all $\omega_1 \in \Omega_1$ and $E \in \mathcal{E}$.”
p196, line 26	Replace $\mathbf{E}[X_s]$ by X_s .
p234, line 5	Replace “ $\mathcal{E}_n =$ ” by “ $\mathcal{E}_n \supset$ ”.
p235, (12.4)	Replace $(N\Xi_N(A_l))^{m_l}$ by $(N\Xi_N(A_l))_{m_l}$.
p236, line 14ff	Replace Y_{-n} by Y_n .
p261, line 17ff	Replace the definitions of V_n and W_{n+1} by $V_n := \bigcup_{i=1}^n U_i$ and $W_{n+1} := V_{N(\bar{W}_n)}$, respectively.
p263, line -4	We also have to show that $F(-\infty) = 0$ in order that F be a distribution function. This however follows from tightness just as in the lines -3ff.
p276, line 9	Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$.
p276, line 16	Replace $E_j \in \mathcal{E}_j$ by $E_j \in \mathcal{E}_j \cup \{\Omega_j\}$.
p289, line -6	Replace $\omega \in \Omega$ by $\omega \in E$.

p297, line 1	Replace $H(x)$ by $H_z(x)$.
p297, line 6	Replace $h(y)$ by $h_z(y)$.
p301, line 8	$\ f\ _2 = \ \varphi\ _2 / (2\pi)^{d/2}$.
p301, line -3	Factor $1/\sqrt{2\pi}$ in front of the integral is missing.
p304, line 18	Replace (t/a) by (t/θ) .
p306, line -7	Replace $\varphi(t)$ by $\varphi_X(t)$.
p316, lines 3, 4	Replace h^n by $ h ^n$ (two instances).
p316, lines 13, 14	Replace $\sqrt{2\pi n}$ by $1/\sqrt{2\pi n}$ (two instances).
p318, line 1	$\mathbf{E}[X^{2k}] = (-1)^k u^{(2k)}(0)$.
p323, line 3	Replace $L_n(\varepsilon)$ by $\varepsilon^{-2}L_n(\varepsilon)$.
p323, line -4	Replace εt by $\varepsilon t $.
p335, lines 25, 27	Replace μ_n by ν_n .
p335, line 27	Replace ν by μ .
p341, line 11	The map f_t is not continuous. At this point, we have to work with $g_{t,\varepsilon}(x) = e^{-itx} - 1 - itx\mathbf{1}_{\{ x <1-\varepsilon\}}$ instead of $g_t(x) = e^{itx} - 1 - itx\mathbf{1}_{ x <1}$. We choose $\varepsilon > 0$ such that ν has no atoms at the points of discontinuity $-1 + \varepsilon$ and $1 - \varepsilon$. By the Portemanteau Theorem (Theorem 13.16(iii)), we get convergence of the integrals. Finally, we let $\varepsilon \rightarrow 0$.
p342, (16.16)	Replace $(0, \infty)$ by $\mathbb{R} \setminus \{0\}$.
p344, (16.20)	Replace $i(c^+ - c^-)$ by $-i \operatorname{sign}(t)(c^+ - c^-)$.
p353, line 11	Replace $\kappa_{t_{n+1}-t_n}$ by $\kappa_{t_{i+1}-t_i}$.
p356, line 12	Replace $t \in \mathbb{N}_0$ by $t \in I$ (twice).
p408, line 13f	Replace p by r (three times).
p408, line 15	Replace ϱ^k by ρ^k .
p437, line 14	Replace ϱ_i by ϱ_k .
p438, lines 5,6,7	Replace ∞ by 0 (three times).
p448, line 14	Replace $\xrightarrow{n \rightarrow \infty}$ by $\xrightarrow{m \rightarrow \infty}$.
p450, line 7	Of course, the convergence $A_n^\varepsilon \uparrow A_n^0$ holds only on the event $\{S_n \rightarrow \infty\}$, which has probability 1.
p450, lines 9, 10	Replace A_n^ε by A_i^ε (twice).
p450, line 10	Replace $S_n \geq \frac{pn\varepsilon}{2}$ by $S_n \geq S^- + \frac{pn\varepsilon}{2}$.
p450, line 11	Replace $\frac{pn\varepsilon}{2}$ by $\frac{p\varepsilon}{2}$.

<p>p450, line 12ff.</p>	<p>The argument could be given in some more detail. For example: There is an $\varepsilon > 0$ such that $\mathbf{P}[X_1 < -2\varepsilon] > \varepsilon$. Let $L := \liminf_{n \rightarrow \infty} S_n$. As shown above, we have $\mathbf{P}[L = \infty] = 0$. The event $\{L > -\infty\}$ is invariant and hence has probability 0 or 1. We assume $\mathbf{P}[L > -\infty] = 1$ and construct a contradiction. Inductively, define stopping times $\tau_1 := \inf\{k \in \mathbb{N} : S_k < L + \varepsilon\}$ and</p> $\tau_{n+1} := \inf\{k > \tau_n : S_k < L + \varepsilon\} \quad \text{for } n \in \mathbb{N}.$ <p>By assumption, we have $\tau_n < \infty$ almost surely for all n. Let $\mathbb{F} = (\mathcal{F}_n)_{n \in \mathbb{N}_0} = \sigma((X_n)_{n \in \mathbb{N}})$ be the filtration generated by $X = (X_n)_{n \in \mathbb{N}}$. Let \mathcal{F}_{τ_n} be the σ-algebra of τ_n-past. Define the events $A_n := \{X_{\tau_{n+1}} < -2\varepsilon\}$, $n \in \mathbb{N}$. On A_n, we have $S_{\tau_{n+1}} < L - \varepsilon$. Clearly, A_n is independent of \mathcal{F}_{τ_n} and thus</p> $\mathbf{P}[A_n \mathcal{F}_{\tau_n}] = \mathbf{P}[A_n] > \varepsilon.$ <p>By the conditional version of the Borel-Cantelli Lemma (see Exercise 11.2.7), we infer</p> $\mathbf{P}\left[\limsup_{n \rightarrow \infty} A_n\right] = 1.$ <p>This shows that almost surely $S_{\tau_{n+1}} < L - \varepsilon$ infinitely often and this in turn contradicts the assumption that L be finite. Consequently, we have $\mathbf{P}[\liminf S_n = -\infty] = 1$. The statement for $\limsup S_n$ is similar.</p>
<p>p460, line -6</p>	<p>Replace <i>Chebyshev's</i> by <i>Markov's</i>.</p>
<p>p489, line -2.</p>	<p>Replace “$a = \lceil (t + s)n \rceil - (t + s)n$ and $a = sn - \lfloor sn \rfloor$” by “$a = (t + s)n - \lfloor (t + s)n \rfloor$ and $a = \lceil sn \rceil - sn$”.</p>
<p>p502, line -2,-1</p>	<p>Replace $\tau_n \leq s$ by $\tau_n > s$ (three times).</p>
<p>p539, line 5</p>	<p>$I^\beta(x) = \beta \cdot (F^\beta(x) - \inf_{y \in \mathcal{M}_1(\Sigma)} F^\beta(y))$.</p>

p551, line 20	$X_i := (\int \delta_{\phi_i(x)} X(dx)) \Big _{(0,\infty)} = (X \circ \phi_i^{-1}) \Big _{(0,\infty)}$
p551, line 23	Replace $G_1 \geq G_2$ by $G_1 \leq G_2$.
p558, line -3	Replace $X^{n,1} = (X_{I_1^n}^n, X_2, \dots$ by $\hat{X}^{n,1} = (X_{I_1^n}^n, X_1^n, X_2^n, \dots$
p559, line 2	Replace $X^{n,1}$ by $\hat{X}^{n,1}$.
p541, line -6	Replace PD by GEM.
p541, line -5	Replace Theorem 25 by Theorem 3.2.
p576, line 18	Replace \mathcal{P}_T by \mathcal{P}_T^n .
p577, line -5	Replace $F(X_t) - F(X_0)$ by $F(X_T) - F(X_0)$.
p577, line -3	Replace $\langle X \rangle_t$ by $\langle X \rangle_T$.
p581, line -1	Replace $\sigma_s^{i,l}$ by $\sigma_s^{l,i}$.
p582, (25.17)	Replace \int_0^t by \int_0^T (three times).
p582, line 17	Replace F by $(F(W_t))_{t \geq 0}$.
p587, line 7	Replace $d = 2$ by $d \leq 2$.
p587, line 11	On the r.h.s. replace $\ W_t\ < r$ by $\ W_t\ \leq s$.
p596, line 18	In the right inequality on the right hand side the factor K is missing.
p599, line 17	Replace $\mathbf{1}_{(0,\infty)}$ by $\mathbf{1}_{[0,\infty)}$.
p599, line -6	Replace \int_0^1 by \int_0^t .